

17 Units of Measure

17.1 Estimating Metric Units of Length, Mass and Capacity

It is very useful to be able to *estimate* lengths, masses, etc. because it may not always be easy to measure them. Some useful hints for estimating are listed below:

The height of a standard door is about 2 m.
The length of an adult pace is about 1 m.
The length of a size 8 shoe is about 30 cm.
Most adults are between 1.5 m and 1.8 m in height.
It takes about 15 minutes to walk one kilometre.
The mass of a standard bag of sugar is 1 kg.
The mass of a family car is about 1 tonne.
1 hectare = 10 000 m² (about 2 football pitches).
A teaspoon holds about 5 ml of liquid.
The volume of a normal can of drink is about 330 cm³.

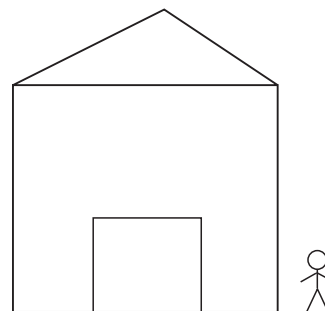


Example 1

The diagram shows a tall man standing beside a factory.

Estimate:

- (a) the height of the factory,
- (b) the height of the door.



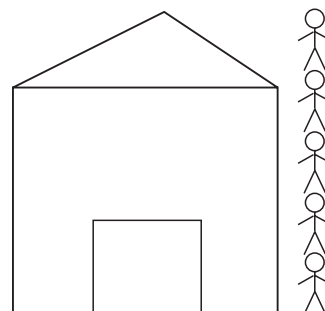
Solution

- (a) The diagram shows that the height of the factory is approximately 5 times the height of the man.

Estimate the man's height as 1.8 m.

An estimate for the height of the factory is

$$5 \times 1.8 \text{ m} = 9 \text{ m}$$



- (b) The height of the door is approximately $1\frac{1}{2}$ times the height of the man.

An estimate for the height of the door is

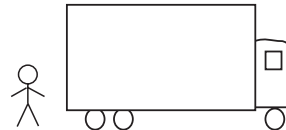
$$1\frac{1}{2} \times 1.8 = 2.7 \text{ m}$$



Example 2

The diagram shows a tall person standing behind a lorry.

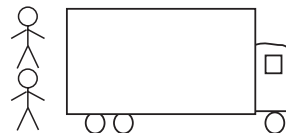
Estimate the length and height of the lorry, assuming that the height of the person is about 1.8 m.



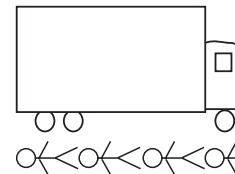
Solution

The diagrams show how to make estimates for the height and length.

$$\begin{aligned} \text{Height} &\approx 2 \times 1.8 \text{ m} \\ &\approx 3.6 \text{ m} \end{aligned}$$



$$\begin{aligned} \text{Length} &\approx 3\frac{1}{2} \times 1.8 \text{ m} \\ &\approx 6.3 \text{ m} \end{aligned}$$



Note If the height of the person was actually 1.6 m, the estimates for the height and length would change to 3.2 m and 5.6 m respectively.



Exercises

1. *Estimate* the following in your classroom:
 - (a) length of room,
 - (b) width of room,
 - (c) height of room,
 - (d) height of door,
 - (e) height of windows,
 - (f) width of black/white board.

2. *Estimate* the following:

- (a) the height of a football goal,
- (b) the width of a hockey pitch,
- (c) the width of a football goal,
- (d) the height of a netball post.

Measure the *actual* heights and widths and compare with your estimates.

3. (a) *Estimate* the size of your text book (width, height and thickness).

(b) *Measure* your text book to see how good your estimates were.

4. *Estimate* the lengths of the following vehicles:

- (a) a car,
- (b) a bus,
- (c) an articulated lorry,
- (d) a motorcycle.

5. Collect together a number of items of various masses.

(a) Copy and complete the table, writing in the *actual* mass after each estimate.

<i>Item</i>	<i>Estimate of Mass</i>	<i>Actual Mass</i>
Text book		
Can of drink		

(b) Do you become more accurate at estimating as you have more practice?

6. *Estimate*, in grams or kilograms, the mass of the following:

- (a) a table tennis ball,
- (b) a chair,
- (c) a large dog,
- (d) your school bag, when full,
- (e) a calculator,
- (f) a pen.

7. *Estimate*, in ml or litres, the volume of milk you would:
- (a) add to a cup of tea,
 - (b) pour on to cereal in a bowl,
 - (c) pour into a mug.
8. *Estimate* the volume of:
- (a) a football,
 - (b) a tennis ball,
 - (c) a table tennis ball,
 - (d) a hockey ball.
9. Jo estimates that the height of a double-decker bus is 9 m. Do you think that this is a reasonable estimate? Explain why.
10. Tony estimates that the capacity of a thermos flask is 1 litre, because it is about the same size and shape as a 1 litre lemonade bottle. Explain whether or not you think he has made a good estimate.
11. Which of the following would be the best estimate for the mass of an apple:
- A 1 kg
 - B 2 grams
 - C 200 grams
 - D 20 grams
 - E 800 grams
12. Which of the following would be the best estimate for the diameter of a saucer:
- A 16 cm
 - B 16 mm
 - C 16 m
 - D 8 mm
 - E 80 cm
13. Which of the following would be the best estimate for the capacity of a tea cup:
- A 15 ml
 - B 1500 ml
 - C 0.5 litres
 - D 5 litres
 - E 150 ml

17.2 The Metric System: Conversion Between Units

The metric (decimal) system uses a number of standard prefixes for units of length, mass, etc.

The three most important are:

$$\textit{kilo} = 1000$$

$$\textit{centi} = \frac{1}{100}$$

$$\textit{milli} = \frac{1}{1000}$$

You will have met many of these already, for example,

$$1 \text{ millimetre} = \frac{1}{1000} \text{ metre} \quad \textit{so} \quad 1 \text{ metre} = 1000 \text{ millimetres}$$

$$1 \text{ kilogram} = 1000 \text{ grams} \quad \textit{so} \quad 1 \text{ gram} = \frac{1}{1000} \text{ kilogram}$$

$$1 \text{ centimetre} = \frac{1}{100} \text{ metre} \quad \textit{so} \quad 1 \text{ metre} = 100 \text{ centimetres}$$

$$1 \text{ millilitre} = \frac{1}{1000} \text{ litre} \quad \textit{so} \quad 1 \text{ litre} = 1000 \text{ millilitres}$$

It is also useful to know that:

$$1 \text{ cm}^3 = 1 \text{ millilitre (ml)}$$

and

$$1000 \text{ kg} = 1 \text{ tonne}$$



Example 1

Complete each of the following statements:

(a) $150 \text{ cm} = \boxed{} \text{ m}$

(b) $360 \text{ mm} = \boxed{} \text{ m}$

(c) $3.6 \text{ tonnes} = \boxed{} \text{ kg}$

(d) $62 \text{ ml} = \boxed{} \text{ litres}$



Solution

- (a) $150 \text{ cm} = 150 \times \frac{1}{100} = 1.5 \text{ m}$
- (b) $360 \text{ mm} = 360 \times \frac{1}{1000} = 0.36 \text{ m}$
- (c) $3.6 \text{ tonnes} = 3.6 \times 1000 = 3600 \text{ kg}$
- (d) $62 \text{ ml} = 62 \times \frac{1}{1000} = 0.062 \text{ litres}$



Example 2

John adds 250 ml of water to a jug that already contains 1.2 litres of water. How much water is now in the jug?



Solution

$$\begin{aligned} 1.2 \text{ litres} &= 1.2 \times 1000 \\ &= 1200 \text{ ml} \end{aligned}$$

$$\begin{aligned} \text{Total volume} &= 1200 + 250 \\ &= 1450 \text{ ml or } 1.45 \text{ litres} \end{aligned}$$



Exercises

1. Change the following lengths into mm:

- | | | | |
|------------|-------------|-------------|-------------|
| (a) 4 cm | (b) 7 cm | (c) 26 cm | (d) 835 cm |
| (e) 6.2 cm | (f) 14.7 cm | (g) 9.25 cm | (h) 0.04 cm |

Change the following lengths into cm:

- | | | | |
|-----------|------------|-------------|-------------|
| (i) 60 mm | (j) 80 mm | (k) 340 mm | (l) 9450 mm |
| (m) 87 mm | (n) 262 mm | (o) 67.9 mm | (p) 6 mm |

2. Change the following lengths into cm:

- | | | | |
|-----------|------------|-------------|------------|
| (a) 7 m | (b) 18 m | (c) 36 m | (d) 904 m |
| (e) 4.3 m | (f) 53.9 m | (g) 28.38 m | (h) 0.09 m |

Change the following lengths into m:

- | | | | |
|------------|------------|-------------|-------------|
| (i) 800 cm | (j) 500 cm | (k) 760 cm | (l) 2150 cm |
| (m) 365 cm | (n) 57 cm | (o) 77.6 cm | (p) 6 cm |

3. Change the following lengths into m:

- (a) 5 km (b) 11 km (c) 63 km (d) 423 km
 (e) 7.4 km (f) 2.56 km (g) 14.321 km (h) 0.07 km

Change the following lengths into km:

- (i) 6000 m (j) 17 000 m (k) 53 000 m (l) 4750 m
 (m) 807 m (n) 62 m (o) 3 m (p) 29.3 m

4. Change the following masses into g:

- (a) 6 kg (b) 8 kg (c) 15 kg (d) 92 kg
 (e) 1.7 kg (f) 5.47 kg (g) 2.925 kg (h) 0.004 kg

Change the following masses into kg:

- (i) 3000 g (j) 40 000 g (k) 8340 g (l) 29 750 g
 (m) 237 g (n) 52 g (o) 9 g (p) 3.6 g

5. Copy and complete each of the following statements:

- (a) 320 mm = m (b) 6420 mm = m
 (c) 642 mm = m (d) 888 cm = m
 (e) 224 cm = mm (f) 45 m = mm
 (g) 320 m = cm (h) 8.73 m = mm

6. Convert the following masses to kg:

- (a) 8.2 tonnes (b) 160 tonnes
 (c) 88 g (d) 3470 g

7. Convert the following masses to g:

- (a) 3.6 kg (b) 3.7 tonnes
 (c) 840 mg (d) 62 mg

8. Convert the following volumes to ml:

- (a) $\frac{1}{4}$ litre (b) 22 litres
 (c) 0.75 litres (d) 450 cm³

The height of a tall adult is about 6 feet.

The width of an adult thumb is about 1 inch.

The length of a size 8 shoe is about 1 foot.

An adult pace is about 1 yard.

The mass of a bag of sugar is just over 2 pounds.

An old-style bottle of milk contains 1 pint.

It takes about 20 minutes to walk one mile.

You will find the following abbreviations used for imperial units:

1 yard = 1 yd

6 feet = 6 ft = 6'

9 inches = 9 in = 9"

8 ounces = 8 oz

7 pounds = 7 lb

but be careful not to use m as an abbreviation for miles because m is a standard abbreviation for metres.



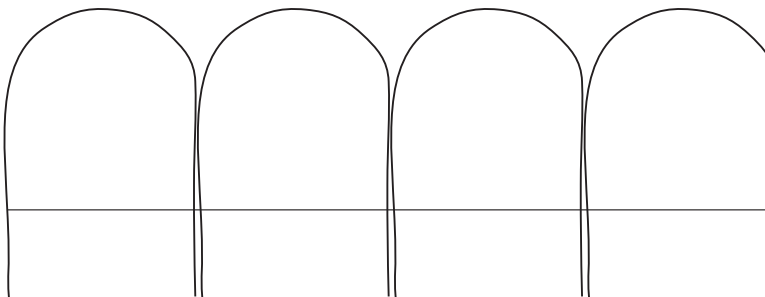
Example 1

Estimate the length of the following line, in inches:



Solution

The diagram shows the line itself and the outline of 4 adult thumbs:



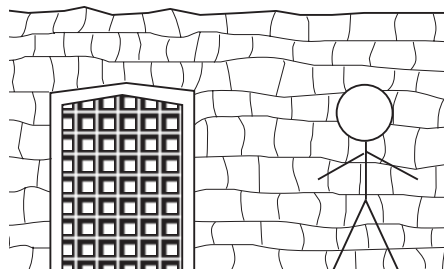
So the length can be estimated as 4 inches.



Example 2

The picture shows a man standing next to a wall with a gate in it:

Estimate the height in feet of both the wall and the gate.





Solution

The wall is about $1\frac{1}{3}$ times the height of the man, so taking the height of the man as 6 feet, gives

$$\text{height of wall} \approx 1\frac{1}{3} \times 6 \approx 8 \text{ feet}$$

The gate is about the same height as the man, so its height can be estimated as six feet.



Exercises

1. *Estimate* the length of each of the lines below, in inches. Then *measure* each line to check your estimate.

(a)



(b)



(c)



(d)



2. (a) *Estimate* the size of the top of your desk, in inches.
 (b) *Measure* your desk and see how accurate your estimate was.
3. (a) *Estimate* the heights of 4 of your friends, in feet and inches.
 (b) *Measure* these friends and see how accurate your estimates were.
4. *Estimate* the length and width of your classroom, in feet.
5. *Estimate* the total mass of 3 maths text books, in pounds.
6. *Estimate* the mass of an apple, in ounces. (Remember that there are 16 ounces in 1 lb.)
7. *Estimate* the capacity of a mug, in pints.

8. *Estimate* the mass of your shoe, in pounds. Check your estimate if possible.
9. *Estimate* the dimensions of a football or hockey pitch, in yards.
10. A fish tank is in the shape of a cube with sides of length 1 foot. *Estimate* the volume of this tank in pints.

17.4 Metric and Imperial Units

As both metric and imperial units are in general use, you need to be able to convert between the two systems. The list below contains a number of useful conversion facts which you will need in the examples and exercises that follow.

8 km	≈	5 miles
1 m	≈	40 inches
30 cm	≈	1 foot
2.5 cm	≈	1 inch
1 kg	≈	2.2 lbs
1 litre	≈	$1\frac{3}{4}$ pints
1 gallon	≈	$4\frac{1}{2}$ litres
1 acre	≈	$\frac{2}{5}$ hectare
450 g	≈	1 lb

The following list reminds you of some of the relationships in the imperial system:

1 lb	=	16 ounces
1 stone	=	14 lb
1 mile	=	1760 yards
1 yard	=	3 feet
1 foot	=	12 inches
1 gallon	=	8 pints
1 chain	=	22 yards
1 furlong	=	220 yards

Also note that $1 \text{ acre} = 4840 \text{ square yards}$ (approximately the area of a football pitch)

Conversions between metric and imperial units are not precise, so we always round the converted figure, taking the context into account (see Examples 1 and 2 below).



Example 1

While on holiday in France, a family see the following road-sign:

PARIS 342 km

How many *miles* are the family from Paris?



Solution

Note $8 \text{ km} \approx 5 \text{ miles}$

$$\text{Distance from Paris} \approx 342 \times \frac{5}{8} \text{ miles}$$

$$\approx 213.75 \text{ miles}$$

The family are therefore about 214 miles from Paris.



Example 2

A bottle contains 2.5 litres of milk. How many *pints* of milk does the bottle contain?



Solution

Note $1 \text{ litre} \approx 1\frac{3}{4} \text{ pints}$

$$\text{Volume of milk} \approx 2.5 \times 1.75 \text{ pints}$$

$$\approx 4.375 \text{ pints}$$

The bottle contains almost $4\frac{1}{2}$ pints of milk.



Example 3

Vera buys 27 litres of petrol for her car. How many *gallons* of petrol does she buy?



Solution

Note 1 gallon \approx 4.5 litres

$$\begin{aligned}\text{Quantity of petrol} &\approx \frac{27}{4.5} \\ &\approx 6 \text{ gallons}\end{aligned}$$

Vera buys approximately 6 gallons of petrol.



Exercises

1. Change the following lengths into inches:

- | | | |
|---------------------|--------------|---------------------|
| (a) 4 feet | (b) 7 feet | (c) 4 feet 2 inches |
| (d) 8 feet 7 inches | (e) 5.5 feet | (f) 2 yards |
| (g) 5 yards 2 feet | (h) 1 mile | |

Change the following lengths into feet or feet and inches:

- | | | |
|----------------|----------------|---------------|
| (i) 60 inches | (j) 48 inches | (k) 17 inches |
| (l) 29 inches | (m) 108 inches | (n) 95 inches |
| (o) 240 inches | (p) 6 inches | |

2. Change the following masses into ounces:

- | | | |
|----------------|-----------------------|----------------|
| (a) 7 pounds | (b) 11 pounds | (c) 36 pounds |
| (d) 904 pounds | (e) 42 pounds | (f) 5.5 pounds |
| (g) 2 stone | (h) 9 stone 12 pounds | |

Change the following masses into pounds or pounds and ounces:

- | | | |
|----------------|----------------|---------------|
| (i) 80 ounces | (j) 128 ounces | (k) 56 ounces |
| (l) 720 ounces | (m) 36 ounces | (n) 77 ounces |
| (o) 8 ounces | (p) 4 ounces | |

3. Change the following volumes into pints:

- | | | |
|----------------------------|-------------------|--------------------------|
| (a) 5 gallons | (b) 11 gallons | (c) 63 gallons |
| (d) 412 gallons | (e) 7.5 gallons | (f) $\frac{1}{2}$ gallon |
| (g) $3\frac{1}{4}$ gallons | (h) 6.875 gallons | |

Change the following volumes into gallons or gallons and pints:

- (i) 56 pints (j) 160 pints (k) 4800 pints
- (l) 528 pints (m) 12 pints (n) 87 pints
- (o) 2 pints (p) 1884 pints

4. Convert the following distances to cm, where necessary giving your answers to 2 significant figures where necessary:

- (a) 6 inches (b) 8 inches (c) $7\frac{1}{2}$ inches
- (d) 8 feet (e) 4 yards (f) $1\frac{1}{4}$ yards

5. The road-sign shown gives distances in km:
Produce a version of the sign with the equivalent distances given in *miles*.

BREST	400
ROSCOFF	384
ST MALO	168
RENNES	162
NANTES	148

6. A recipe requires $\frac{1}{2}$ lb of flour. What is the equivalent amount of flour in:

- (a) grams, (b) kilograms, (c) ounces ?

7. The capacity of a fuel tank is 30 gallons. What is the capacity of the tank in:

- (a) litres, (b) pints ?

8. A cow produces an average of 18 pints of milk each time she is milked.
Convert this to *litres*, giving your answer to 1 decimal place.

9. The mass of a parcel is 4 lb 4 oz. Calculate its mass in kilograms, giving your answer to 1 decimal place.

10. Copy and complete the table shown, which can be used to convert speeds between mph and km/h. Where necessary, express your answers to 3 significant figures.

<i>mph</i>	<i>km/h</i>
30	
	50
40	
	70
	80
60	
	100
70	
	120

17.5 Problems in Context

In this section we look at a variety of problems where the context requires us to deal with more than one type of unit. The units may be only metric, or only imperial, or a mixture of both.



Example 1

A school canteen buys a 1 gallon can of fruit juice. The canteen sells the fruit juice in paper cups that each contain 150 ml of drink. How many cups can be filled?



Solution

$$\begin{aligned} 1 \text{ gallon} &\approx 4.5 \text{ litres} \\ &\approx 4500 \text{ ml} \end{aligned}$$

$$\text{So about } \frac{4500}{150} = 30 \text{ cups can be filled from one can.}$$



Example 2

Some students take part in a 20-mile sponsored relay run, where each student runs 3000 m and then another student takes over. If each student runs only once, how many students are needed to complete the run?



Solution

$$\begin{aligned} 20 \text{ miles} &\approx 20 \times \frac{8}{5} \text{ km} \\ &\approx 32 \text{ km} \\ &\approx 32\,000 \text{ m} \end{aligned}$$

$$32\,000 \div 3000 = 10 \text{ remainder } 2000,$$

so 11 students are needed to complete the run, but one of them will run only about 2000 m.



Example 3

A technology teacher has a 50-yard roll of glass fibre tape. For a project, each student in the class will need 80 cm of tape. There are 30 students in the class. What length of tape will be left over?



Solution

$$\begin{aligned}
 50 \text{ yards} &= 50 \times 3 \\
 &= 150 \text{ feet} \\
 &\approx 150 \times 0.3 \text{ m} \\
 &\approx 45 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Tape used} &= 30 \times 80 \text{ cm} \\
 &= 2400 \text{ cm} \\
 &= 24 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Tape left} &\approx 45 - 24 \text{ m} \\
 &\approx 21 \text{ m}
 \end{aligned}$$



Exercises

- A glass holds 50 ml of drink. How many glasses can be filled from:

 - a 1 litre bottle,
 - a 1 gallon can,
 - a 3 pint carton ?
- A sheet of wood measures 4 feet by 8 feet. A teacher cuts up the sheet into smaller pieces that measure 10 cm by 20 cm. How many of these smaller sheets can the teacher make?
- A baker buys a 25 kg sack of flour. He uses 1 lb of flour for each loaf. How many loaves can he make with 1 sack of flour?
- How many 125 ml glasses can be filled from a can that contains 2 pints of milk?
- How many books of width 2.5 cm can be put on a shelf of length 3 feet?
- A ball of wool contains 75 yards. If 22 m are needed for a knitting pattern, what length of wool is left? Give your answer in:

 - metres,
 - yards,
 - feet and inches.

