# 4 Fractions and Percentages
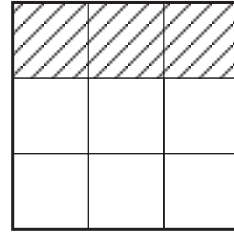
## 4.1 Equivalent Fractions

Equivalent fractions are revisited in this section.
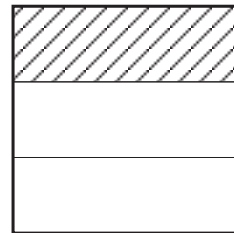
### Example 1

Write down in 2 different ways, the fraction of this large square which been shaded.

### Solution

$\dfrac{3}{9}$, as 3 of the 9 squares are shaded.

$\dfrac{1}{3}$, as the shape could have been drawn like this:

### Example 2

Complete each of the following expressions:

(a) $\dfrac{3}{4} = \dfrac{\square}{12}$

(b) $\dfrac{2}{3} = \dfrac{\square}{15}$

(c) $\dfrac{5}{6} = \dfrac{\square}{18}$

(d) $\dfrac{4}{12} = \dfrac{\square}{3}$

### Solution

(a) $\dfrac{3}{4} = \dfrac{3 \times 3}{4 \times 3} = \dfrac{9}{12}$

(b) $\dfrac{2}{3} = \dfrac{2 \times 5}{3 \times 5} = \dfrac{10}{15}$

(c) $\dfrac{5}{6} = \dfrac{5 \times 3}{6 \times 3} = \dfrac{15}{18}$

(d) $\dfrac{4}{12} = \dfrac{4 \div 4}{12 \div 4} = \dfrac{1}{3}$

## Example 3

Write each of the following fractions in their simplest form:

(a)  $\dfrac{8}{18}$

(b)  $\dfrac{5}{40}$

(c)  $\dfrac{12}{32}$

## Solution

(a)  $\dfrac{8}{18} = \dfrac{4}{9}$            *(dividing top and bottom by  2)*
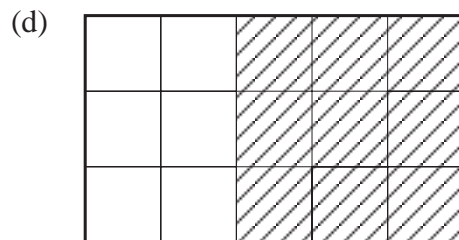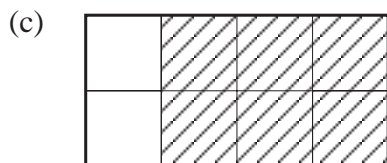
(b)  $\dfrac{5}{40} = \dfrac{1}{8}$            *(dividing top and bottom by  5)*

(c)  $\dfrac{12}{32} = \dfrac{3}{8}$            *(dividing top and bottom by  4)*

## Exercises

1.    Write, in two different ways the fraction of each shape which has been shaded:

(a)

(b)

(c)

(d)
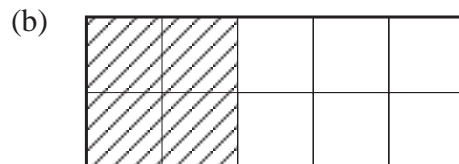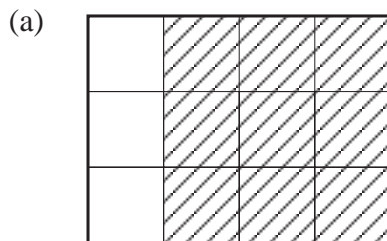
2.    Fill in the missing number in each of the following statements:

(a)  $\dfrac{3}{5} = \dfrac{\square}{20}$

(b)  $\dfrac{3}{4} = \dfrac{\square}{12}$

(c)  $\dfrac{4}{7} = \dfrac{\square}{35}$

(d)  $\dfrac{5}{9} = \dfrac{\square}{18}$

(e) $\dfrac{3}{7} = \dfrac{\square}{28}$

(f) $\dfrac{3}{8} = \dfrac{\square}{40}$

(g) $\dfrac{4}{5} = \dfrac{\square}{30}$

(h) $\dfrac{2}{9} = \dfrac{\square}{36}$

(i) $\dfrac{9}{10} = \dfrac{\square}{60}$

(j) $\dfrac{4}{7} = \dfrac{\square}{28}$

(k) $\dfrac{7}{11} = \dfrac{\square}{66}$

(l) $\dfrac{5}{8} = \dfrac{\square}{64}$

3. Fill in the missing numbers in the following statements:

(a) $\dfrac{10}{15} = \dfrac{\square}{3}$

(b) $\dfrac{11}{44} = \dfrac{\square}{4}$

(c) $\dfrac{20}{60} = \dfrac{\square}{3}$

(d) $\dfrac{10}{16} = \dfrac{\square}{8}$

(e) $\dfrac{30}{36} = \dfrac{\square}{6}$

(f) $\dfrac{10}{50} = \dfrac{\square}{5}$

(g) $\dfrac{4}{28} = \dfrac{\square}{7}$

(h) $\dfrac{18}{24} = \dfrac{\square}{4}$

(i) $\dfrac{14}{100} = \dfrac{\square}{50}$

(j) $\dfrac{24}{56} = \dfrac{\square}{7}$

4. Write each of the following fractions in its simplest form:

(a) $\dfrac{4}{8}$     (b) $\dfrac{6}{9}$     (c) $\dfrac{20}{25}$     (d) $\dfrac{3}{18}$

(e) $\dfrac{20}{100}$     (f) $\dfrac{20}{50}$     (g) $\dfrac{16}{40}$     (h) $\dfrac{32}{40}$

(i) $\dfrac{21}{28}$     (j) $\dfrac{16}{24}$     (k) $\dfrac{15}{21}$     (l) $\dfrac{28}{35}$

5. Write each of the following fractions in *two* different ways:

(a) $\dfrac{2}{7}$          (b) $\dfrac{3}{8}$          (c) $\dfrac{5}{9}$

6.   Is each of the following statements *true* or *false*:

(a)   $\dfrac{4}{7} = \dfrac{16}{21}$                    (b)   $\dfrac{3}{8} = \dfrac{12}{32}$

(c)   $\dfrac{4}{5} = \dfrac{16}{20}$                    (d)   $\dfrac{5}{9} = \dfrac{25}{45}$

7.   (a)   Fill in the missing number in each of the following statements:

$$\dfrac{4}{5} = \dfrac{\Box}{40} \qquad\qquad \dfrac{5}{8} = \dfrac{\Box}{40}$$

(b)   Which of the fractions $\dfrac{4}{5}$ and $\dfrac{5}{8}$ is the *larger*?

8.   (a)   Fill in the missing number in each of the following statements:

$$\dfrac{5}{7} = \dfrac{\Box}{21} \qquad\qquad \dfrac{2}{3} = \dfrac{\Box}{21}$$

(b)   Which of the fractions $\dfrac{5}{7}$ and $\dfrac{2}{3}$ is the *smaller*?

9.   Which of these fractions is the *largest*?

$$\dfrac{1}{2} \qquad \dfrac{3}{5} \qquad \dfrac{4}{7}$$

10.   Write the following fractions in order of size, with the *smallest* first:

$$\dfrac{1}{5} \qquad \dfrac{1}{4} \qquad \dfrac{2}{9} \qquad \dfrac{1}{2} \qquad \dfrac{5}{9}$$

# 4.2 Fractions of Quantities

In this section we review how to find fractions of quantities; for example, $\dfrac{3}{4}$ of 60.

## Example 1

Calculate:

(a)  $\dfrac{1}{3}$ of £60,     (b)  $\dfrac{1}{5}$ of £40.

**Solution**

(a)    $60 \div 3 = 20$

So $\dfrac{1}{3}$ of £60 $= £20$.

(b)    $40 \div 5 = 8$

So $\dfrac{1}{5}$ of £40 $= £8$.

## Example 2

Calculate:

(a)  $\dfrac{3}{4}$ of 700,     (b)  $\dfrac{5}{7}$ of 21.

**Solution**

(a)    $700 \div 4 = 175$

$175 \times 3 = 525$

So $\dfrac{3}{4}$ of 700 $= 525$.

(b)    $21 \div 7 = 3$

$5 \times 3 = 15$

So $\dfrac{5}{7}$ of 21 $= 15$.

## Exercises

1. Calculate:

   (a) $\dfrac{1}{5}$ of 10

   (b) $\dfrac{1}{3}$ of 12

   (c) $\dfrac{1}{4}$ of 20

   (d) $\dfrac{1}{7}$ of 28

   (e) $\dfrac{1}{6}$ of 24

   (f) $\dfrac{1}{5}$ of 30

   (g) $\dfrac{1}{9}$ of 18

   (h) $\dfrac{1}{3}$ of 24

   (i) $\dfrac{1}{8}$ of 32

2. Calculate:

   (a) $\dfrac{3}{4}$ of 20

   (b) $\dfrac{2}{5}$ of 15

   (c) $\dfrac{3}{8}$ of 24

   (d) $\dfrac{2}{3}$ of 24

   (e) $\dfrac{3}{7}$ of 28

   (f) $\dfrac{3}{5}$ of 40

   (g) $\dfrac{5}{8}$ of 32

   (h) $\dfrac{4}{5}$ of 30

   (i) $\dfrac{5}{9}$ of 36

3. In a class there are 28 pupils; $\dfrac{1}{2}$ of these pupils are girls.

   How many girls are in the class?

4. A can holds 330 ml of drink. Javinda drinks $\dfrac{1}{3}$ of the contents of the can.

   (a) How much has Javinda drunk?

   (b) How much drink is left in the can?

5. There are 320 sweets in a large tin. Laura eats $\dfrac{3}{8}$ of the sweets.

   (a) How many sweets does she eat?

   (b) How many sweets are left?

6. A car journey is 120 miles. Richard has driven $\dfrac{3}{5}$ of this distance.

   (a) How far has Richard driven?

   (b) How much further does he have to drive to complete the journey?

7.  There are 300 passengers on a train.  At a station, $\dfrac{3}{5}$ of the passengers get off.

    (a)    How many people get off the train?

    (b)    How many passengers are left on the train?

8.  Alison has £30.  She decides to save $\dfrac{2}{5}$ of this and to spend $\dfrac{1}{6}$ on books.

    (a)    How much money does she save?

    (b)    How much does she spend on books?

    (c)    How much does she have left?

9.  A farmer owns 360 hectares of land.  He plants potatoes on $\dfrac{3}{10}$ of his land.

    How many hectares are planted with potatoes?

10. An engineer tests a box of 120 floppy disks.  He finds that $\dfrac{1}{20}$ of the disks are damaged.  How many of the disks are damaged?

11. Sue and Ben each have 12 biscuits.

    (a)    Sue eats a quarter of her biscuits.  How many biscuits does Sue eat?

    (b)    Ben eats 6 of his biscuits.  What fraction of his biscuits does Ben eat?

    (c)    How many biscuits are left altogether?

    <div align="right">(KS3/97/Ma/Tier 3-5/P1)</div>

## 4.3  Operations with Fractions

Here we review how to *add, subtract, multiply* and *divide* fractions.

### Example 1

Calculate:

(a)    $\dfrac{3}{5} + \dfrac{1}{4}$                     (b)    $\dfrac{5}{7} - \dfrac{2}{3}$

### Solution

Before fractions can be added or subtracted, they must each have the same denominator (known as a *common denominator*).

(a)    $\dfrac{3}{5} + \dfrac{1}{4} = \dfrac{12}{20} + \dfrac{5}{20}$

                  $= \dfrac{17}{20}$

(b) $\quad \dfrac{5}{7} - \dfrac{2}{3} \;=\; \dfrac{15}{21} - \dfrac{14}{21}$

$\qquad\qquad\quad =\; \dfrac{1}{21}$

## Example 2

Calculate:

(a) $\quad \dfrac{4}{5} \times \dfrac{3}{7}$
 
(b) $\quad \dfrac{5}{8} \times \dfrac{2}{7}$

## Solution

(a) $\quad \dfrac{4}{5} \times \dfrac{3}{7} \;=\; \dfrac{4 \times 3}{5 \times 7}$

$\qquad\qquad\quad =\; \dfrac{12}{35}$

(b) $\quad \dfrac{5}{8} \times \dfrac{2}{7} \;=\; \dfrac{5 \times 2}{8 \times 7}$      *OR*      $\dfrac{5}{\overset{}{\underset{4}{\cancel{8}}}} \times \dfrac{\overset{1}{\cancel{2}}}{7} \;=\; \dfrac{5 \times 1}{4 \times 7}$

$\qquad\qquad\quad =\; \dfrac{10}{56}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad =\; \dfrac{5}{28}$

$\qquad\qquad\quad =\; \dfrac{5}{28}$

## Example 3

Calculate:

(a) $\quad \dfrac{3}{5} \div \dfrac{2}{3}$
 
(b) $\quad \dfrac{5}{7} \div \dfrac{3}{4}$

## Solution

(a) $\quad \dfrac{3}{5} \div \dfrac{2}{3} \;=\; \dfrac{3}{5} \times \dfrac{3}{2}$

$\qquad\qquad\quad =\; \dfrac{9}{10}$

(b) $\quad \dfrac{5}{7} \div \dfrac{3}{4} \;=\; \dfrac{5}{7} \times \dfrac{4}{3}$

$\qquad\qquad\quad =\; \dfrac{20}{21}$

## Example 4

Calculate:

(a)    $1\frac{1}{2} \times 1\frac{1}{4}$               (b)    $1\frac{1}{5} \div 2\frac{1}{4}$

**Solution**

(a)    $1\frac{1}{2} \times 1\frac{1}{4} = \frac{3}{2} \times \frac{5}{4}$

$$= \frac{15}{8}$$

$$= 1\frac{7}{8}$$

(b)    $1\frac{1}{5} \div 2\frac{1}{4} = \frac{6}{5} \div \frac{9}{4}$

$$= \frac{6}{5} \times \frac{4}{9}$$      (You could cancel at this stage to give

$$= \frac{24}{45}$$                  $\frac{2}{5} \times \frac{4}{3}$, etc.)

$$= \frac{8}{15}$$

## Exercises

1. Calculate:

(a)   $\frac{1}{7} + \frac{4}{7}$        (b)   $\frac{3}{8} + \frac{5}{8}$        (c)   $\frac{3}{10} + \frac{1}{10}$

(d)   $\frac{1}{5} + \frac{3}{5}$        (e)   $\frac{4}{9} + \frac{2}{9}$        (f)   $\frac{1}{6} + \frac{5}{6}$

2. Calculate:

(a)   $\frac{1}{2} + \frac{1}{3}$        (b)   $\frac{1}{5} + \frac{1}{4}$        (c)   $\frac{1}{7} + \frac{1}{3}$

(d)   $\frac{2}{5} + \frac{3}{4}$        (e)   $\frac{1}{7} + \frac{3}{8}$        (f)   $\frac{1}{6} + \frac{2}{3}$

(g) $\dfrac{3}{4} + \dfrac{2}{3}$　　　　(h) $\dfrac{3}{5} + \dfrac{2}{3}$　　　　(i) $\dfrac{4}{7} + \dfrac{2}{5}$

(j) $\dfrac{5}{6} + \dfrac{2}{3}$　　　　(k) $\dfrac{1}{8} + \dfrac{2}{3}$　　　　(l) $\dfrac{4}{5} + \dfrac{5}{6}$

3. Calculate:

(a) $\dfrac{1}{2} \times \dfrac{1}{3}$　　　　(b) $\dfrac{4}{5} \times \dfrac{2}{3}$　　　　(c) $\dfrac{1}{8} \times \dfrac{2}{3}$

(d) $\dfrac{5}{6} \times \dfrac{3}{4}$　　　　(e) $\dfrac{4}{5} \times \dfrac{5}{7}$　　　　(f) $\dfrac{3}{8} \times \dfrac{1}{4}$

(g) $\dfrac{4}{5} \times \dfrac{1}{2}$　　　　(h) $\dfrac{2}{3} \times \dfrac{3}{4}$　　　　(i) $\dfrac{5}{8} \times \dfrac{2}{3}$

(j) $\dfrac{3}{7} \times \dfrac{2}{3}$　　　　(k) $\dfrac{4}{8} \times \dfrac{3}{4}$　　　　(l) $\dfrac{7}{8} \times \dfrac{2}{3}$

4. Calculate:

(a) $\dfrac{1}{2} \div \dfrac{1}{3}$　　　　(b) $\dfrac{3}{4} \div \dfrac{2}{3}$　　　　(c) $\dfrac{4}{5} \div \dfrac{2}{3}$

(d) $\dfrac{2}{3} \div \dfrac{2}{5}$　　　　(e) $\dfrac{3}{7} \div \dfrac{3}{4}$　　　　(f) $\dfrac{5}{8} \div \dfrac{3}{4}$

(g) $\dfrac{4}{15} \div \dfrac{2}{3}$　　　　(h) $\dfrac{2}{3} \div \dfrac{5}{7}$　　　　(i) $\dfrac{3}{7} \div \dfrac{3}{5}$

(j) $\dfrac{4}{9} \div \dfrac{2}{3}$　　　　(k) $\dfrac{3}{8} \div \dfrac{6}{7}$　　　　(l) $\dfrac{7}{9} \div \dfrac{2}{3}$

5. Calculate:

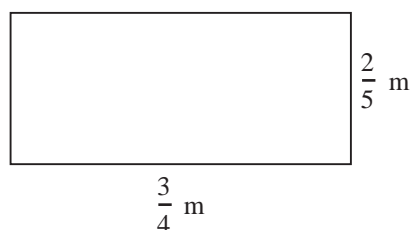(a) $1\dfrac{1}{2} \times 2\dfrac{1}{4}$　　　　(b) $2\dfrac{1}{2} \times 1\dfrac{1}{3}$　　　　(c) $2\dfrac{1}{3} \times 1\dfrac{3}{4}$

(d) $3\dfrac{1}{4} \times 1\dfrac{1}{3}$　　　　(e) $2\dfrac{1}{2} \times 1\dfrac{1}{2}$　　　　(f) $1\dfrac{1}{5} \times 1\dfrac{1}{2}$

6. Calculate the area and perimeter of the rectangle shown:

$\dfrac{2}{5}$ m

$\dfrac{3}{4}$ m

7

7. Julie has a vegetable plot that has an area of $\frac{2}{3}$ of an acre.

   She plants potatoes on $\frac{1}{4}$ of the plot.

   What fraction of an acre does she plant with potatoes?

8. Which is the *larger*

$$\frac{3}{4} \times \frac{1}{2} \qquad \text{or} \qquad \frac{3}{4} \div \frac{1}{2} \ ?$$

9. Solve these equations:

   (a) $\frac{2}{3}x = \frac{4}{9}$ (b) $\frac{3}{5}x = \frac{9}{4}$

10. If the area of the rectangle shown is $1\frac{1}{2}$ m$^2$, what is the length of the rectangle?

$\frac{2}{3}$ m

11. (a) In a magazine there are three adverts on the same page.

   | | |
   |---|---|
   | Advert 1 uses | $\frac{1}{4}$ of the page |
   | Advert 2 uses | $\frac{1}{8}$ of the page |
   | Advert 3 uses | $\frac{1}{16}$ of the page |

   In total, what fraction of the page do the three adverts use? Show your working.

   (b) The cost of an advert is £10 for each $\frac{1}{32}$ of a page.

   An advert uses $\frac{3}{16}$ of a page. How much does the advert cost?

(KS3/99/Ma/Tier 4-6/P1)

12. (a)    Alan had this special rectangle.

He cut off $\frac{1}{3}$ of the rectangle.

↓    subtract $\frac{1}{3}$

He had this square left.

↓    add on  ?

Alan put back the piece he had cut off.
He said:

"*I've added on $\frac{1}{3}$ of the square.*"

He was wrong.  Explain why.

What fraction of the square did he add on?

(b)    Look at shape 1 and shape 2.          shape 1
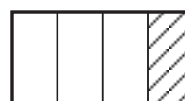
↓    subtract $\frac{1}{4}$
     of shape 1

shape 2

What fraction of shape 2 is added on
to get back to shape 1?

↓    add on  . . . .
     of shape 2

shape 1

(c)    Look at the numbers on the bottom of the fractions in (a) and (b).

Suppose you subtract $\frac{1}{8}$ of a shape.

You want to get back to the shape you started with.
What fraction of the new shape would you add on?

(d)    Suppose you subtract $\frac{1}{n}$ of a shape.

You want to get back to the shape you started with.

What fraction of the new shape would you add on?

(KS3/94/Ma/5-7/P1)

# 4.4 Fraction, Decimal and Percentage Equivalents

In this section we revisit the equivalence of fractions, decimals and percentages; for example, we could write $\dfrac{1}{2}$ as 0.5 or as 50%.

## Example 1

Write each of the following percentages as decimals and fractions in their simplest form:

(a)  75%                                     (b)  32%

## Solution

(a)  $75\% \;=\; \dfrac{75}{100}$

$\qquad\qquad =\; 0.75 \quad$ as a decimal

$\quad\;\; 75\% \;=\; \dfrac{75}{100}$

$\qquad\qquad =\; \dfrac{3}{4} \qquad$ as a fraction in its simplest form

(b)  $32\% \;=\; \dfrac{32}{100}$

$\qquad\qquad =\; 0.32 \quad$ as a decimal

$\quad\;\; 32\% \;=\; \dfrac{32}{100}$

$\qquad\qquad =\; \dfrac{8}{25} \qquad$ as a fraction in its simplest form

## Example 2

Write each of the following decimals as a percentage and as a fraction in its simplest form:

(a)  0.72                                     (b)  0.08

## Solution

(a)    $0.72 = \dfrac{72}{100}$

$= 72\%$   as a percentage

$0.72 = \dfrac{72}{100}$

$= \dfrac{18}{25}$   as a fraction in its simplest form

(b)    $0.08 = \dfrac{8}{100}$

$= 8\%$   as a percentage

$0.08 = \dfrac{8}{100}$

$= \dfrac{2}{25}$   as a fraction in its simplest form

## Example 3

Write each of the following fractions as a decimal and as a percentage:

(a)    $\dfrac{3}{10}$                    (b)    $\dfrac{4}{25}$                    (c)    $\dfrac{3}{8}$

## Solution

(a)    $\dfrac{3}{10} = \dfrac{30}{100}$        *(multiply top and bottom by 10)*

$= 0.3$   as a decimal

$= 30\%$   as a percentage

$\dfrac{4}{25} = \dfrac{16}{100}$        *(multiply top and bottom by 4)*

$= 0.16$   as a decimal

$= 16\%$   as a percentage

(b)    $\dfrac{3}{8} = \dfrac{37.5}{100}$        *(multiply top and bottom by 12.5)*

$= 0.375$   as a decimal

$= 37.5\%$  as a percentage

## Exercises

1. Write each of the following percentages as a decimal:

   (a)  60%              (b)  70%              (c)  20%

   (d)  45%              (e)  31%              (f)  82%

   (g)  14%              (h)  4%               (i)  63%

   (j)  2%               (k)  1%               (l)  19%

2. Write each of the following percentages as a fraction in its simplest form:

   (a)  80%              (b)  25%              (c)  40%

   (d)  35%              (e)  65%              (f)  4%

   (g)  64%              (h)  82%              (i)  28%

   (j)  6%               (k)  7%               (l)  92%

3. Write each of the following decimals as a percentage:

   (a)  0.74             (b)  0.99             (c)  0.5

   (d)  0.06             (e)  0.26             (f)  0.02

   (g)  0.3              (h)  0.002            (i)  0.042

4. Write each of the following decimals as a fraction in its simplest form:

   (a)  0.5              (b)  0.25             (c)  0.4

   (d)  0.7              (e)  0.62             (f)  0.44

   (g)  0.37             (h)  0.04             (i)  0.05

   (j)  0.24             (k)  0.1              (l)  0.74

5. Write each of the following fractions as a decimal:

   (a)  $\dfrac{1}{2}$          (b)  $\dfrac{3}{4}$          (c)  $\dfrac{4}{5}$

   (d)  $\dfrac{9}{20}$         (e)  $\dfrac{7}{10}$         (f)  $\dfrac{3}{100}$

   (g)  $\dfrac{19}{100}$       (h)  $\dfrac{23}{50}$        (i)  $\dfrac{7}{25}$

   (j)  $\dfrac{8}{25}$         (k)  $\dfrac{1}{8}$          (l)  $\dfrac{5}{8}$

6.    Write each of the following fractions as a percentage:

(a) $\dfrac{9}{10}$          (b) $\dfrac{17}{100}$          (c) $\dfrac{14}{25}$

(d) $\dfrac{3}{20}$          (e) $\dfrac{2}{5}$          (f) $\dfrac{3}{5}$

(g) $\dfrac{9}{20}$          (h) $\dfrac{9}{100}$          (i) $\dfrac{1}{100}$

(j) $\dfrac{3}{50}$          (k) $\dfrac{7}{8}$          (l) $\dfrac{7}{200}$

7.    Copy and complete this table of equivalent fractions, decimals and percentages:

| Fraction | Decimal | Percentage |
|---|---|---|
| $\dfrac{4}{5}$ | | |
| | 0.68 | |
| | | 85% |
| | 0.76 | |
| $\dfrac{8}{25}$ | | |
| | | 3% |
| | 0.005 | |

8.    In a survey, 400 people were asked how they would vote at the next election. The results are listed below:

|  |  |
|---|---|
| *Labour* | 220 |
| *Conservative* | 160 |
| *Other* | 20 |

Express these results as percentages.

9.    In a school there are 50 Manchester City supporters out of a total of 2000 pupils.

(a)    What percentage of the pupils support Manchester City?

(b)    What percentage of the pupils do *not* support Manchester City?

10. In a group of 40 pupils there are 7 who cannot swim.

   What percentage of the pupils *can* swim?

11. Simon is growing vegetables in three vegetable patches.

   (a)   About 50% of this vegetable patch is for *carrots*.
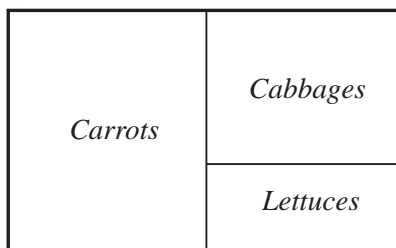


   Write down the missing *percentages*:

   (i)    about  . . . % of the patch is for *cabbages*,

   (ii)   about  . . . % of the patch is for *lettuces*.

   (b)   About $\frac{1}{8}$ of this vegetable patch is for *beetroot*.
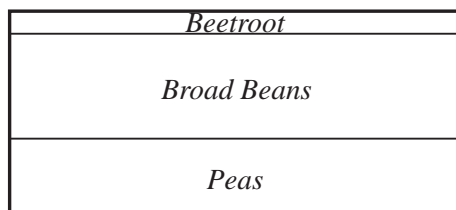


   Write down the missing *fractions*:

   (i)    about  . . .  of the patch is for *broad beans*.

   (ii)   about  . . .  of the patch is for *peas*.

   (c)   About $\frac{4}{5}$ of this vegetable patch is for *potatoes*.

   Copy the diagram below and draw a straight line to show how much
   of the patch is for *potatoes*.  Shade in the area for potatoes.
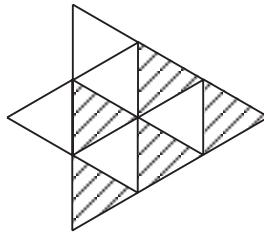


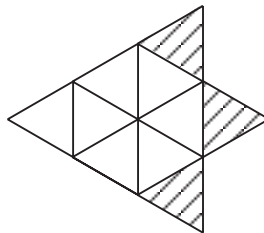   The rest of the patch is for *turnips*.

   About what fraction of the patch is for *turnips*?

   (KS3/96/Ma/Tier 4-6/P1)
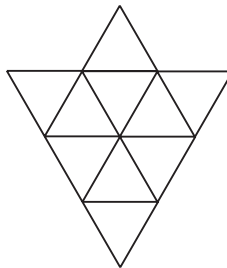
12.  $\frac{1}{2}$ of the diagram below is shaded.



(a)    Look at this diagram:



What *fraction* is shaded?

What *percentage* is shaded?

(b)    Copy the diagram below and shade $\frac{2}{5}$ of it.



What *percentage* of the diagram have you shaded?

(KS3/97/Ma/Tier 3-5/P1)

# 4.5  Percentage Increases and Decreases

Often prices are increased or decreased by a percentage.  In this section we consider how to increase or decrease quantities by using percentages.

## Example 1

Katie earns £40 per week for her part-time job.  She is to be given a 5% pay rise. How much will she earn per week after the pay rise?

## Solution

$$5\% \text{ of } £40 = \frac{5}{100} \times £40 \qquad OR \qquad 100\% + 5\% = 105\%$$

$$= £2 \qquad\qquad\qquad\qquad \text{which is } 1.05 \text{ as a decimal}$$

$$\text{New pay} = £40 + £2 \qquad\qquad \text{New pay} = £40 \times 1.05$$

$$= £42 \qquad\qquad\qquad\qquad\qquad = £42$$

## Example 2

The prices of all the televisions in a shop are to be increased by 8%. Calculate the new price of a television that originally cost £150.

## Solution

$$8\% \text{ of } £150 = \frac{8}{100} \times £150 \qquad OR \qquad 100\% + 8\% = 108\%$$

$$= £12 \qquad\qquad\qquad\qquad \text{which is } 1.08 \text{ as a decimal}$$

$$\text{New price} = £150 + £12 \qquad\qquad \text{New price} = £150 \times 1.08$$

$$= £162 \qquad\qquad\qquad\qquad\qquad = £162$$

## Example 3

In a sale the cost of a computer is reduced by 30%. The normal price of the computer was £900. Calculate the sale price of the computer.

## Solution

$$30\% \text{ of } £900 = \frac{30}{100} \times £900 \qquad OR \qquad 100\% - 30\% = 70\%$$

$$= £270 \qquad\qquad\qquad\qquad \text{which is } 0.7 \text{ as a decimal}$$

$$\text{Sale price} = £900 - £270 \qquad\qquad \text{New price} = £900 \times 0.7$$

$$= £630 \qquad\qquad\qquad\qquad\qquad = £630$$

## Exercises

1. (a)   Increase £100 by 20%.     (b)   Increase £400 by 30%.

   (c)   Increase £80 by 25%.     (d)   Increase £50 by 6%.

   (e)   Increase 40 kg by 3%.     (f)   Increase 250 m by 7%.

2. (a)   Decrease £60 by 30%.     (b)   Decrease 8 m by 5%.

   (c)   Decrease 80 kg by 10%.     (d)   Decrease £44 by 20%.

   (e)   Decrease 90 m by 2%.     (f)   Decrease 420 kg by 25%.

3. A company increases the cost of all its products by 5%. Calculate the new price of each of the items listed below:

   (a)   a tent that previously cost £60.

   (b)   a rucksack that previously cost £15,

   (c)   a sleeping bag that previously cost £24.

4. Joe was paid £30 per week for delivering papers. He was given a 3% pay rise. How much will he now earn each week?

5. A small firm employs 4 staff. They are all given a 4% pay rise. The original salaries are as follows:

   | | |
   |---|---|
   | John Smith | £24 000 |
   | Alice Holland | £22 500 |
   | Graham Hall | £14 000 |
   | Emma Graham | £8500 |

   Calculate the new salary for each member of staff.

6. Rachel puts £50 into a bank account. After one year 5% interest is added to her money. How much does she have then?

7. Add $17\frac{1}{2}$% VAT to each of the following prices:

   (a)   £200     (b)   £70     (c)   £42

8. A rope is 8 m long but it shrinks when it gets wet. What would be the new length of the rope if its length is reduced by:

   (a)   2%     (b)   7%     (c)   12% ?

**4.5**

9. In a sale the prices of each of the items listed below is to be reduced by 35%.

|  |  |  |  |
|---|---|---|---|
| *Coat* | £28 | *Jeans* | £42 |
| *Trainers* | £36 | *Shirt* | £14 |

Calculate the sale price of each item.

10. A mountain bike was priced at £180. Its price was increased by 8%. Later, this increased price was reduced by 20% in a sale.
Calculate the sale price of the bike.

11. This is how Caryl works out 15% of 120 in her head.

10% of 120 is 12,

5% of 120 is 6,

so 15% of 120 is 18.

(a) Copy and complete the following calculations to show how Caryl can work out $17\frac{1}{2}$% of 240 in her head.

....... % of 240 is .........

....... % of 240 is .........

....... % of 240 is .........

so $17\frac{1}{2}$% of 240 is .........

(b) Work out 35% of 250. Show your working.

(KS3/98/Ma/Tier 3-5/P1)

12. Look at this table:

*Birth rate per 1000 population*

|  | 1961 | 1994 |
|---|---|---|
| *England* | 17.6 |  |
| *Wales* | 17.0 | 12.2 |

(a) In England, from 1961 to 1994, the birth rate fell by 26.1%
What was the birth rate in England in 1994 ? Show your working.

(b) In Wales, the birth rate also fell.
Calculate the percentage fall from 1961 to 1994. Show your working.

(KS3/98/Ma/Tier 5-7/P2)

13.  The table shows the land area of each of the World's continents.

| Continent | Land Area (in 1000 $km^2$) |
|---|---|
| Africa | 30 264 |
| Antarctica | 13 209 |
| Asia | 44 250 |
| Europe | 9 907 |
| North America | 24 398 |
| Oceania | 8 534 |
| South America | 17 793 |
| World | 148 355 |

(a)  Which continent is approximately 12% of the World's land area?

(b)  What percentage of the World's land area is Antarctica?  Show your working.

(c)  About 30% of the World's area is land.  The rest is water.  The amount of land in the World is about 150 million $km^2$.

Work out the approximate total area (land and water) of the World. Show your working.

(KS3/98/Ma/Tier 6-8/P2)

14.  In 1995, the Alpha Company employed 4000 people. For each of the next 2 years, the number of people employed increased by 10%.

| | |
|---|---|
| 1995 | employed 4000 people |
| 1996 | employed 20% fewer people |
| 1997 | employed 10% more people |

(a)  Tony said:
    "*Each year, the Alpha Company employed another 400 people.*"

Tony was wrong.  Explain why.

(b)  Which of the calculations below shows how many people worked for the company in 1997:

   (i)   $4000 \times 0.1 \times 2$      (ii)   $4000 \times 0.1^2$      (iii)   $(4000 \times 0.1)^2$

   (iv)   $4000 \times 1.1 \times 2$      (v)   $4000 \times 1.1^2$      (vi)   $(4000 \times 1.1)^2$

(c)    Look at these figures for the Beta Company:

| 1995 | employed $n$ people |
|------|---------------------|
| 1996 | employed 20% fewer people |
| 1997 | employed 10% more people |

Write an expression using $n$ to show how many people the company employed in 1997. Show your working and write your expression as simply as possible.

(KS3/99/Ma/Tier 6-8/P1)

15.    A clothes shop had a closing down sale. The sale started on Tuesday and finished on Saturday. For each day of the sale, prices were reduced by 15% of the prices on the day before.

(a)    A shirt had a price of £19.95 on Monday. Kevin bought it on Wednesday. How much did he pay? Show your working.

(b)    Ghita bought a dress on Tuesday for £41.48. What was its price on Monday? Show your working.

(c)    A jacket had a price of £49.95 on Monday. What was its price on Friday? Show your working.

(d)    Another shop is reducing its prices each day by 12% of the prices on the day before. How many days would it take for its original prices to be reduced by more than 50%? Show your working.

(KS3/96/Ma/Tier 6-8/P2)