20 Arithmetic: Fractions

20.1 Revision: Whole Numbers and Decimals

In this section we revise *addition, subtraction, multiplication* and *division* of whole numbers and decimals, before starting to work with *fractions.*

Example 1

Calculate:

(a) $18 + 49$ (b) $1.6 + 0.84$ (c) $3.82 - 1.6$

Solution

(a)
$$\begin{array}{r} 18 \\ +\ 49 \\ \hline 67 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 1.60 \\ +\ 0.84 \\ \hline 2.44 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 3.82 \\ -\ 1.60 \\ \hline 2.22 \\ \hline \end{array}$$

Example 2

Calculate:

(a) 18×34 (b) 1.7×2.6

Solution

(a)
$$\begin{array}{r} 18 \\ \times\ 34 \\ \hline 72 \\ 540 \\ \hline 612 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 17 \\ \times\ 26 \\ \hline 102 \\ 340 \\ \hline 442 \\ \hline \end{array}$$

Hence $1.7 \times 1.6 = 4.42$

Example 3

Calculate:

(a) $165 \div 5$ (b) $4.26 \div 3$

Solution

(a)
$$5\overline{)\begin{array}{r} 33 \\ {\scriptstyle 1}\,165 \end{array}}$$

so $165 \div 5 = 33$

(b)
$$3\overline{)\begin{array}{r} 1.42 \\ {\scriptstyle 1}\,4.26 \end{array}}$$

so $4.26 \div 3 = 1.42$

Exercises

1. Calculate:

(a)	182 + 57	(b)	32 + 168	(c)	1807 + 94
(d)	3.2 + 4.7	(e)	18.2 + 1.9	(f)	3.71 + 4.2
(g)	0.26 + 1.2	(h)	11.4 + 6.21	(i)	0.09 + 0.123
(j)	38 + 4.7	(k)	0.71 + 2.8	(l)	4.52 + 9.89

2. Calculate:

(a)	192 – 71	(b)	486 – 234	(c)	620 – 108
(d)	0.9 – 0.2	(e)	1.8 – 0.3	(f)	2.42 – 1.23
(g)	0.8 – 0.11	(h)	8.9 – 1.12	(i)	3.7 – 2.15
(j)	28 – 3.7	(k)	52 – 6.9	(l)	4.07 – 3.88

3. Calculate:

(a)	18 × 3	(b)	42 × 5	(c)	63 × 7
(d)	12 × 15	(e)	26 × 14	(f)	39 × 23
(g)	0.7 × 5	(h)	1.9 × 6	(i)	4.29 × 3
(j)	1.8 × 2.9	(k)	3.5 × 2.6	(l)	1.42 × 1.6

4. Calculate:

(a)	468 ÷ 2	(b)	578 ÷ 2	(c)	145 ÷ 5
(d)	345 ÷ 5	(e)	78 ÷ 3	(f)	981 ÷ 3
(g)	6.84 ÷ 4	(h)	14.7 ÷ 7	(i)	7.92 ÷ 6

5. There were 52 people on a bus and 17 got off. How many people were still on the bus?

6. Floppy disks cost 34p each. How much would 6 floppy disks cost?

7. It costs £5.20 for one adult to go into a theme park. How much would it cost in total for 24 adults to go into the theme park?

8. Tickets for a show cost £3 each. To cover the cost of putting on the show, £378 is needed. How many tickets must be sold to cover the cost of the show?

9. An 8 m length of rope is cut into 5 pieces of equal length. How long is each of the 5 pieces?

10. A PE department has £30 to spend on footballs which cost £4 each.
    (a) How many footballs can they buy?
    (b) How much money will they have left?

20.2 Addition and Subtraction of Fractions

In this section we consider how to add and subtract fractions. The key step in this process is to make sure that both fractions have the *same denominator.*

Example 1

Calculate:

(a) $\frac{1}{5}+\frac{2}{5}$ (b) $\frac{5}{6}-\frac{1}{6}$

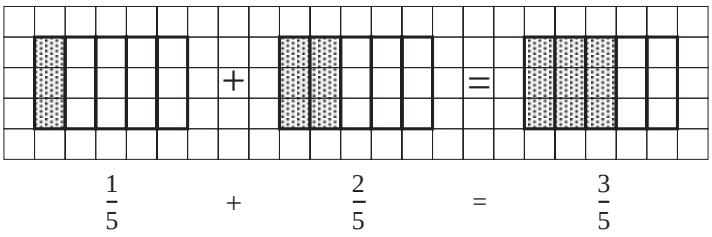
Solution

(a) As the denominator is the same in both fractions, we simply add the numbers on the top of the fraction to give

$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}$$

$$=\frac{3}{5}$$

This addition is shown in the diagram below:



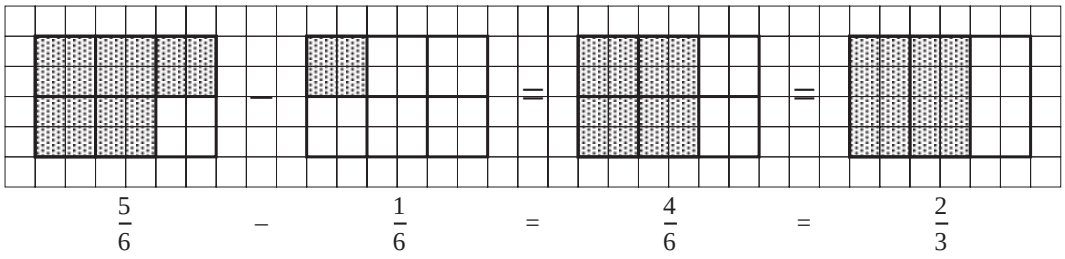
(b) The denominator is the same in both fractions, so

$$\frac{5}{6}-\frac{1}{6}=\frac{5-1}{6}$$

$$=\frac{4}{6}$$

$$=\frac{2}{3}$$

This is shown in the diagram below:



Example 2

Calculate:

(a) $\frac{1}{4}+\frac{2}{5}$ (b) $\frac{2}{3}-\frac{1}{4}$

Solution

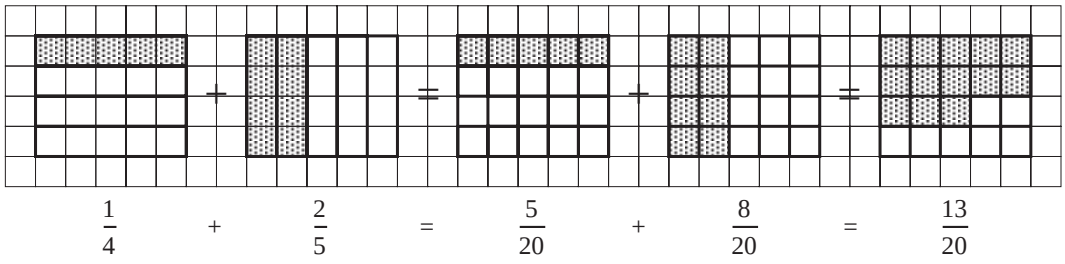
(a) These fractions do not have the same denominator, so the first step is to change them so that they do. In this case, we can use 20 as the common denominator.

$$\frac{1}{4}+\frac{2}{5} = \frac{5}{20}+\frac{8}{20}$$

$$= \frac{5+8}{20}$$

$$= \frac{13}{20}$$

This is illustrated in the diagram below:



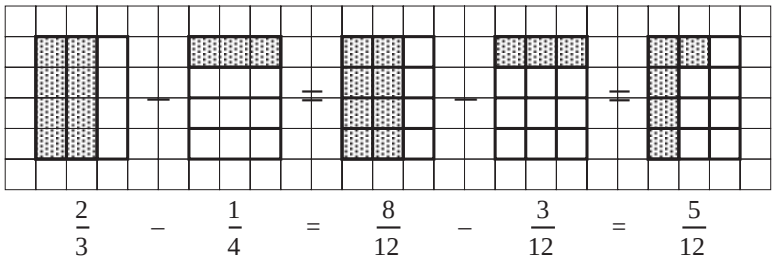
(b) In this case we can use a common denominator of 12.

$$\frac{2}{3}-\frac{1}{4} = \frac{8}{12}-\frac{3}{12}$$

$$= \frac{8-3}{12}$$

$$= \frac{5}{12}$$

This is illustrated in the diagram below:



Example 3

Calculate:

(a) $1\frac{1}{8}+3\frac{1}{3}$ (b) $4\frac{3}{8}-1\frac{3}{4}$ (c) $2\frac{2}{3}+1\frac{1}{2}$

Solution

(a) $1+3=4$

$$\frac{1}{8}+\frac{1}{3}=\frac{3}{24}+\frac{8}{24}$$

$$=\frac{3+8}{24}$$

$$=\frac{11}{24}$$

So $1\frac{1}{8}+3\frac{1}{3}=4\frac{11}{24}$

(b) $4\frac{3}{8}-1\frac{3}{4}=\frac{35}{8}-\frac{7}{4}$

$$=\frac{35}{8}-\frac{14}{8}$$

$$=\frac{35-14}{8}$$

$$=\frac{21}{8}$$

$$=2\frac{5}{8}$$

Note: It is usually easier to convert the mixed numbers into improper fractions.

(c) $2\frac{2}{3}+1\frac{1}{2} = \frac{8}{3}+\frac{3}{2}$

$= \frac{16}{6}+\frac{9}{6}$

$= \frac{16+9}{6}$

$= \frac{25}{6}$

$= 4\frac{1}{6}$

Exercises

1. Calculate:

(a) $\frac{3}{7}+\frac{1}{7}$ (b) $\frac{3}{8}+\frac{1}{8}$ (c) $\frac{1}{9}+\frac{7}{9}$

(d) $\frac{3}{10}+\frac{7}{10}$ (e) $\frac{1}{5}+\frac{3}{5}$ (f) $\frac{2}{7}+\frac{4}{7}$

(g) $\frac{1}{4}+\frac{3}{4}$ (h) $\frac{5}{8}-\frac{3}{8}$ (i) $\frac{7}{9}-\frac{5}{9}$

(j) $\frac{9}{10}-\frac{7}{10}$ (k) $\frac{8}{11}-\frac{3}{11}$ (l) $\frac{4}{15}-\frac{2}{15}$

(m) $\frac{6}{13}-\frac{3}{13}$ (n) $\frac{4}{7}-\frac{3}{7}$ (o) $\frac{6}{25}-\frac{2}{25}$

2. Fill in the missing numbers:

(a) $\frac{1}{2}+\frac{1}{5} = \frac{?}{10}+\frac{?}{10} = \frac{?}{10}$

(b) $\frac{4}{5}+\frac{2}{3} = \frac{?}{15}+\frac{?}{15} = \frac{?}{15}$

(c) $\frac{1}{6}+\frac{4}{5} = \frac{5}{?}+\frac{24}{?} = \frac{?}{?}$

(d) $\frac{4}{7}-\frac{1}{3} = \frac{?}{21}-\frac{?}{21} = \frac{?}{21}$

(e) $\frac{5}{6}-\frac{2}{3} = \frac{5}{6}-\frac{?}{6} = \frac{?}{6}$

3. Calculate:

(a) $\frac{1}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{2}{3}$ (c) $\frac{1}{5}+\frac{1}{4}$

(d) $\frac{3}{5}+\frac{2}{3}$ (e) $\frac{5}{8}+\frac{1}{4}$ (f) $\frac{1}{3}+\frac{1}{6}$

(g) $\frac{4}{5}+\frac{2}{7}$ (h) $\frac{1}{7}+\frac{2}{3}$ (i) $\frac{1}{2}+\frac{1}{10}$

(j) $\frac{6}{7}+\frac{2}{3}$ (k) $\frac{5}{6}-\frac{1}{2}$ (l) $\frac{7}{8}-\frac{3}{4}$

(m) $\frac{8}{9}-\frac{3}{4}$ (n) $\frac{3}{7}-\frac{1}{3}$ (o) $\frac{4}{5}-\frac{3}{4}$

4. A birthday cake is divided into 12 equal parts. Andrew eats $\frac{3}{12}$ of the cake and Timothy eats $\frac{1}{12}$ of the cake.

(a) What fraction of the cake is left?

(b) How many pieces of cake are left?

5. A garden has an area of $\frac{3}{4}$ hectare. The owner buys an extra $\frac{3}{5}$ of a hectare of land.

(a) What is the area of the garden now?

(b) How much more land would the owner need to have a garden with an area of 2 hectares?

6. Steve and Sheila buy a computer. Steve fills $\frac{2}{5}$ of the hard disk with his programs. Sheila fills $\frac{1}{3}$ of the hard disk with her programs.

(a) What fraction of the hard disk is full?

(b) What fraction of the hard disk is empty?

(c) Steve deletes one of his programs that takes up $\frac{1}{10}$ of the hard disk. What fraction of the hard disk do his programs fill now?

7. If $\frac{9}{10}$ of all men in the UK own cars, and $\frac{2}{3}$ of all men in the UK own *more than one* car, what fraction of men in the UK:

(a) do *not* own a car,

(b) own only *one* car?

8. Calculate:

(a) $1\frac{1}{2}+1\frac{1}{3}$ (b) $1\frac{3}{4}+2\frac{1}{2}$ (c) $4\frac{2}{5}+3\frac{1}{2}$

(d) $1\frac{4}{7}+1\frac{3}{8}$ (e) $1\frac{1}{2}-\frac{2}{3}$ (f) $3\frac{1}{4}-1\frac{3}{5}$

(g) $2\frac{1}{2}-1\frac{5}{8}$ (h) $4\frac{1}{7}+3\frac{2}{3}$ (i) $4\frac{3}{5}-2\frac{7}{8}$

(j) $6\frac{1}{4}-1\frac{2}{5}$ (k) $3\frac{1}{2}-1\frac{3}{4}$ (l) $5\frac{1}{4}-2\frac{1}{2}$

9. Ron wins $£1\frac{1}{4}$ million. He gives $£\frac{3}{5}$ million to his daughter and $£\frac{1}{3}$ million to his wife. How much does he have left?

10. An old-fashioned gardener measures the height of a plant as $6\frac{3}{8}$ inches. A week later the height is measured as $8\frac{3}{5}$ inches. How much did the plant grow during the week?

20.3 Multiplying Fractions

In this section we extend the ideas of Unit 10, where you multiplied fractions by numbers, to now include multiplying fractions by fractions.

Example 1

Calculate:

(a) $\frac{1}{3}$ of £24, (b) $\frac{2}{5}$ of £40, (c) $\frac{3}{7}$ of 35 m.

Solution

(a) $\frac{1}{3}$ of £24 $= \frac{24}{3}$

$= £8$

(b) $\frac{1}{5}$ of £40 $= \frac{40}{5}$

$= £8$

$\frac{2}{5}$ of £40 $= 2 \times 8$

$= £16$

or $\frac{2}{5}$ of £40 $= \frac{2 \times 40}{5}$

$= \frac{80}{5}$

$= £16$

(c) $\frac{1}{7}$ of 35 m $= \frac{35}{7}$

$= 5$ m

$\frac{3}{7}$ of 35 m $= 3 \times 5$

$= 15$ m

or $\frac{3}{7}$ of 35 m $= \frac{3 \times 35}{7}$

$= \frac{105}{7}$

$= 15$ m

Example 2

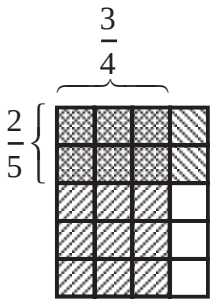
Calculate $\frac{2}{5} \times \frac{3}{4}$ and illustrate this on a diagram.

Solution

(c) $\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4}$

$= \frac{6}{20}$

$= \frac{3}{10}$



Note that 6 of the small squares are shaded twice, so $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

Note that we are using the rule:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Example 3

Calculate:

(a) $\frac{4}{7} \times \frac{3}{5}$ (b) $1\frac{3}{4} \times \frac{2}{3}$ (c) $1\frac{1}{2} \times 3\frac{1}{3}$

Solution

(a) $\frac{4}{7} \times \frac{3}{5} = \frac{4 \times 3}{7 \times 5}$

$= \frac{12}{35}$

(b) $1\frac{3}{4} \times \frac{2}{3} = \frac{7}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{3}$

(*Note*: it is usually quicker to cancel down at this stage rather than at the end.)

$= \frac{7 \times 1}{2 \times 3}$

$= \frac{7}{6}$

$= 1\frac{1}{6}$

(c) $2\frac{1}{4} \times 3\frac{1}{3} = \frac{\cancel{9}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{10}^{5}}{\cancel{3}_{1}}$

$= \frac{3 \times 5}{2 \times 1}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Exercises

1. Calculate:

(a) $\frac{1}{5} \times 15$ (b) $\frac{1}{8} \times 32$ (c) $\frac{7}{8} \times 16$

(d) $\frac{3}{7} \times 14$ (e) $\frac{3}{4} \times 28$ (f) $\frac{4}{5} \times 30$

(g) $\frac{5}{7} \times 21$ (h) $24 \times \frac{5}{8}$ (i) $18 \times \frac{5}{9}$

(j) $66 \times \frac{2}{3}$ (k) $34 \times \frac{4}{17}$ (l) $\frac{5}{19} \times 57$

2. Calculate:

(a) $\frac{1}{2} \times \frac{1}{3}$ (b) $\frac{1}{2} \times \frac{1}{2}$ (c) $\frac{1}{3} \times \frac{1}{4}$

(d) $\frac{2}{3} \times \frac{3}{4}$ (e) $\frac{3}{7} \times \frac{4}{5}$ (f) $\frac{3}{8} \times \frac{3}{4}$

(g) $\frac{4}{7} \times \frac{2}{9}$ (h) $\frac{6}{7} \times \frac{3}{8}$ (i) $\frac{5}{6} \times \frac{5}{7}$.

(j) $\frac{3}{10} \times \frac{3}{7}$ (k) $\frac{1}{2} \times \frac{3}{19}$ (l) $\frac{4}{11} \times \frac{2}{3}$

3. Calculate:

(a) $1\frac{1}{2} \times \frac{3}{4}$ (b) $4\frac{1}{2} \times \frac{2}{3}$ (c) $1\frac{3}{4} \times \frac{2}{5}$

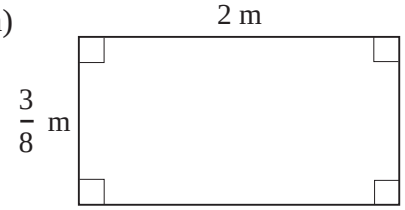
(d) $1\frac{3}{7} \times \frac{1}{2}$ (e) $4\frac{1}{4} \times \frac{1}{5}$ (f) $3\frac{1}{7} \times \frac{1}{3}$

(g) $4\frac{1}{2} \times \frac{3}{5}$ (h) $1\frac{1}{2} \times 1\frac{1}{2}$ (i) $1\frac{1}{3} \times 1\frac{1}{2}$

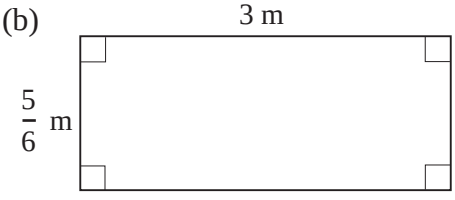
(j) $1\frac{1}{4} \times 2\frac{1}{2}$ (k) $3\frac{1}{4} \times 2\frac{1}{3}$ (l) $1\frac{1}{4} \times 2\frac{1}{5}$

4. Calculate the area of each of these rectangles:

(a)



(b)



5. A cake recipe requires $\frac{3}{4}$ kg of flour. How much flour is needed to make:

   (a) 2 cakes,

   (b) 6 cakes,

   (c) 10 cakes?

6. Jan buys $\frac{3}{4}$ kg cheese. She keeps $\frac{2}{3}$ of it and gives $\frac{1}{3}$ to her sister.
   What is the weight of:

   (a) the cheese Jan keeps,

   (b) the cheese Jan gives to her sister?

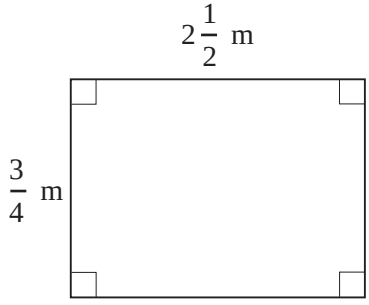
7. A large company makes £$\frac{3}{5}$ million profit. They spend $\frac{1}{4}$ of this on new equipment.

   (a) How much does the company spend on new equipment?

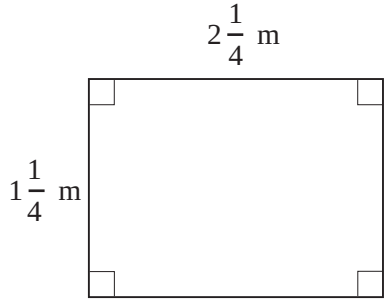
   (b) How much is left?

8. Calculate the area of each of these rectangles:

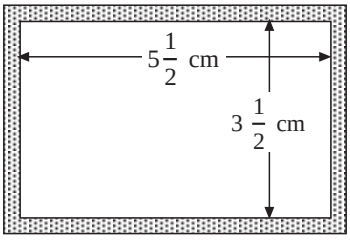
   (a)



   (b)



9. The diagram shows a small picture frame. The shaded border is $\frac{3}{4}$ cm wide.



   What is the area of the shaded border?

10. A petrol can holds $3\frac{1}{2}$ litres. Sanjit fills up a lawn mower and uses $\frac{1}{3}$ of the petrol from the full can.

    (a) How much petrol does the lawn mower hold?

    (b) How much petrol is left in the can?

    Later, Sanjit uses another $\frac{3}{4}$ litres of petrol from the can.

    (c) How much petrol has he now used?

20.4 Dividing Fractions

In this section we consider how to divide fractions and whole numbers by either whole numbers or fractions.

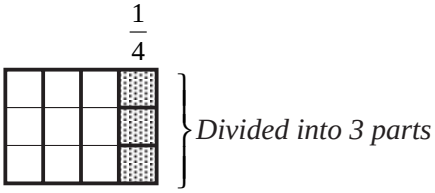
Example 1

Calculate $\frac{1}{4} \div 3$.

Solution

You can deal with this problem by thinking about the fraction being divided into 3 parts.

$\frac{1}{4}$ of the diagram has been divided into 3 parts:



Each of these parts is $\frac{1}{12}$ of the whole, so

$$\frac{1}{4} \div 3 = \frac{1}{12}$$

We can also obtain the result in this way:

$$\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$

which uses the rule:

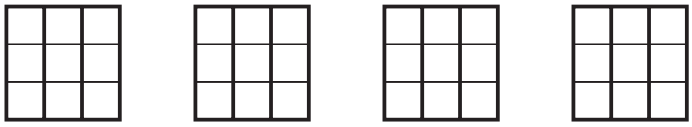
$$\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$$

Example 2

Calculate: (a) $4 \div \frac{1}{3}$, (b) $4 \div \frac{2}{5}$.

Solution

(a) The problem is to calculate how many $\frac{1}{3}$s there are in 4 whole units. The four whole units are shown below, and each is divided into $\frac{1}{3}$s.



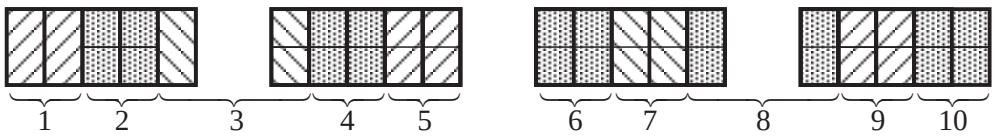
The diagram shows 12 $\frac{1}{3}$s, so

$$4 \div \frac{1}{3} = 12$$

We can obtain this result from

$$4 \div \frac{1}{3} = 4 \times 3$$
$$= 12$$

(b) The problem is to calculate how many $\frac{2}{5}$s there are in 4 whole units.



The diagram shows 10 $\frac{2}{5}$s, so

$$4 \div \frac{2}{5} = 10$$

We can also obtain this result from

$$4 \div \frac{2}{5} = 4 \times \frac{5}{2}$$
$$= \frac{20}{2}$$
$$= 10$$

using the rule:

$$a \div \frac{b}{c} = \frac{a \times c}{b}$$

Example 3

Calculate: (a) $\frac{3}{4} \div \frac{1}{5}$ (b) $\frac{5}{7} \div \frac{2}{3}$ (c) $\frac{3}{4} \div \frac{9}{10}$

Solution

These problems can be tackled using the same approach as when a whole number is divided by a fraction.

(a) $\frac{3}{4} \div \frac{1}{5} = \frac{3}{4} \times \frac{5}{1}$

$= \frac{15}{4}$

$= 3\frac{3}{4}$

(b) $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2}$

$= \frac{15}{14}$

$= 1\frac{1}{14}$

(c) $\frac{3}{4} \div \frac{9}{10} = \frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{10}^{5}}{\cancel{9}_{3}}$ [*Note:* You can cancel *only* when the 2nd fraction has been turned upside-down.]

$= \frac{5}{6}$

We are using the rule:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Exercises

1. Calculate:

(a) $\frac{1}{2} \div 3$ (b) $\frac{3}{4} \div 2$ (c) $\frac{1}{8} \div 2$

(d) $\frac{3}{4} \div 4$ (e) $\frac{5}{8} \div 3$ (f) $\frac{4}{5} \div 2$

(g) $\frac{6}{7} \div 3$ (h) $\frac{4}{5} \div 9$ (i) $\frac{1}{8} \div 3$

(j) $\frac{5}{6} \div 4$ (k) $\frac{9}{10} \div 6$ (l) $\frac{4}{5} \div 7$

2. Calculate:

(a) $6 \div \frac{1}{2}$ (b) $9 \div \frac{1}{3}$ (c) $8 \div \frac{1}{3}$

(d) $2 \div \frac{1}{4}$ (e) $5 \div \frac{2}{3}$ (f) $4 \div \frac{3}{4}$

(g) $8 \div \frac{1}{7}$ (h) $5 \div \frac{5}{7}$ (i) $9 \div \frac{3}{7}$

(j) $6 \div \frac{2}{3}$ (k) $14 \div \frac{7}{9}$ (l) $11 \div \frac{1}{13}$

3. Calculate:

(a) $\frac{1}{2} \div \frac{1}{3}$ (b) $\frac{3}{8} \div \frac{1}{2}$ (c) $\frac{3}{4} \div \frac{2}{3}$

(d) $\frac{4}{5} \div \frac{2}{3}$ (e) $\frac{3}{4} \div \frac{1}{8}$ (f) $\frac{3}{8} \div \frac{1}{2}$

(g) $\frac{5}{7} \div \frac{2}{5}$ (h) $\frac{5}{7} \div \frac{5}{9}$ (i) $\frac{1}{8} \div \frac{2}{9}$

(j) $\frac{3}{4} \div \frac{1}{9}$ (k) $\frac{1}{7} \div \frac{1}{3}$ (l) $\frac{4}{5} \div \frac{5}{8}$

4. By using *improper fractions*, calculate:

(a) $1\frac{1}{2} \div 3\frac{1}{4}$ (b) $3\frac{1}{2} \div 1\frac{1}{4}$ (c) $1\frac{5}{8} \div \frac{5}{7}$

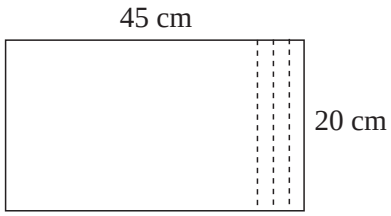
(d) $3\frac{1}{2} \div 1\frac{1}{2}$ (e) $5\frac{1}{2} \div \frac{2}{3}$ (f) $4\frac{1}{5} \div \frac{5}{7}$

5. Ahmed has $\frac{3}{4}$ kg of sweets. He divides these into 3 equal parts so that he can share them with his two brothers. What fraction of a kg does each boy get?

6. Sandra has $\frac{1}{4}$ litre of orange squash to make 10 drinks. How much orange squash should she put in each drink?

7. A large cake uses 3 times as much flour as a small cake. A large cake needs $1\frac{1}{8}$ kg of flour. How much flour does a small cake need?

8. A piece of leather is 20 cm wide and 45 cm long.



How many bookmarks, $2\frac{1}{2}$ cm wide, can be made if the leather is:

(a) cut as shown above, to make bookmarks 20 cm long,

(b) cut the other way to make bookmarks 45 cm long?

9. A recipe for a cake requires $\frac{1}{4}$ kg of sugar. How many cakes can be made with:

(a) $1\frac{1}{4}$ kg of sugar.,

(b) $2\frac{3}{4}$ kg of sugar,

(c) $3\frac{1}{3}$ kg of sugar?

10. A car uses $1\frac{1}{4}$ litres of petrol for every 10 miles it travels. How far can the car travel on:

(a) 5 litres of petrol,

(b) $7\frac{1}{2}$ litres of petrol,

(c) 9 litres of petrol?