16 Circles and Cylinders

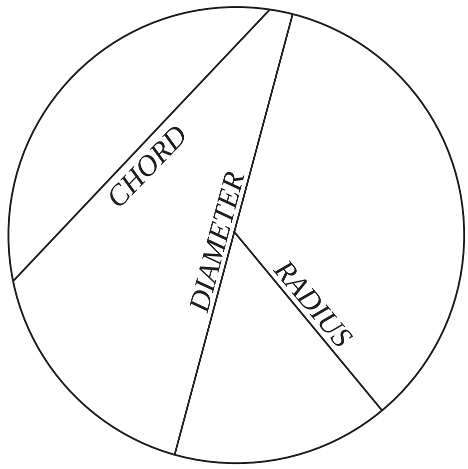
16.1 Introduction to Circles

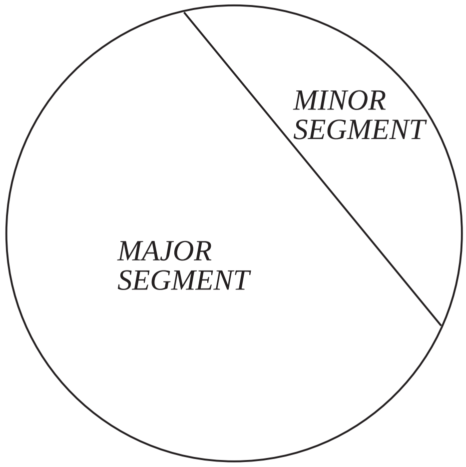
In this section we consider the circle, looking at drawing circles and at the lines that split circles into different parts.

A *chord* joins any two points on the circumference of a circle.

A *diameter* is a chord that passes through the centre of the circle.

A *radius* joins the centre of the circle to any point on the circumference of the circle.



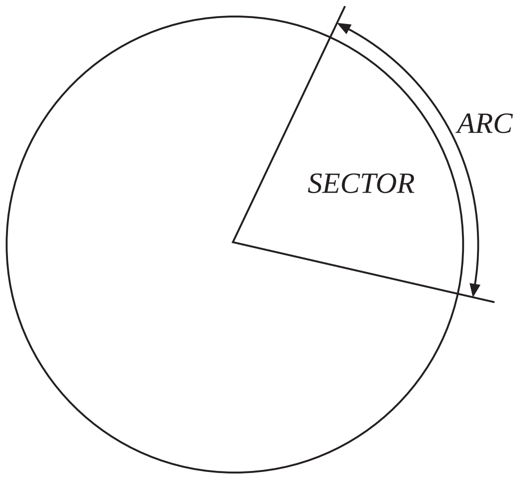


A chord splits a circle into two *segments*.

The larger one is called a *major segment*: the smaller one is the *minor segment*.

The part of a circle between two radii is called a *sector*.

The part of the circle that forms the curved side of the sector is called an *arc*.



Example 1

(a) Draw a circle of radius 5 cm.

(b) Draw a radius of the circle.

(c) Draw a chord that is perpendicular to the radius and is 3.3 cm from the centre of the circle.

(d) Measure the length of the chord.

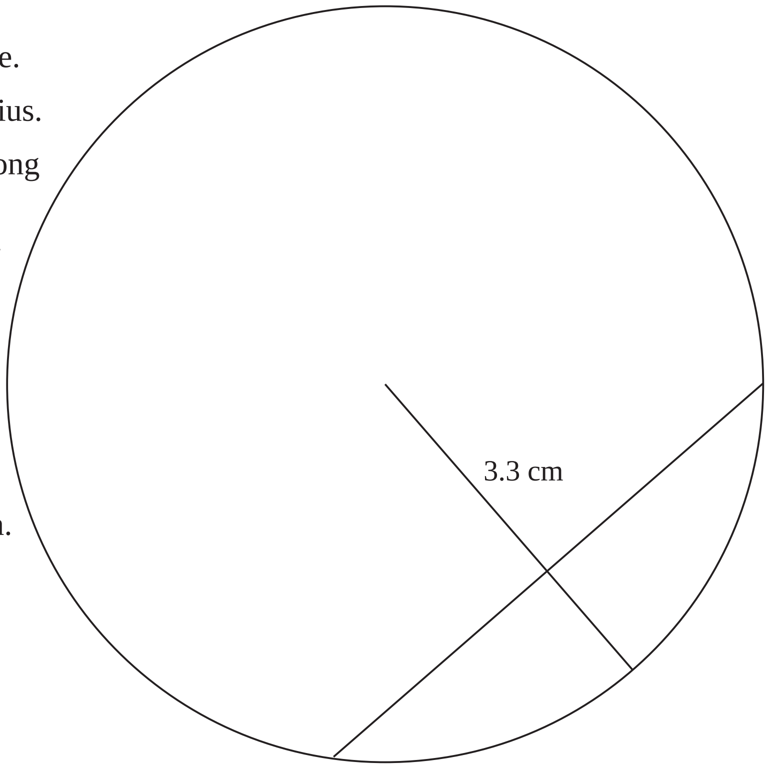
Solution

(a) First draw the circle.

(b) Then draw in a radius.

(c) Measure 3.3 cm along the radius from the centre of the circle. Draw a chord at right angles to the radius and through this point.

(d) Measure the chord, which gives 7.5 cm.



Example 2

(a) Draw a circle of radius 3 cm.

(b) Draw a chord of length 5 cm, inside the circle.

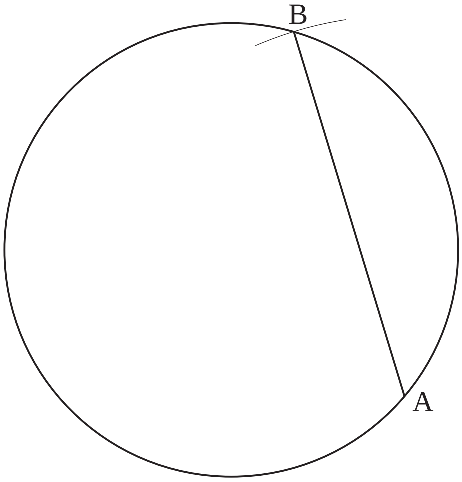
(c) Draw the perpendicular bisector of the chord.

(d) What is the length of the new chord that is formed by the perpendicular bisector?

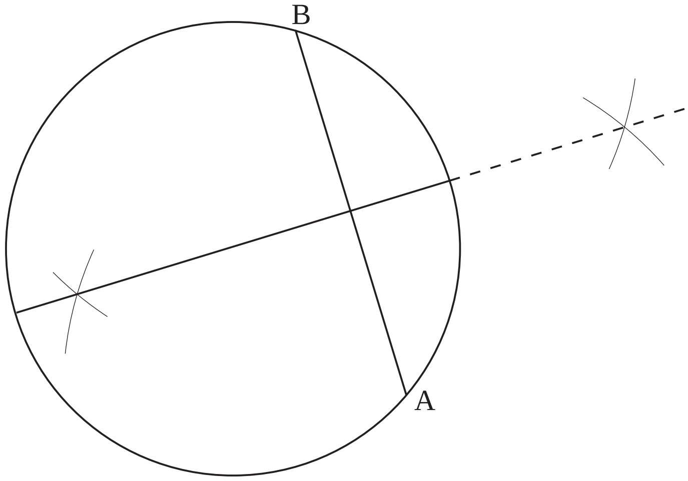
Solution

(a) First draw the circle.

(b) Put your compass point at A, with your compass set at 5 cm. Draw an arc to find the point B. Then join the points A and B.



(c) To draw a perpendicular bisector, place your compass point at A and draw two arcs. Repeat with your compass point at B, drawing arcs of the same radius as before, so that they intersect. Draw a line through the two intersections.

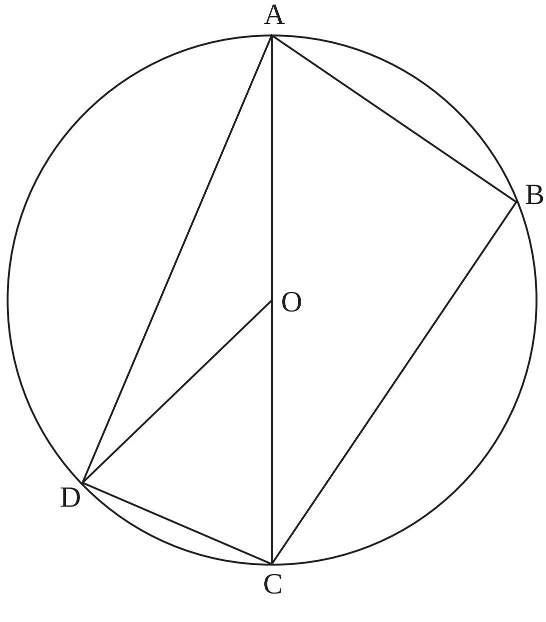


(d) Measure the length of the new chord as 6 cm. The new chord is, in fact, a diameter of the circle.

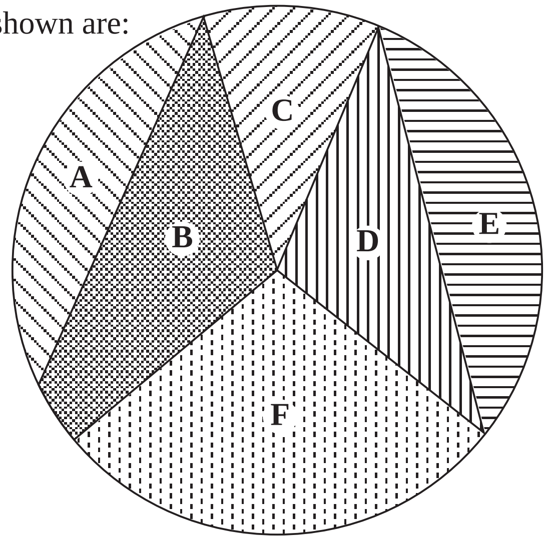
Exercises

1. The diagram shows a circle with centre, O. What is the name given to each of the following lines:

   (a) O A (b) A B (c) B C (d) O D
   (e) C D (f) A C (g) A D



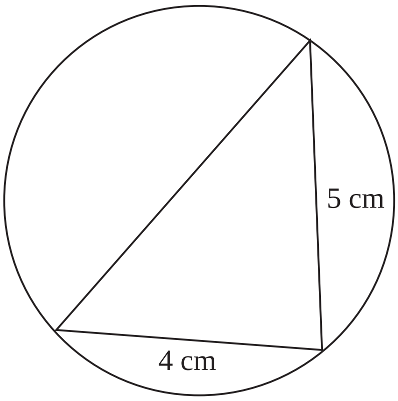
2. Which of the parts of the circle shown are:
   (a) sectors,
   (b) segments,
   (c) triangles?



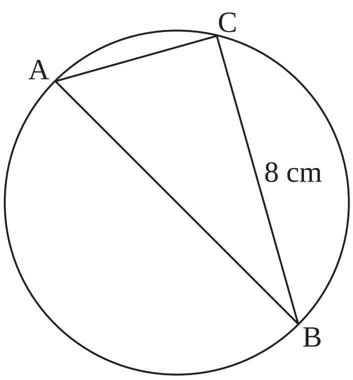
3. (a) Draw a circle of radius 5 cm.
   (b) Draw any chord in this circle.
   (c) Draw the perpendicular bisector of the chord.
   (d) Draw 2 other chords and their perpendicular bisectors.
   (e) Comment on the perpendicular bisectors of the chords.

4. (a) Draw a circle of radius 4 cm and a chord of length 3 cm, in the circle.
   (b) Join the ends of the chord to the centre of the circle, to form a triangle.
   (c) What length is the perimeter of the triangle?

5. The diagram shows a sketch of a triangle which is drawn inside a circle of radius 3 cm.
   (a) Draw this triangle accurately.
   (b) Determine the perimeter of the triangle.



6. The diagram shows a triangle drawn inside a circle of radius 5 cm. The line A B is a diameter.
   (a) Draw this triangle accurately.
   (b) Determine the length of A C.
   (c) Determine the size of the angle A C B.
   (d) Calculate the area of the triangle.



7. In a circle of radius 2.5 cm, draw a radius and the chord that is a perpendicular bisector to the radius. What is the length of this chord?

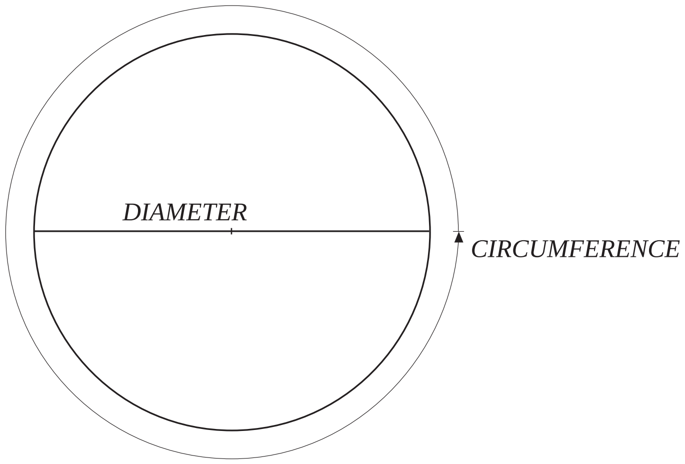
8. A triangle is drawn so that the 3 corners are on a circle of radius 3 cm. Two of the sides have length 5 cm.
   (a) Draw the triangle.
   (b) Determine the length of the third side.
   (c) Draw the perpendicular bisector of each side of the triangle.
   (d) How far is it along each perpendicular bisector from the side to the centre of the circle?

9. (a) Draw any triangle.
   (b) Draw the perpendicular bisector of each side.
   (c) Draw a circle with its centre at the point where the perpendicular bisectors intersect, and that passes through the three corners of the triangle.

10. Draw a circle of radius 3 cm. A chord in the circle has length 4 cm. Determine the distance from the centre of the chord to the centre of the circle.

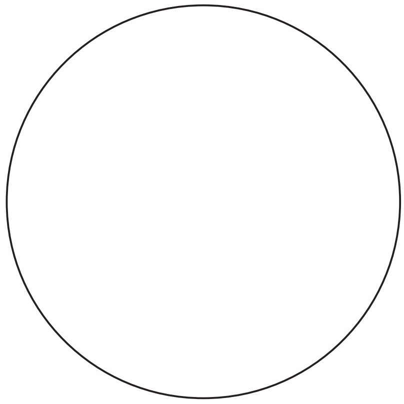
16.2 Estimating the Circumference of a Circle

In this section we investigate the relationship between the *diameter* and the *circumference* of a circle. The circumference is the distance round the outside of a circle.



Example 1

Measure the diameter and circumference of the circle shown.

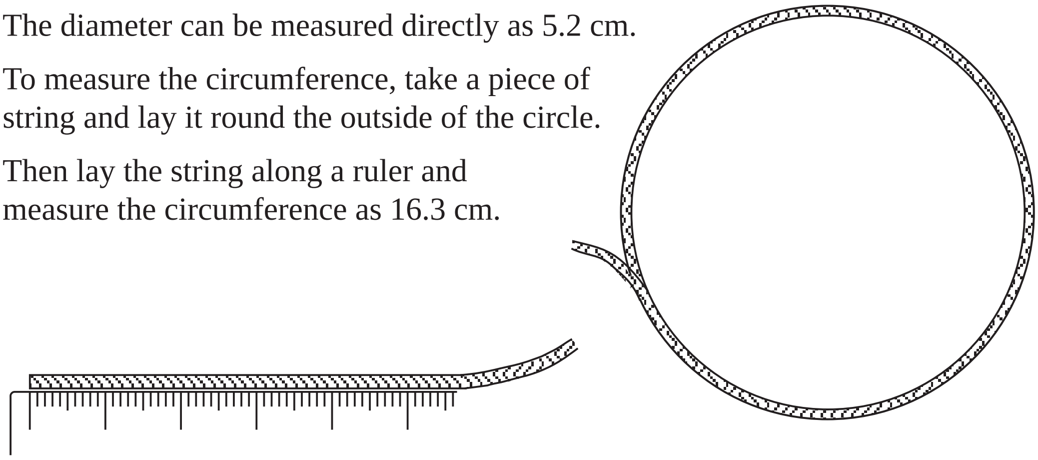


Solution

The diameter can be measured directly as 5.2 cm.

To measure the circumference, take a piece of string and lay it round the outside of the circle.

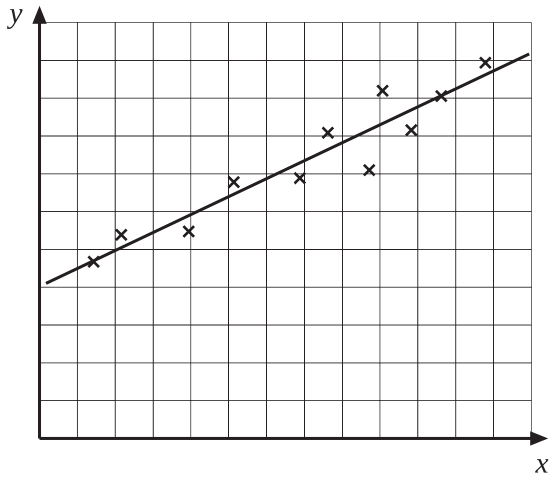
Then lay the string along a ruler and measure the circumference as 16.3 cm.



Reminder

The points shown below form a *scatter graph*.

The line drawn on the graph is called the 'line of best fit': it is the straight line which best fits the points plotted. It does not have to go through *every* point; just as close to them as possible. The line should be positioned so that approximately the same number of points are above it as below it.



You will need to draw lines of best fit as you work through this unit.

Exercises

1. (a) Draw circles of radius 1 cm, 2 cm, 3 cm, 4 cm, 5 cm and 6 cm.

   (b) Measure the circumference and diameter of each circle.

2. (a) Obtain a number of circular objects, for example,

   bottle of squash,

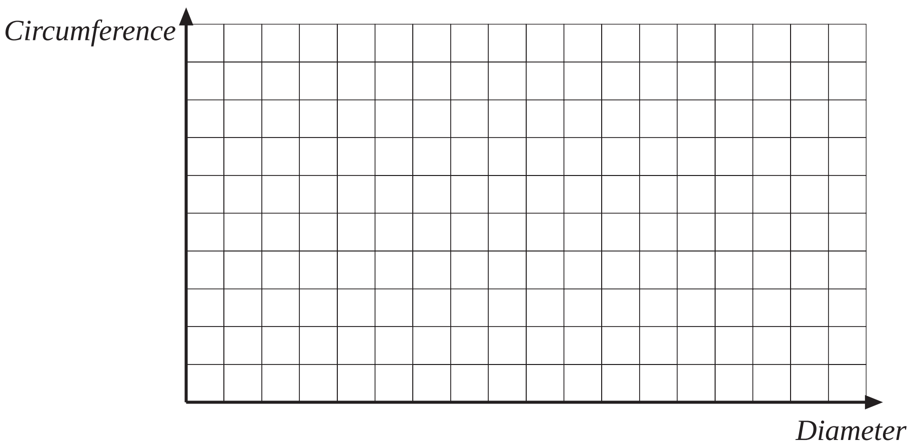
   tin of baked beans,

   bottle of Tippex,

   roll of Sellotape, etc.

   (b) For each object, measure the diameter and the circumference.

3. (a) Draw a scatter graph to show your results from questions 1 and 2, on a set of axes like those that follow:



   (b) Explain why a line of best fit should pass through the point with coordinates (0, 0).

   (c) Draw a line of best fit.

   (d) The relationship between the circumference, C, and the diameter, d, is given by $C = k\,d$, where k is a constant number. Use your line of best fit to determine k.

16.3 Estimating the Area of a Circle

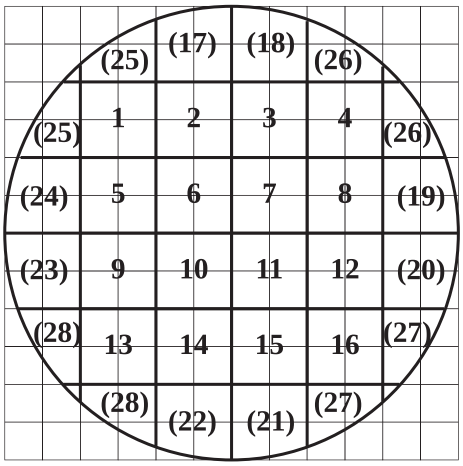
In this section we investigate how the *area* of a circle depends on the *radius* of the circle.

Example 1

Estimate the area of a circle of radius 3 cm.

Solution

The following diagram shows the circle drawn on squared paper. Complete squares are numbered 1 to 16 whilst partial squares are numbered in brackets, e.g. (17), and joined together where possible to approximately make a square, e.g. the two part squares marked (26) add to make about one whole square.



Counting the squares shows that the area is approximately 28 cm^2.

Exercises

1. Draw circles of radius 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, 6 cm and 7 cm on squared paper. Determine the approximate area of each of the circles.

2. What is the area of a circle of radius 0 cm ?

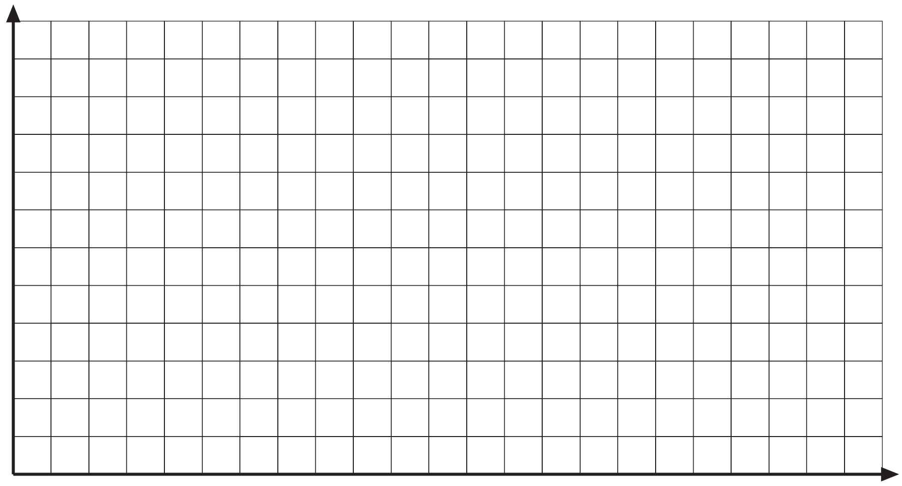
3. (a) Draw a scatter graph to show area against radius for the results that you have obtained in question 1.

   (b) Explain why it would not be sensible to draw a line of best fit through the points that you have plotted.

4. (a) Copy and complete the following table:

Radius (cm)	0	1	2	3	4	5	6	7
$(Radius)^2$ (cm^2)	0	1		9				
Approximate Area (cm^2)	0			28				

(b) On a set of axes like those shown below, draw a scatter graph using the data from your table.



(c) Draw a line of best fit through your data points.

(d) The area, A, of a circle of radius r, can be found using the formula $A = kr^2$, where k is a constant number. Use your line of best fit to determine the value of k.

16.4 Formulae for Circumference and Area

In the previous two sections you have found *approximate* formulae for both the circumference and the area of a circle. The *exact* formulae are given below:

$$\text{Circumference} = \pi d$$

or

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

The symbol π (lower case Greek letter p) represents a special number called 'pi'. The value of π has been calculated to over 1000 million decimal places; its value correct to 5 decimal places is 3.14159.

Compare this with the gradient you obtained from your scatter graphs in the last two sections, to see how close you were.

There is a button on your calculator which you can use when doing calculations involving π, as the next examples illustrate.

Example 1

A circle has radius 6 cm. Calculate:

(a) its *circumference*,

(b) its *area*.

Solution

(a) Circumference $= 2\pi r$

$= 2\pi \times 6$

$= 37.7$ cm to 3 significant figures.

(b) Area $= \pi r^2$

$= \pi \times 6^2$

$= 113\ \text{cm}^2$ to 3 significant figures

Example 2

A circle has diameter 7 cm. Calculate:

(a) its *circumference*,

(b) its *area*.

Solution

(a) Circumference $= \pi d$

$= \pi \times 7$

$= 22.0$ cm to 3 significant figures.

(b) Radius $= 3.5$ cm

Area $= \pi r^2$

$= \pi \times 3.5^2$

$= 38.5\ \text{cm}^2$ to 3 significant figures

Example 3

The circumference of a circle is 18.2 cm. Calculate the length of the diameter, *d*, of the circle.

Solution

$$C = \pi d$$

$$18.2 = \pi d$$

$$\frac{18.2}{\pi} = d$$

$d = 5.79$ cm to 3 significant figures.

Example 4

The area of a circle is 22.8 cm^2. Calculate the length of the radius, r, of the circle.

Solution

$$A = \pi r^2$$

$$22.8 = \pi r^2$$

$$\frac{22.8}{\pi} = r^2$$

$$r = \sqrt{\frac{22.8}{\pi}}$$

$= 2.69$ cm to 3 significant figures.

Exercises

1. A circle has radius 11 cm. Calculate:
   (a) its diameter,
   (b) its circumference,
   (c) its area.

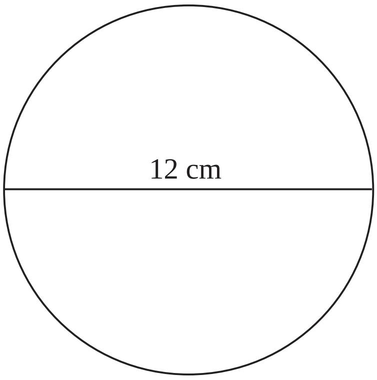
2. Calculate the circumference and area of a circle with radius 8 cm.

3. Calculate the circumference and area of a circle with diameter 19 cm.

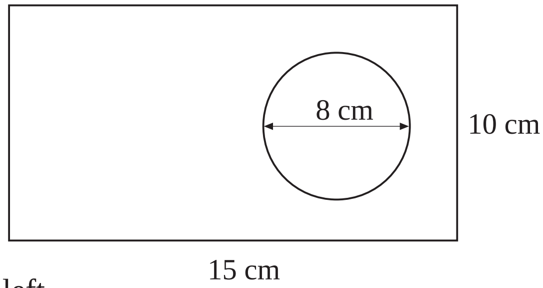
4. Copy and complete the following table:

Radius	*Diameter*	*Circumference*	*Area*
	24 cm		
1 cm			
	6 mm		
	9 m		
	23 km		

5. Determine the circumference and area of the circle shown:



6. A circle is cut out of a rectangular piece of card, as shown:

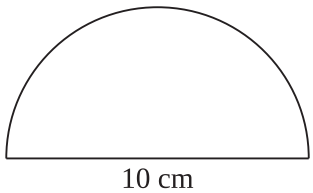


(a) Calculate the area of the rectangle.

(b) Calculate the area of the circle.

(c) Calculate the area of the card left, when the circle has been cut out.

7. Calculate the area and perimeter of the semicircle shown:



8. The circumference of a circle is 29 cm.

(a) Calculate the radius of the circle.

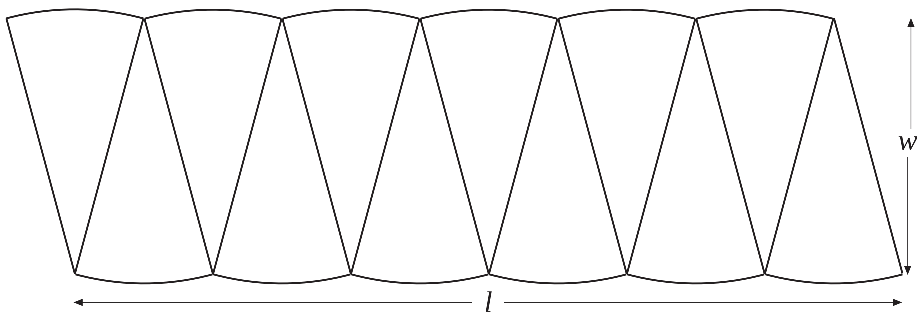
(b) Calculate the area of the circle.

9. The area of a circle is 48 cm^2. Calculate the radius and circumference of the circle.

10. Copy and complete the following table:

Radius	*Diameter*	*Circumference*	*Area*
		82 cm	
			19 m^2
	33 m		
		44 mm	
			36 mm^2

11. A circle is cut up into sectors that can be placed side-by-side as shown in the following diagram:



If the angles of the sectors are *very* small, the shape formed almost becomes a rectangle. In this case, w is nearly equal to r, the radius of the circle.

(a) Explain why l is approximately πr.

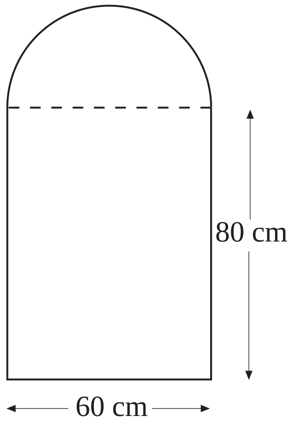
(b) Use the fact that the shape is close to a rectangle to derive a formula for the area of the circle.

16.5 Problems in Context

In this section we apply the formulae for area and circumference to some problems.

Example 1

The diagram shows an arched window made in the shape of a semicircle on top of a rectangle. Calculate the area of the window.



Solution

Area of rectangle $= 80 \times 60$

$= 4800 \text{ cm}^2$

Radius of semicircle $= 30$ cm

Area of semicircle $= \frac{1}{2} \times \pi \times 30^2$

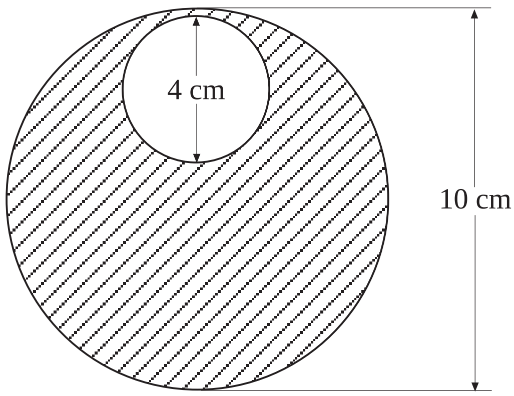
$= 1414 \text{ cm}^2$ (4 s.f.)

Total area $= 4800 + 1414$

$= 6214 \text{ cm}^2$ (4 s.f.)

Example 2

A circular disc of diameter 10 cm has a hole of diameter 4 cm cut in it. Calculate the area remaining of the large disc, as shaded in the diagram.



Solution

Radius of large disc $= 5$ cm

Area of large disc $= \pi \times 5^2$

$= 78.54 \text{ cm}^2$ (2 d.p.)

Radius of hole $= 2$ cm

Area of hole $= \pi \times 2^2$

$= 12.57 \text{ cm}^2$ (2 d.p.)

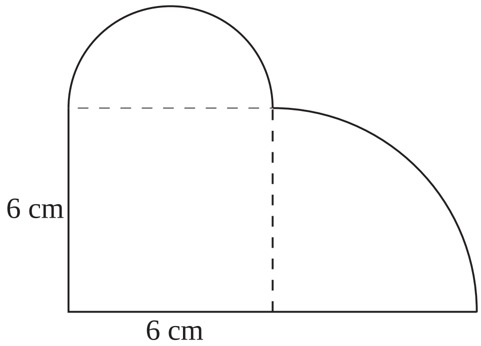
Shaded area $= 78.54 - 12.57$

$= 65.97 \text{ cm}^2$

$\approx 66.0 \text{ cm}^2$

Example 3

The diagram shows a square with sides of length 6 cm. A semicircle has been added to one side of a the square and a quarter of a circle (quadrant) added to another side. Calculate the area of the shape.



Solution

Area of square $= 6^2$

$= 36 \text{ cm}^2$

Radius of semicircle $= 3$ cm

Area of semicircle $= \frac{1}{2} \times \pi \times 3^2$

$= 14.1 \text{ cm}^2$ (3 s.f.)

Radius of quarter circle $= 6$ cm

Area of quadrant $= \frac{1}{4} \times \pi \times 6^2$

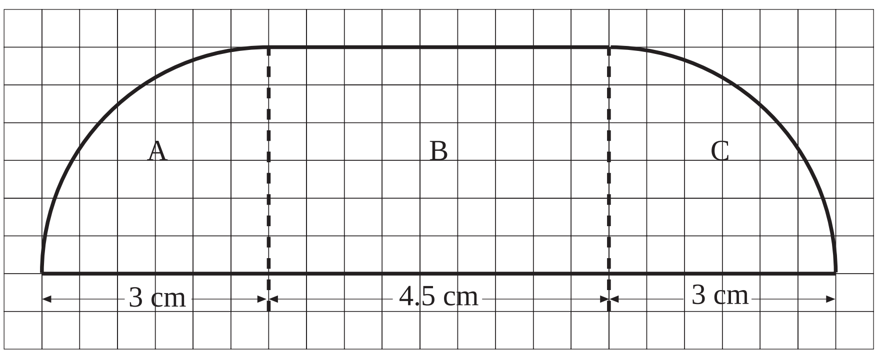
$= 28.3 \text{ cm}^2$ (3 s.f.)

Total area $= 36 + 14.1 + 28.3$

$= 78.4 \text{ cm}^2$ (3 s.f.)

Exercises

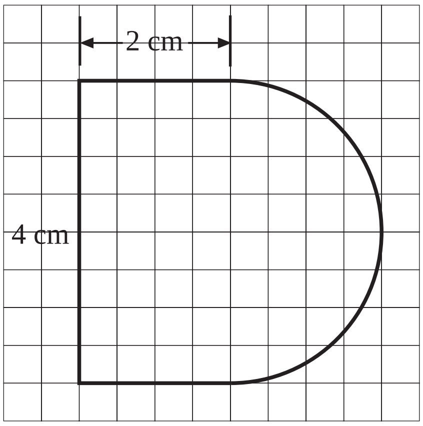
1. (a) Calculate the area of each part of the following shape:



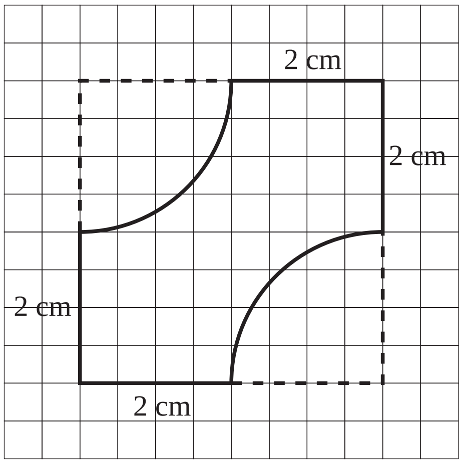
(b) What is the *total* area of the shape?

2. Calculate the area of each of the following shapes:

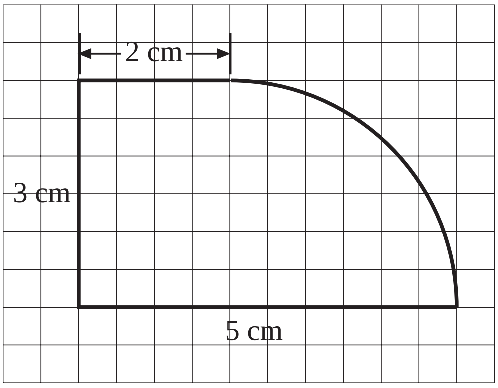
(a)



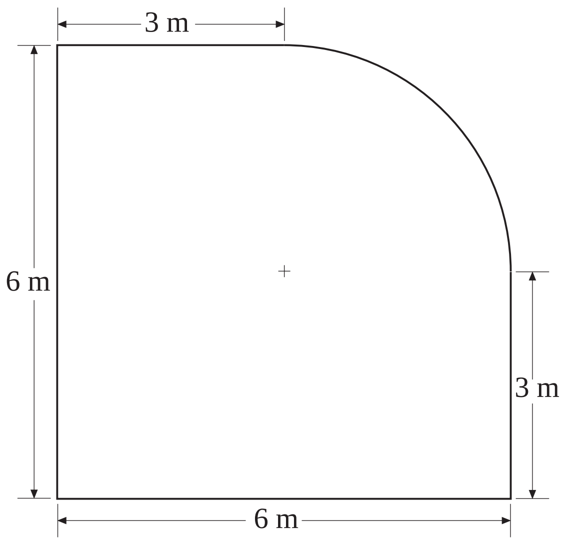
(b)



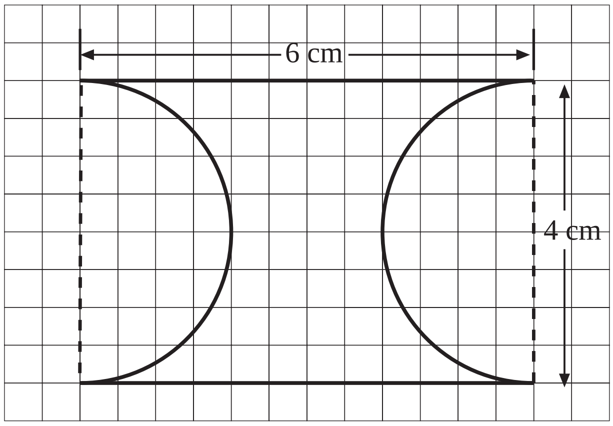
(c)



3. The following diagram shows the plan of a patio. Calculate the area of the patio.



4. Calculate the area and perimeter of the following shape:



5. A Christmas decoration consists of a disc with two holes cut in it, as shown.

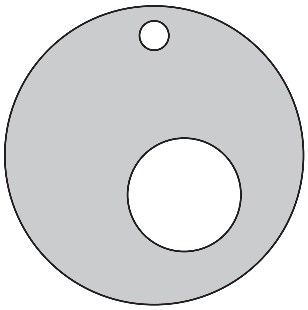
The disc has radius 3.8 cm.

The large hole has radius 1.2 cm.

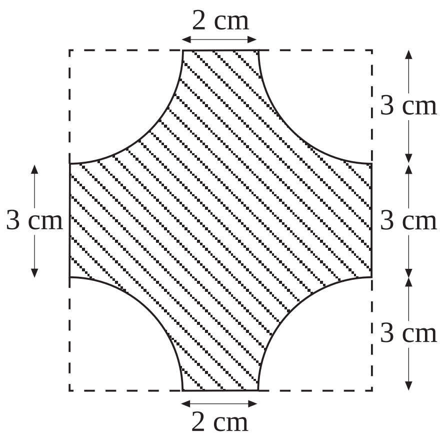
The small hole has radius 0.2 cm.

Both sides of the decoration are painted.

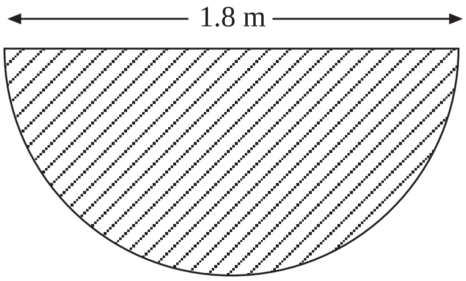
Calculate the area that is painted.

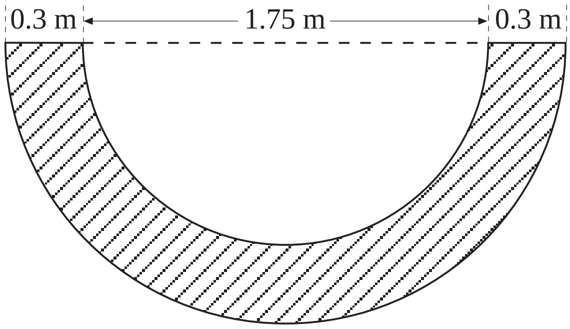


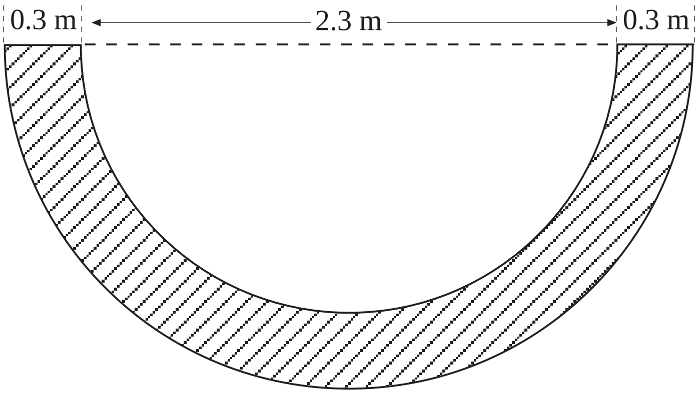
6. Calculate the area and perimeter of the shape shown:



7. A set of steps is to be built with a semicircular shape. Three of the steps are shown in the following diagrams. Calculate the area of each of these three steps.







8. A car wheel has radius 0.25 m. How far does the car travel if the wheel goes round:

   (a) 10 times,

   (b) 600 times?

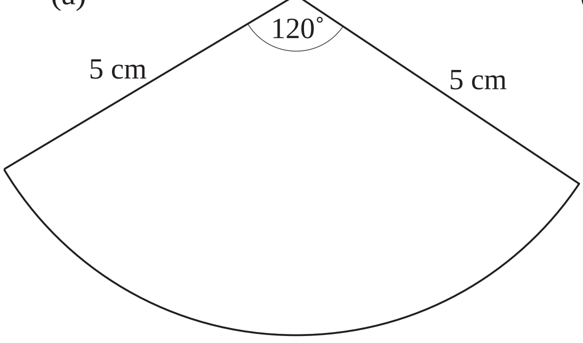
9. A wheel of a bicycle has diameter 60 cm. How many times does the wheel revolve on a journey of length:

   (a) 500 m,

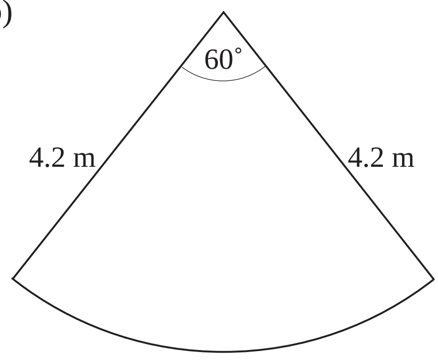
   (b) 2.6 km?

10. Calculate the area and perimeter of the following shapes:

   (a)



   (b)

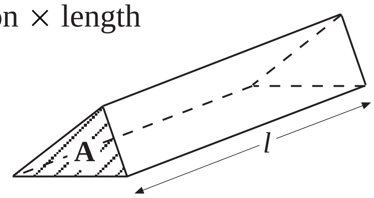


16.6 Volume and Surface Area of a Cylinder

In an earlier unit you will have considered the volume of a *triangular prism*. The formula used for this can be applied to determine the volume of a *cylinder*.

Volume of prism = area of cross-section $\times$ length

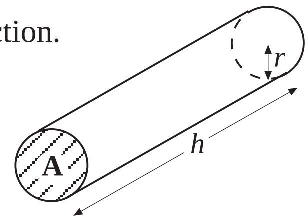
$= Al$



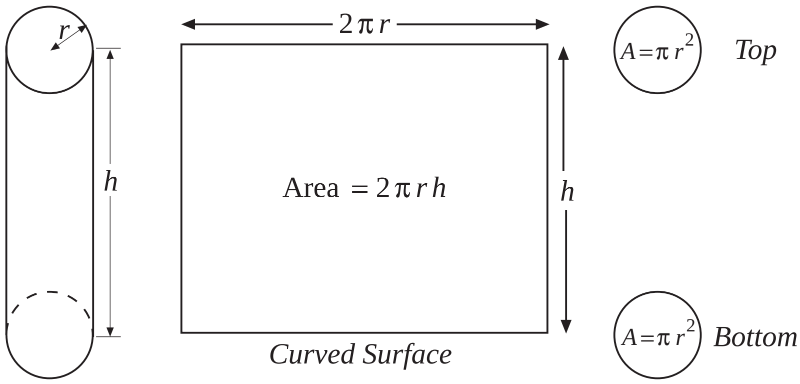
A cylinder is a prism with a circular cross-section.

Volume of cylinder $= A \times h$

$= \pi r^2 h$



The total surface area of the cylinder can be determined by splitting it into 3 parts as below:



The curved surface can be opened out to form a rectangle. The length of one side is equal to the height, *h*, of the cylinder; the other is equal to the circumference of the cross-section, $2\pi r$.

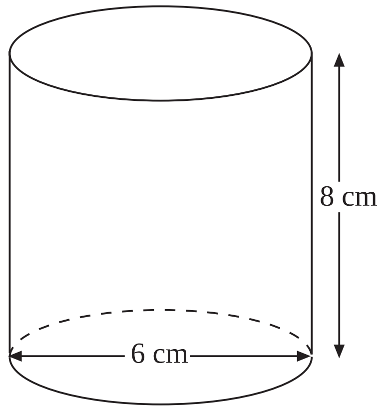
Total area = area of curved surface + area of top + area of bottom

$= 2\pi r h + \pi r^2 + \pi r^2$

$= 2\pi r h + 2\pi r^2$

Example 1

Calculate the volume and surface area of the cylinder shown in the diagram.



Solution

The radius of the base of the cylinder is 3 cm.

$$\text{Volume} = \pi r^2 h$$
$$= \pi \times 3^2 \times 8$$
$$= 226 \text{ cm}^3 \text{ (3 s.f.)}$$

$$\text{Surface area} = 2\pi r h + 2\pi r^2$$
$$= 2 \times \pi \times 3 \times 8 + 2 \times \pi \times 3^2$$
$$= 207 \text{ cm}^2 \text{ (3 s.f.)}$$

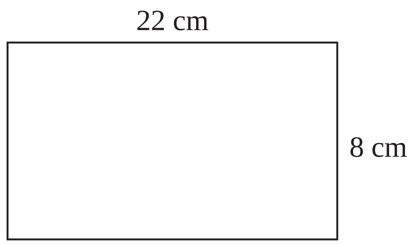
Example 2

The diagram shows a sheet of card that is to be used to make the curved surface of a cylinder of height 8 cm.

(a) Calculate the radius of the cylinder.

(b) Use your answer to part (a) to calculate the area of card that would be needed to make ends for the cylinder.

(c) Calculate the volume of the cylinder.



Solution

(a) The circumference of the cross-section is 22 cm, so

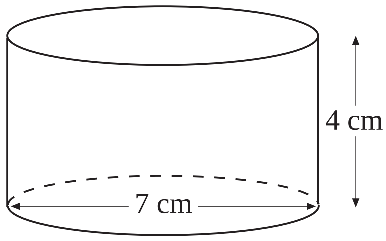
$$2\pi r = 22$$
$$r = \frac{22}{2\pi}$$
$$= \frac{11}{\pi}$$
$$= 3.50 \text{ cm (3 s.f.)}$$

(b) $$\text{Area of ends} = 2 \times \pi r^2$$
$$= 2 \times \pi \times 3.50^2$$
$$= 77.0 \text{ cm}^2 \text{ (3 s.f.)}$$

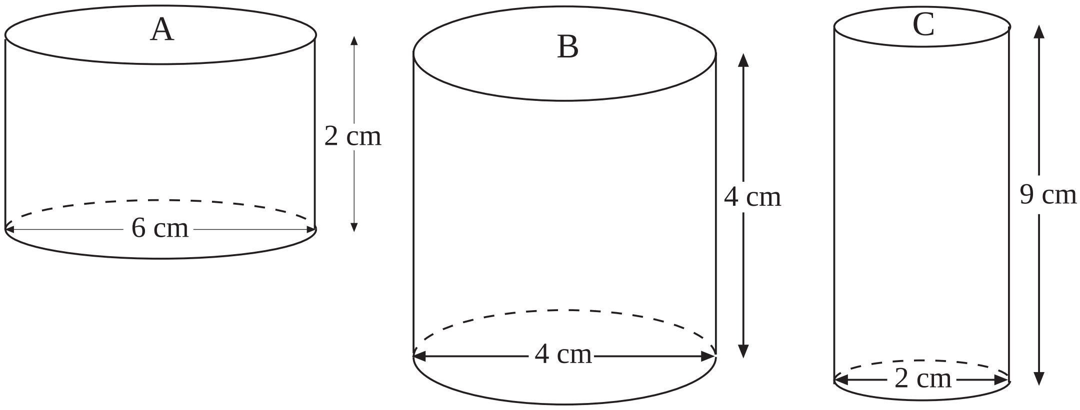
(b) $$\text{Volume of cylinder} = \pi r^2 h$$
$$= \pi \times 3.5^2 \times 8$$
$$= 308 \text{ cm}^3 \text{ (3 s.f.)}$$

Exercises

1. Calculate the volume of the cylinder shown.



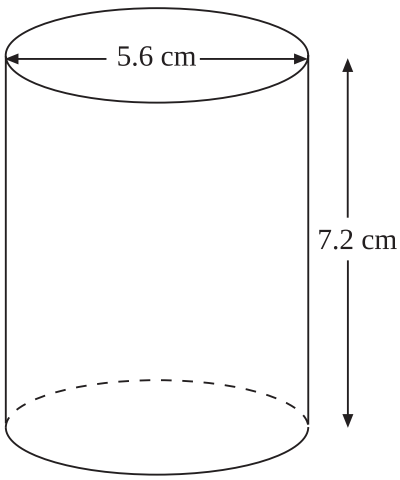
2. Look at the dimensions of the following cylinders:



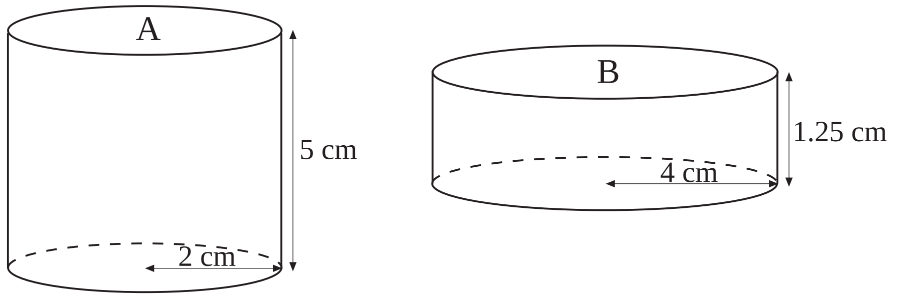
   (a) Without doing any calculations, decide which cylinder you think has the greatest volume.

   (b) Determine the volume of each cylinder and see if you were correct.

3. Calculate the total surface area of the following cylinder:



4. The following diagrams show two cylinders, A and B:



(a) Show that both cylinders have the same volume.

(b) Calculate the total surface area of each cylinder.

5. A cylinder has volume 250 cm^3 and base radius 6 cm.

(a) Calculate the height of the cylinder.

(b) Calculate the total surface area of the cylinder.

6. A cylinder has volume 300 cm^3 and height 9 cm. Calculate the diameter of the cylinder.

7. The curved surface of a cylinder is to be made from a rectangular sheet of material which is 18 cm by 32 cm.

(a) Explain why two different cylinders could be made from this sheet.

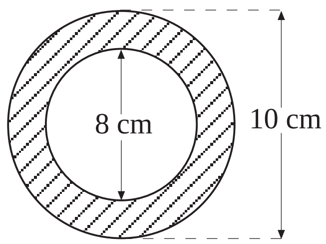
(b) Calculate the radius of each of the cylinders.

(c) Calculate the volume of each cylinder.

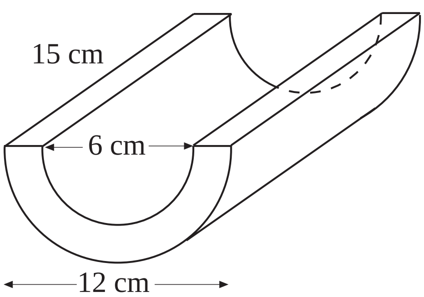
8. A cylinder has height 11 cm. The area of the curved surface of the cylinder is 40 cm^2. Calculate the volume of the cylinder.

9. The diagram shows the cross-section of a clay pipe. The length of the pipe is 40 cm.

Calculate the volume of clay needed to make the pipe.



10.



Calculate the volume and the total surface area of the shape shown.