

8 Algebra: Brackets

8.1 Expansion of Single Brackets

In this section we consider how to expand (multiply out) brackets to give two or more terms, as shown below:

$$3(x + 6) = 3x + 18$$

First we revise *negative numbers* and *order of operations*.



Example 1

Evaluate:

(a) $-6 + 10$

(b) $-7 + (-4)$

(c) $(-6) \times (-5)$

(d) $6 \times (4 - 7)$

(e) $4(8 + 3)$

(f) $6(8 - 15)$

(g) $3 - (-5)$

(h) $\frac{(-2) - (-3)}{-1}$



Solution

(a) $-6 + 10 = 4$

(b) $-7 + (-4) = -7 - 4$
 $= -11$

(c) $(-6) \times (-5) = 30$

(d) $6 \times (4 - 7) = 6 \times (-3)$
 $= -18$

(e) $4(8 + 3) = 4 \times 11$
 $= 44$

(f) $6(8 - 15) = 6 \times (-7)$
 $= -42$

$$\begin{aligned} \text{(g)} \quad 3 - (-5) &= 3 + 5 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad \frac{(-2) - (-3)}{-1} &= \frac{(-2) + 3}{-1} \\ &= \frac{1}{-1} \\ &= -1 \end{aligned}$$

When a bracket is expanded, *every term* inside the bracket must be multiplied by the number outside the bracket. Remember to think about whether each number is positive or negative!



Example 2

Expand $3(x + 6)$ using a table.



Solution

×	x	6
3	$3x$	18

From the table,

$$3(x + 6) = 3x + 18$$



Example 3

Expand $4(x - 7)$.



Solution

$$\begin{aligned} 4(x - 7) &= 4 \times x - 4 \times 7 \\ &= 4x - 28 \end{aligned}$$

Remember that every term inside the bracket must be multiplied by the number outside the bracket.



Example 4

Expand $x(8 - x)$.

**Solution**

$$\begin{aligned}x(8-x) &= x \times 8 - x \times x \\ &= 8x - x^2\end{aligned}$$

**Example 5**Expand $(-3)(4 - 2x)$.**Solution**

$$\begin{aligned}(-3)(4 - 2x) &= (-3) \times 4 - (-3) \times 2x \\ &= -12 - (-6x) \\ &= -12 + 6x\end{aligned}$$

**Exercises**

1. Calculate:

- | | | |
|---------------------|--------------------|------------------------|
| (a) $-6 + 17$ | (b) $6 - 14$ | (c) $-6 - 5$ |
| (d) $6 - (-9)$ | (e) $-11 - (-4)$ | (f) $(-6) \times (-4)$ |
| (g) $8 \times (-7)$ | (h) $88 \div (-4)$ | (i) $6(8 - 10)$ |
| (j) $5(3 - 10)$ | (k) $7(11 - 4)$ | (l) $(-4)(6 - 17)$ |

2. Copy and complete the following tables, and write down each of the expansions:

(a)

\times	x	2
4		

$$4(x + 2) =$$

(b)

\times	x	-7
5		

$$5(x - 7) =$$

(c)

\times	x	3
4		

$$4(x + 3) =$$

(d)

\times	$2x$	5
5		

$$5(2x + 5) =$$

3. Expand:

(a) $4(x + 6)$

(b) $3(x - 4)$

(c) $5(2x + 6)$

(d) $7(3x - 4)$

(e) $3(2x + 4)$

(f) $8(3x - 9)$

(g) $(-2)(x - 4)$

(h) $(-3)(8 - 2x)$

(i) $5(3x - 4)$

(j) $9(2x + 8)$

4. Jordan writes $3(4x - 8) = 12x - 8$.

Explain why his expansion is *not* correct.

5. Copy and complete the following tables and write down each of the expansions:

(a)

\times	x	-2
x		

$$x(x - 2) =$$

(b)

\times	x	$-y$
x		

$$x(x - y) =$$

6. Copy the following expansions, filling in the missing terms:

(a) $4(x + 8) = 4x^2 + ?$

(b) $(-3)(2x - 7) = ? + 21$

(c) $4x(x - 9) = 4x^2 - ?$

(d) $6x(x - 7) = 6x^2 - ?$

(e) $3x(x - y) = 3x^2 - ?$

(f) $(-4x)(2x + 8) = ? - 32x$

7. Expand:

(a) $x(x - 7)$

(b) $x(8 - 2x)$

(c) $6x(x + 2)$

(d) $4x(3x - 5)$

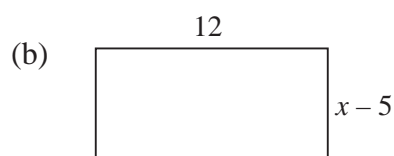
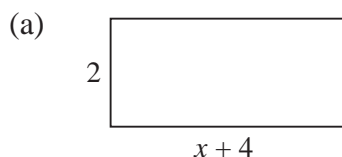
(e) $x(x + y)$

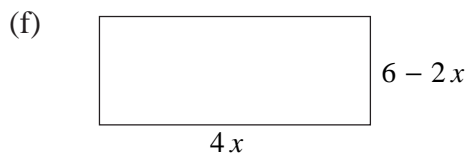
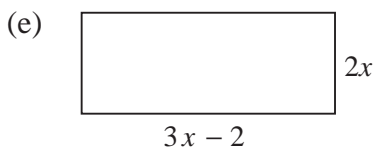
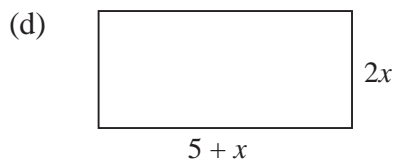
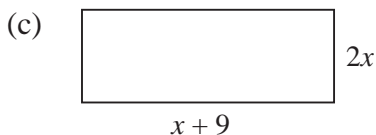
(f) $x(4y - 3x)$

(g) $2x(2x + 3y)$

(h) $5x(2y - 1)$

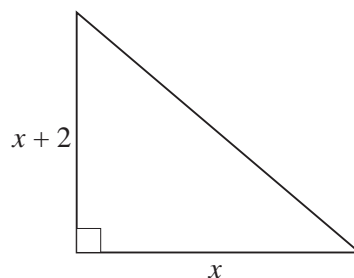
8. Write down expressions for the area of each of these rectangles, and then expand the brackets:





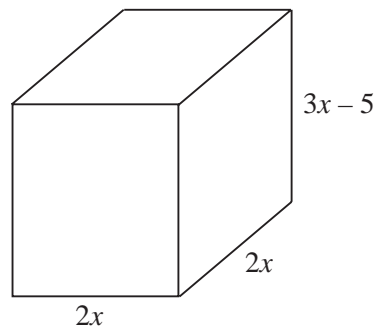
9. Write down an expression for the area of this triangle, that:

- (a) contains brackets,
(b) does *not* contain brackets.



10 Write down an expression for the volume of this cuboid, that:

- (a) contains brackets,
(b) does *not* contain brackets.



8.2 Linear Equations

Expanding a bracket will usually be the first step when solving an equation like

$$4(x + 3) = 20$$



Example 1

Solve

$$5(x - 3) = 35$$



Solution

$$5(x - 3) = 35$$

Expanding brackets gives: $5x - 15 = 35$

Adding 15 to both sides gives: $5x = 50$

Dividing by 5 gives: $x = 10$



Example 2

Solve

$$6(x + 7) = 50$$



Solution

$$6(x + 7) = 50$$

Expanding brackets gives: $6x + 42 = 50$

Subtracting 42 from both sides gives: $6x = 8$

Dividing by 6 gives:

$$x = \frac{8}{6}$$

$$= 1\frac{1}{3}$$



Example 3

Gilda thinks of a number and adds 7 to it. She then multiplies her answer by 4 and gets 64.

- Write down an equation that can be used to calculate the number with which Gilda started.
- Solve your equation to give the number.



Solution

- Start with x .

Add 7 to give $x + 7$

Multiply by 4 to give $4(x + 7)$

This expression equals 64, so the equation is $4(x + 7) = 64$

- $$4(x + 7) = 64$$

Expanding brackets gives; $4x + 28 = 64$

Subtracting 28 from both sides gives: $4x = 36$

Dividing by 4 gives:

$$x = \frac{36}{4}$$

$$= 9$$



Exercises

1. Solve these equations:

(a) $2(x + 6) = 14$

(b) $5(x - 8) = 40$

(c) $3(x + 5) = 12$

(d) $7(x + 4) = 42$

(e) $2(x + 7) = 19$

(f) $3(x - 4) = 11$

(g) $5(x - 4) = 12$

(h) $10(x + 7) = 82$

2. Solve these equations:

(a) $5(2x - 7) = 8$

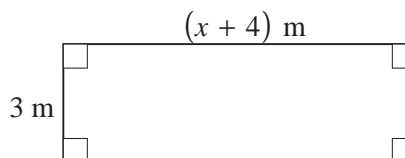
(b) $3(3x + 6) = 27$

(c) $3(2x + 1) = 30$

(d) $8(2x - 12) = 24$

3. A rectangle has sides of length 3 m and $(x + 4)$ m.

Find the value of x , if the area of the rectangle is 18 m^2 .

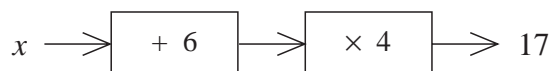


4. Feti chooses a number, adds 7, multiplies the result by 5 and gets the answer 55.

(a) If x is the number Feti first chose, write down an equation that can be used to determine the number.

(b) Solve the equation to determine the value of x .

5. The following flow chart is used to form an equation:



(a) Write down the equation.

(b) Solve the equation to find the value of x .

6. Solve the following equations:

(a) $4(7 - x) = 20$

(b) $3(9 - x) = 15$

(c) $6(5 - 2x) = 18$

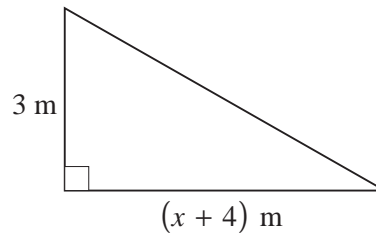
(d) $5(7 - 3x) = 20$

(e) $2(10 - 3x) = 17$

(f) $6(9 - 5x) = 4$

7. Alice thinks of a number, subtracts it from 11 and then multiplies her answer by 5 to get 45. What was the number that Alice started with?
8. Solve the following equations:
- (a) $2(x + 1) = 6(x - 3)$ (b) $3(x + 4) = 11x$
- (c) $5(x + 4) = 2(10x + 1)$ (d) $4(7 - x) = 5(x + 2)$

9.



- (a) Write down an expression for the area of the triangle.
- (b) What is x if the area is 15 m^2 ?

8.3 Common Factors

As well as being able to remove brackets by expanding expressions, it is also important to be able to write expressions so that they include brackets; this is called *factoring* or *factorisation*.



Example 1

Factorise

$$4x + 6$$



Solution

First write each term as a product of factors:

$$4x + 6 = 2 \times 2 \times x + 2 \times 3$$

$$4x + 6 = 2(2x + 3)$$

[Note that 2 is the only factor common to both terms and is placed outside the brackets.]

Now you can check your answer by expanding it.

**Example 2**

Factorise

$$18n + 24$$

**Solution**

$$\begin{aligned}
 18n + 24 &= \underbrace{2 \times 3}_{\text{circled}} \times \underbrace{3 \times n}_{\text{bracketed}} + \underbrace{2 \times 2}_{\text{bracketed}} \times \underbrace{2 \times 3}_{\text{circled}} \\
 &= 6(3n + 4)
 \end{aligned}$$

Note that both 2 and 3 are factors of both terms, and so $2 \times 3 = 6$ is placed outside the brackets.

**Example 3**

Factorise

$$4x^2 + 6x$$

**Solution**

$$\begin{aligned}
 4x^2 + 6x &= 2 \times 2 \times x \times x + 2 \times 3 \times x \\
 &= 2x(2x + 3)
 \end{aligned}$$

Note that both 2 and x are factors of both terms, and so $2 \times x = 2x$ is placed outside the brackets.

**Example 4**

Factorise

$$5x + 20x^2$$

**Solution**

$$\begin{aligned}
 5x + 20x^2 &= 5 \times x + 4 \times 5 \times x \times x \\
 &= 5x(1 + 4x)
 \end{aligned}$$

Note that because 5 and x are factors of both terms, a 1 must be introduced in the bracket when the $5x$ is placed outside the brackets.

You can check the calculation 'backwards':

$$\begin{aligned}
 5x(1 + 4x) &= 5x \times 1 + 5x \times 4x \\
 &= 5x + 20x^2
 \end{aligned}$$



Example 5

Factorise

$$3xy^2 + 12x^2y$$



Solution

$$\begin{aligned} 3xy^2 + 12x^2y &= 3 \times x \times y \times y + 3 \times 4 \times x \times x \times y \\ &= 3xy(y + 4x) \end{aligned}$$

Note that 3, x and y are factors of both terms, and so $3 \times x \times y = 3xy$ is placed outside the brackets.



Exercises

1. Factorise:

(a) $2x + 4$

(b) $5x + 15$

(c) $6x + 18$

(d) $5x - 25$

(e) $3x - 21$

(f) $7x + 35$

(g) $9x - 12$

(h) $15x + 20$

(i) $42x + 15$

2. Factorise:

(a) $3x^2 + x$

(b) $5x^2 + 10$

(c) $6x - 3x^2$

(d) $6x^2 - 4x$

(e) $21x^2 + 14x$

(f) $15x - 25x^2$

3. Denise states that

$$4x + 6x^2 = x(4 + 6x)$$

(a) Is her statement true?

(b) Describe how it could be improved.

4. For each statement below, decide if it has been fully factorised and if not, complete the factorisation:

(a) $x^2 + x = x(x + 1)$

(b) $3x^2 + 9x = 3(x^2 + 3x)$

(c) $5x - 30x^2 = x(5 - 30x)$

(d) $8x^2 - 32x = 4(2x^2 - 8x)$

(e) $6x^2 - 18x = 3x(2x - 6)$

(f) $15x - 6x^2 = 3(5x - 2x^2)$

5. Explain why the following factorisation is *incorrect*:

$$15x + 24x^2 = 3x(5 + 24x)$$

6. Factorise:

(a) $xy + xz$

(b) $xyz + 3yz$

(c) $4pq - 8qr$

(d) $5xyz + 20uxy$

(e) $5xy - 4py$

(f) $7xy + 12xz$

7. Factorise:

(a) $x^2y + xy^2$

(b) $3x^2y^2 + 6xy^2$

(c) $5x^2y - 35xy$

(d) $22xy + 4xy^2$

(e) $x^2yz + xy^2z$

(f) $x^2y - x^3z$

(g) $x^6y^2 + xy^3$

(h) $x^4y^3 + x^2y^6$

8. (a) Expand $x(x + y + z)$.

(b) Factorise $5x^2 + 2xy + 4xz$.

9. Factorise:

(a) $3x + 9y + 18z$

(b) $4x^2 + 2x + 8xy$

(c) $6x - 3xy + 12xz$

(d) $5xz + 20x - 35xy$

(e) $7x^2 + 14xy - 21xy^2$

(f) $4x + 6xz + 15xy$

10. Factorise:

(a) $4x^2y + 12x^3y^2 + x^2$

(b) $6x^7y^2 - 4x^5y - x^4y^2$

(c) $3x^2y^2 - 4xy^3 + x^4y$

(d) $5x^7y - x^2y^3 + 4x^3z$

8.4 Expansion of Two Brackets

When two brackets are multiplied together, for example,

$$(x + 2)(x + 3)$$

every term in the *first* bracket must be multiplied by every term in the *second* bracket.



Example 1

Use a table to determine

$$(x + 2)(x + 3)$$



Solution

		$(x + 3)$	
×		x	3
	x	x^2	$3x$
	2	$2x$	6

$(x + 2)$ →

The multiplication table is formed using the two brackets.

The contents of the table give the expansion.

$$\begin{aligned}
 (x + 2)(x + 3) &= x^2 + 3x + 2x + 6 & \text{or} & & x^2 + 3x \\
 &= x^2 + 5x + 6 & & & \underline{+ 2x + 6} \\
 & & & & = \underline{x^2 + 5x + 6}
 \end{aligned}$$



Example 2

Use a table to determine

$$(x - 6)(x + 2)$$



Solution

		$(x + 2)$	
×		x	2
	x	x^2	$2x$
	-6	$-6x$	-12

$(x - 6)$ →

So,

$$\begin{aligned}
 (x - 6)(x + 2) &= x^2 + 2x - 6x - 12 & \text{or} & & x^2 + 2x \\
 &= x^2 - 4x - 12 & & & \underline{- 6x - 12} \\
 & & & & = \underline{x^2 - 4x - 12}
 \end{aligned}$$

An alternative method for expanding two brackets is shown in the next example.



Example 3

Determine

$$(x + 2)(x - 7)$$



Solution

$$\begin{aligned} (x + 2)(x - 7) &= x(x - 7) + 2(x - 7) \\ &= x^2 - 7x + 2x - 14 && \text{or} && x^2 - 7x \\ &= x^2 - 5x - 14 && && + 2x - 14 \\ & && && \hline & && && = x^2 - 5x - 14 \end{aligned}$$

Note how each term in the first bracket multiplies the whole of the second bracket.



Exercises

1. Copy and complete the following tables and write down each of the expansions:

(a)

×	x	5
x		
4		

$(x + 4)(x + 5)$

(b)

×	x	-7
x		
4		

$(x + 4)(x - 7)$

(c)

×	x	4
x		
-1		

$(x - 1)(x + 4)$

(d)

×	x	-5
x		
-2		

$(x - 2)(x - 5)$

2. Expand:

(a) $(x + 3)(x + 4)$

(b) $(x - 2)(x + 5)$

(c) $(x - 5)(x - 1)$

(d) $(x + 7)(x - 3)$

(e) $(x + 2)(x - 3)$

(f) $(x + 4)(x - 1)$

3. Expand:

(a) $(x - 1)(x + 1)$

(b) $(x + 2)(x - 2)$

(c) $(x - 5)(x + 5)$

(d) $(x - 7)(x + 7)$

How are the answers to this question different from the others you have done?

4. Explain what is wrong with this statement:

$$(x + 5)^2 = x^2 + 25$$

5. Expand:

(a) $(x + 1)^2$

(b) $(x - 1)^2$

(c) $(x + 3)^2$

(d) $(x - 5)^2$

6. (a) Copy and complete this table:

\times	x	6
$2x$		
1		

(b) What is the expansion of

$$(2x + 1)(x + 6) ?$$

7. Expand:

(a) $(2x + 1)(2x + 4)$

(b) $(3x + 1)(4x + 1)$

(c) $(2x - 1)(3x + 2)$

(d) $(4x - 1)(5x + 1)$

(e) $(2x + 1)^2$

(f) $(4x - 3)^2$

8. Write out the following expansions, filling in the missing terms:

(a) $(x + 7)(x + 6) = x^2 + ? + 42$

(b) $(x + 6)^2 = x^2 + ? + 36$

(c) $(x - 2)(x - 5) = x^2 + ? + 10$

(d) $(x - 1)(2x + 1) = 2x^2 - x - ?$

(e) $(x + 3)(2x + 1) = ? + 7x + 3$

(f) $(x - 7)^2 = x^2 - ? + 49$

9. Explain what is wrong with this statement:

$$(x + 4)(x - 5) = x^2 - 20$$

10. Write out the following expansions, filling in the missing terms:

(a) $(x + ?)(x - 1) = x^2 + x - 2$

(b) $(x + 4)(x - ?) = x^2 - 2x - 24$

(c) $(2x + 3)(x + ?) = 2x^2 + 9x + ?$

(d) $(x - ?)(x + 5) = x^2 - 2x - ?$

(e) $(x + ?)(x + ?) = x^2 + 4x + 4$

(f) $(x + ?)(x + ?) = x^2 + 6x + 8$

11. The following example shows how to determine $(x + 1)^3$.

×	x	1
x	x^2	x
1	x	1

$$\begin{aligned}(x + 1)^2 &= x^2 + x + x + 1 \\ &= x^2 + 2x + 1\end{aligned}$$

×	x^2	$2x$	1
x	x^3	$2x^2$	x
1	x^2	$2x$	1

$$\begin{aligned}(x + 1)^3 &= (x + 1)(x^2 + 2x + 1) \\ &= x^3 + 2x^2 + x + x^2 + 2x + 1 \\ &= x^3 + 3x^2 + 3x + 1\end{aligned}$$

Use the same method to determine:

(a) $(x + 1)^4$,

(b) $(x + 1)^5$.

Compare your answers with Pascal's Triangle and describe any connections that you see.