13 Searching for Pattern

13.1 Pictorial Logic

In this section we will see how to continue patterns involving simple shapes.

Example

Continue these patterns by drawing the next 5 shapes in each case:

(a) 

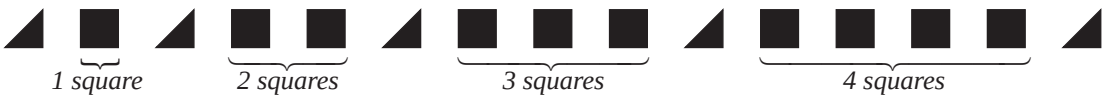
(b) 

Solution

(a) This pattern consists of four shapes that repeat in the same order. The repeating pattern is:

The pattern can now be extended:

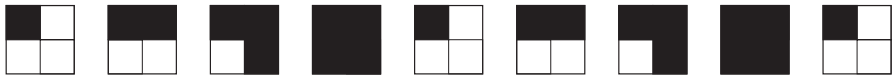
(b) This pattern consists of increasing numbers of squares separated by triangles. The pattern can be extended by adding 4 squares and another triangle:



Exercises

1. Add the next 5 shapes to each of the repeating patterns below:

(a) 

(b) 

(c) 

(d) 

2. Add the next 5 shapes to each of these patterns:

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

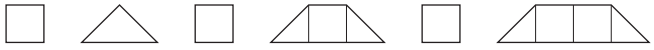
(h) 

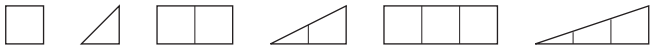
3. Extend each pattern until you obtain a shape that is 5 squares long:

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

4. Consider the pattern of shapes shown below:

(a) What is the 3rd shape in the pattern?

(b) What is the 15th shape in the pattern?

(c) What is the 30th shape in the pattern?

(d) What is the 31st shape in the pattern?

5. Consider this pattern of shapes:

(a) Draw the 8th shape.

(b) Draw the 20th shape.

(c) Draw the 21st shape.

(d) Draw the 19th shape.

6. Consider this pattern of shapes:

(a) What is the 11th shape?

(b) What is the 21st shape?

(c) What is the 41st shape?

(d) How long is the 4th shape?

(e) How long is the 6th shape?

(f) How long is the 20th shape?

7. Look at this pattern:

(a) Draw the 4th and 8th shapes.

(b) Draw the 16th shape.

(c) Draw the 17th shape.

(d) Draw the 40th shape.

(e) Draw the 38th shape.

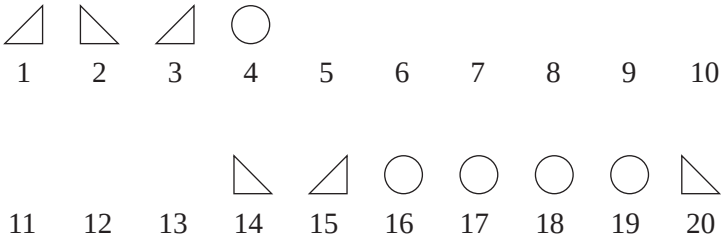
8. The diagram shows the 5th to 14th shapes in a pattern:

Draw and label the first 4 shapes in the correct order.

9. The diagram shows the 6th to 15th shapes of a pattern:

Draw and label the first 5 shapes of the pattern.

10. Fill in the missing shapes in this pattern. There should be one shape for each number.

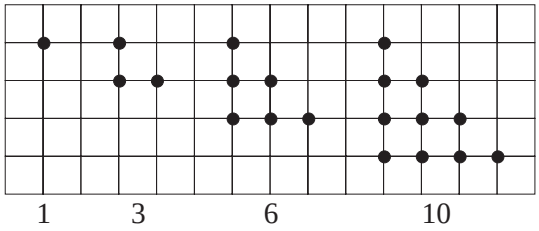


13.2 Extending Number Sequences

You will have studied some sequences in Unit 7. This section takes these ideas further and introduces some other types of sequences.

Example

The first 4 triangular numbers are represented by the diagrams below:

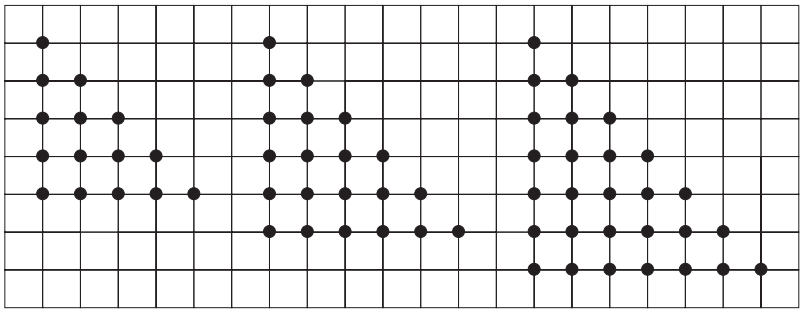


(a) Draw the next 3 triangular numbers.

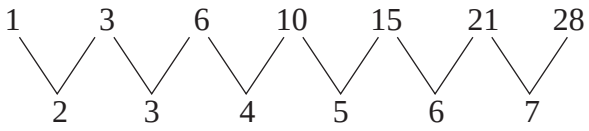
(b) Describe how to find the 8th, 9th and 10th triangular numbers without drawing the diagrams.

Solution

(a) Note that an extra row of dots is added to each triangle and that the extra row has one more dot than the previous row. The next 3 triangular numbers are shown below:



(b) To extend the sequence of triangular numbers, look at the difference between the terms:



Note that the difference between each term increases by 1 as you move along the sequence.

So,

$$\begin{aligned} 8\text{th term} &= 28 + 8 \\ &= 36 \end{aligned}$$

$$\begin{aligned} 9\text{th term} &= 36 + 9 \\ &= 45 \end{aligned}$$

$$\begin{aligned} 10\text{th term} &= 45 + 10 \\ &= 55 \end{aligned}$$

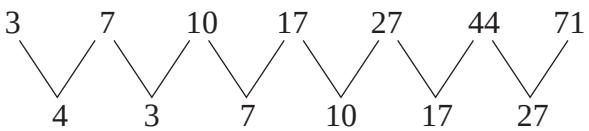
Example

Write down the next 3 terms of the sequence:

$$3,\ 7,\ 10,\ 17,\ 27,\ 44,\ 71,\ \ldots$$

Solution

Look at the differences between each term:



The first difference is not very helpful, but then note how the sequence of differences is the same as the original sequence.

For example, $10 + 7 = 17$

To find the next 5 terms:

$$\begin{aligned} \text{8th term} &= 71 + 44 \\ &= 115 \\ \text{9th term} &= 115 + 71 \\ &= 186 \\ \text{10th term} &= 186 + 115 \\ &= 301 \end{aligned}$$

In this type of sequence, called a *Fibonacci* sequence, each term is the sum of the two previous terms. For example, this sequence begins:

3, 7, 10 where $3 + 7 = 10$

and the next term is $10 + 7 = 17$.

Triangular Numbers	1, 3, 6, 10, 15, 21, 28, . . .
Square Numbers	1, 4, 9, 16, 25, 36, 49, . . .
Cubic Numbers	1, 8, 27, 64, 125, . . .
Fibonacci Sequence	1, 1, 2, 3, 5, 8, 13, . . .
(formed by adding the two previous terms to get the next one)	

Exercises

1. Write down the next 4 terms of each of these sequences:

   (a) 4, 7, 10, 13, 16, 19, . . .

   (b) 5, 11, 17, 23, 29, 35, . . .

   (c) 6, 8, 11, 15, 20, 26, . . .

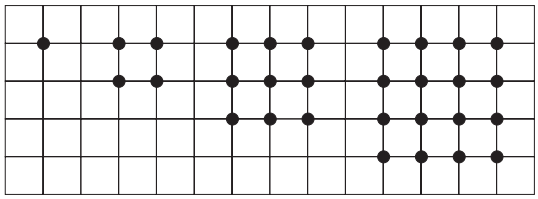
   (d) 8, 10, 14, 20, 28, 38, . . .

   (e) 24, 23, 21, 18, 14, 9, . . .

   (f) 2, 12, 21, 29, 36, 42, . . .

   (g) 1, 1, 2, 4, 7, 11, . . .

2. The diagram shows the first 4 square numbers:



(a) Draw the next 2 square numbers, and write their actual value underneath.

(b) What is the 10th square number?

(c) What is the 20th square number?

(d) Find the differences between each of the first 6 square numbers in turn. What would be the difference between the 6th and 7th square numbers? Check that your answer is correct by drawing the 7th square number.

3. (a) Write down the next 3 terms in each of these sequences:

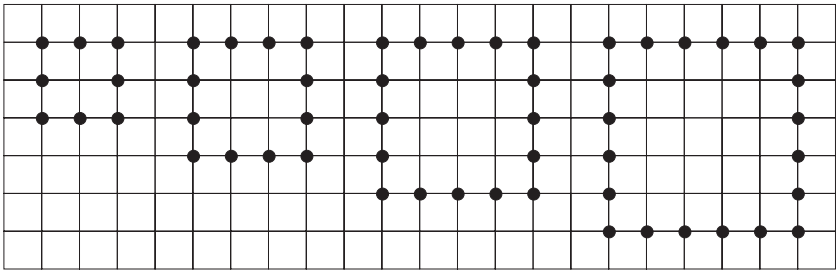
(i) 0, 3, 8, 15, 24, . . . (ii) 2, 5, 10, 17, 26, . . .

(iii) 11, 14, 19, 26, 35, . . . (iv) 6, 9, 14, 21, 30, . . .

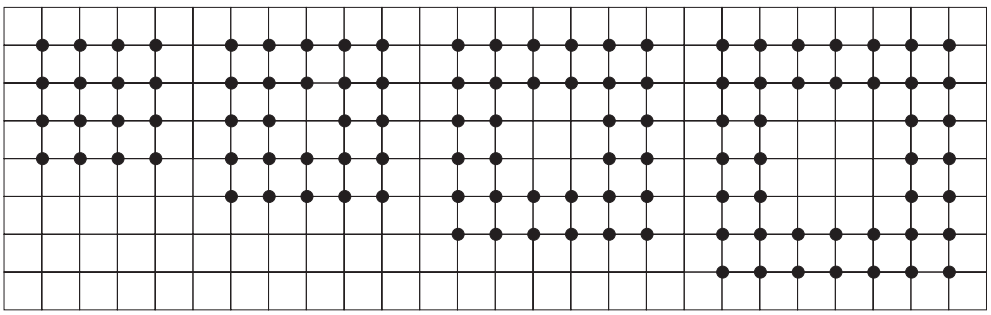
(b) In each case above, explain how the sequence is related to the sequence of square numbers 1, 4, 9, 16, 25, . . . , . . .

4. For each sequence below, draw the next two diagrams and write down the number of dots in each of the first 10 diagrams:

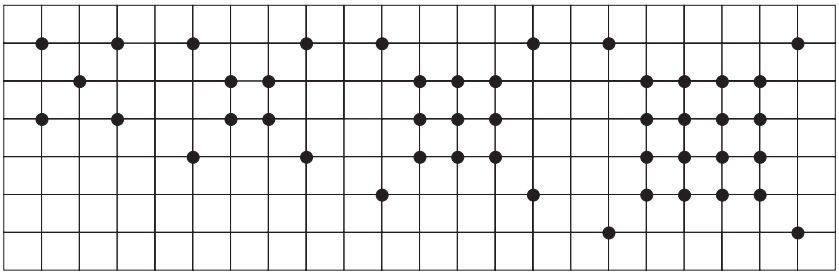
(a)



(b)

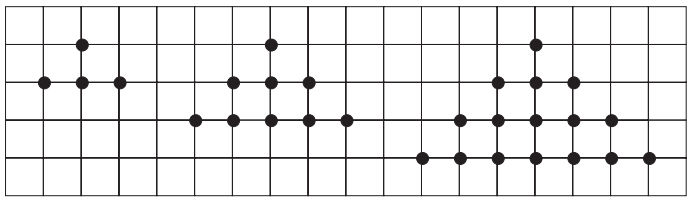


(c)

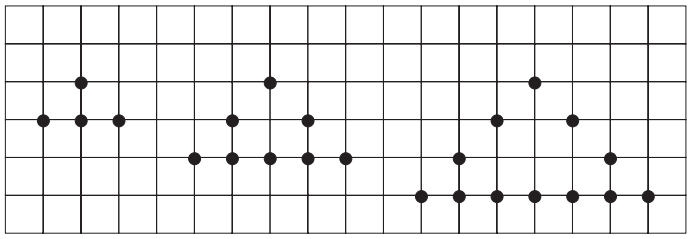


5. For each sequence below, draw the next 3 diagrams and write down the number represented by each of the first 8 diagrams:

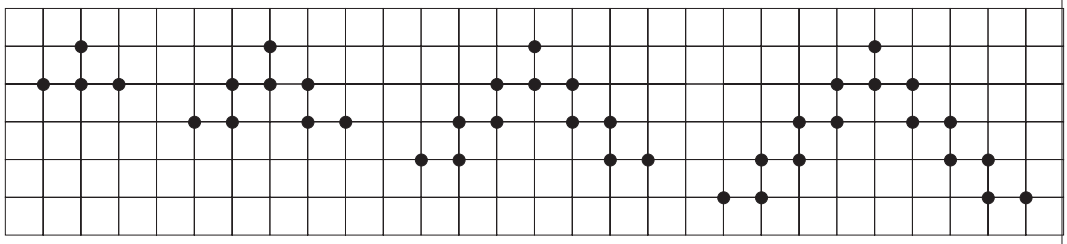
(a)



(b)

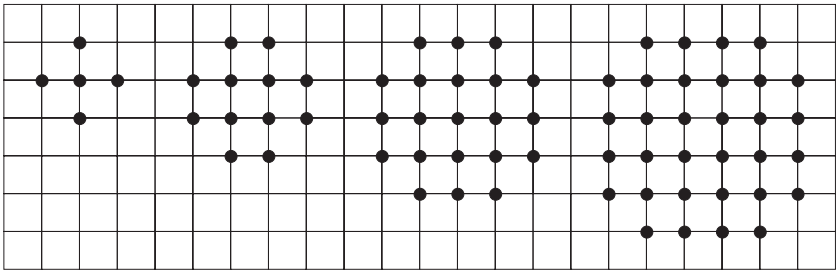


(c)

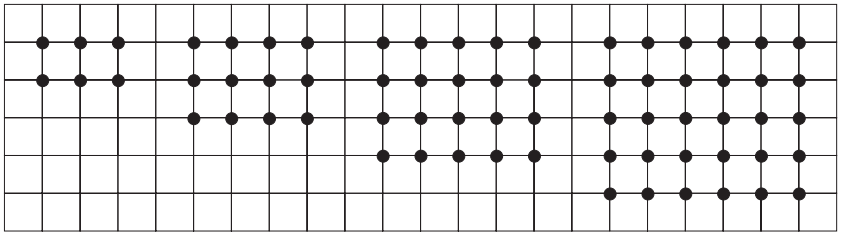


6. What number is represented by the 10th diagram in each of the sequences illustrated in the following diagrams:

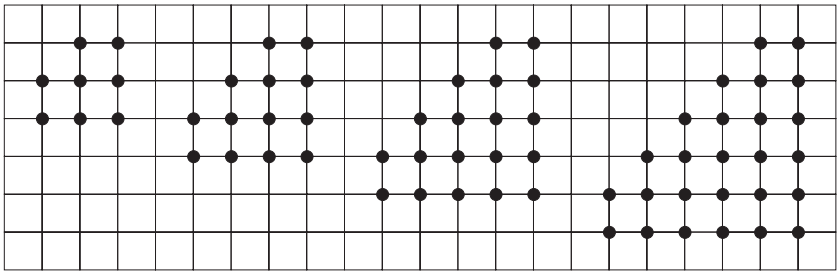
(a)



(b)



(c)



7. The Fibonacci sequence begins:

1, 1, 2, 3, 5, 8

Calculate the 10th and 20th terms in this sequence.

8. Write down the next 5 terms in each of these sequences:

(a) 2, 2, 4, 6, 10, . . .

(b) 1, 3, 4, 7, 11, . . .

(c) 2, 5, 7, 12, 19, . . .

(d) 1, 9, 10, 19, 29, . . .

9. Write down the missing terms in each sequence:

(a) ☐, ☐, 5, 9, 14, 23, 37, ☐, ☐, . . .

(b) ☐, ☐, ☐, ☐, 20, 33, 53, 86, 139, . . .

(c) ☐, ☐, ☐, ☐, 7, 11, 18, 29, 47, . . .

10. A sequence begins:

1, 2, 3, 6, 11, 20, 37, 68, . . .

(a) What do you get if you add: (i) the first three terms,

(ii) the 2nd, 3rd and 4th terms,

(iii) the 3rd, 4th and 5th terms?

(b) What are the next 3 terms in the sequence?

(c) A similar sequence is given below. Write down the missing terms.

☐, ☐, ☐, 14, 26, 48, 88, 162, . . .

(d) A sequence begins:

1, 1, 3, 5, 9, 17, 31, . . .

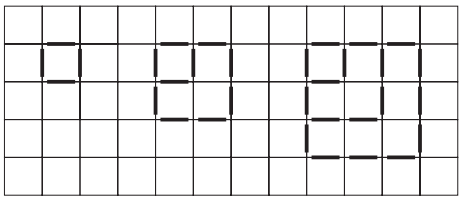
Write down the next 3 terms in the sequence.

13.3 Patterns and Matchsticks

In this section we look at forming patterns with matches, to generate sequences. We then look at how to describe these sequences.

Example 1

(a) Draw the next three shapes in this sequence:

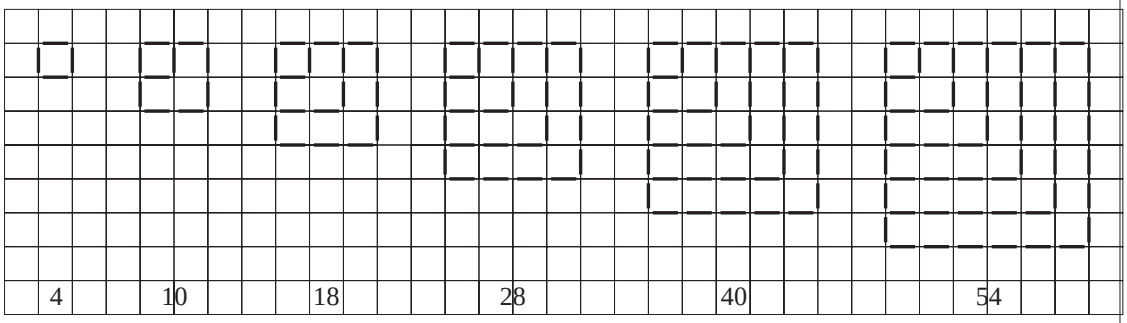


(b) How many matches are used in each shape?

(c) How many matches are used in the 10th shape?

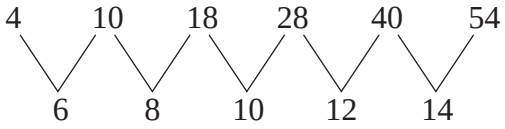
Solution

(a) Here is the sequence with the next 3 shapes:



(b) The number of matches is written under each shape.

(c) The sequence is listed here with the differences between terms:



Note how the differences increase by 2 as the sequence continues.

The 6th term is 54.

The 7th term is $54 + 16 = 60$.

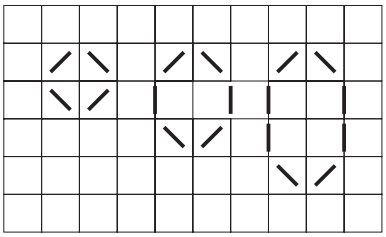
The 8th term is $60 + 18 = 78$.

The 9th term is $78 + 20 = 98$.

The 10th term is $98 + 22 = 120$.

Example 2

The diagram shows the first 3 shapes in a pattern made from matches:



(a) Draw the next 3 shapes and state how many matches are used to make each shape.

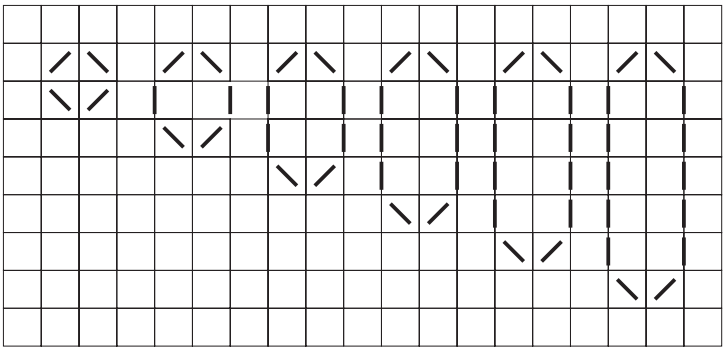
(b) Write down the 10th and 20th terms in this sequence.

(c) What is the *n*th term in this sequence?

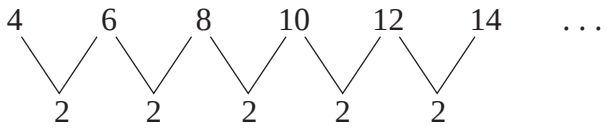
(d) One shape needs 20 matches. Which one is it?

Solution

(a) The diagram shows the next 3 shapes:



The number of matches in each shape is listed below:



Notice that the difference between each term is 2.

(b) Note that:

1st term $4 = 2 + 2 \times 1$

2nd term $6 = 2 + 2 \times 2$

3rd term $8 = 2 + 2 \times 3$

4th term $10 = 2 + 2 \times 4$

So to find the 10th term,

$$2 + 2 \times 10 \quad = \quad 22$$

and the 20th term,

$$2 + 2 \times 20 \quad = \quad 42$$

(c) The nth term is $2 + 2 \times n \; = 2 + 2n$.

(d) For the shape that needs 20 matches, we need to find the missing number in the calculation:

$$2 + 2 \times \square = 20$$

The missing number is 9.

We can write this in steps:

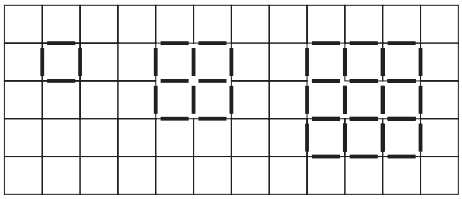
$$2 + 2 \times \square = 20$$

$$2 \times \square = 18$$

$$\square = 9$$

Exercises

1. Here is a pattern formed with matches:

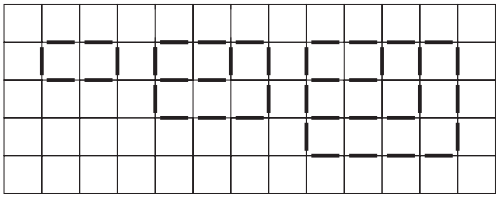


(a) Draw the next 3 shapes.

(b) How many matches are used in each of the first 6 shapes?

(c) How many matches are needed for each of the 7th and 8th shapes?

2. Here is a pattern of shapes made with matches:

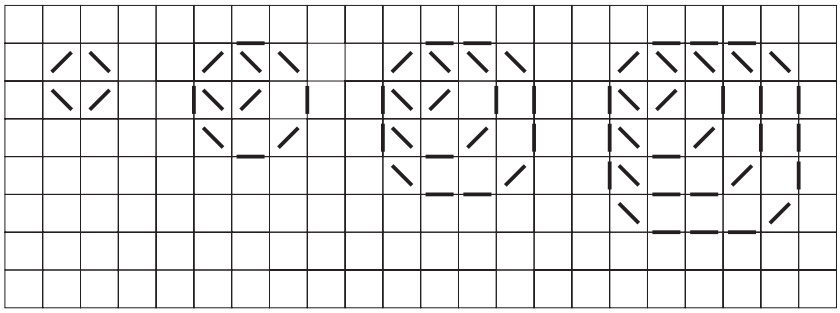


(a) Draw the next 3 shapes.

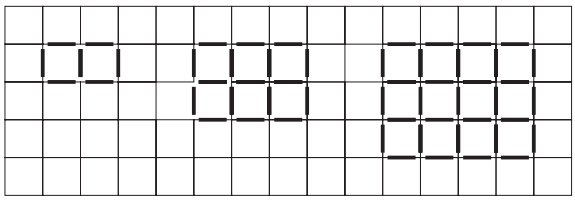
(b) How many matches are needed for the 10th shape?

(c) Which shape needs 97 matches?

3. How many matches are needed to make the 8th shape in this pattern?



4. A pattern of rectangles is made using matches:

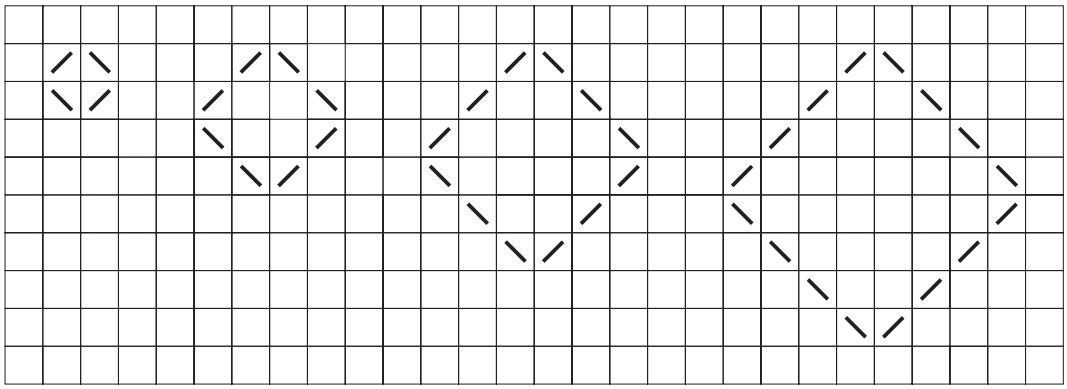


(a) Draw the next two rectangles.

(b) How many matches would be needed for the 7th rectangle?

(c) Which rectangle requires 199 matches?

5. The shapes below are made using matches:

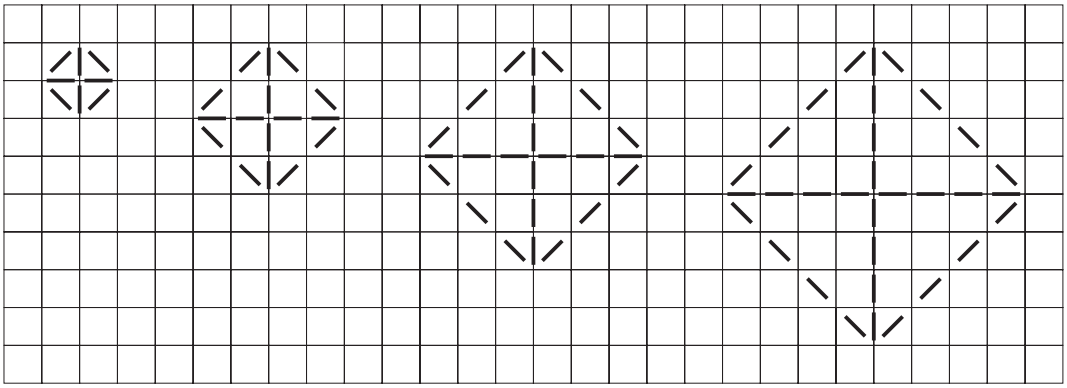


(a) How many matches would be needed for each of the 5th and 6th shapes?

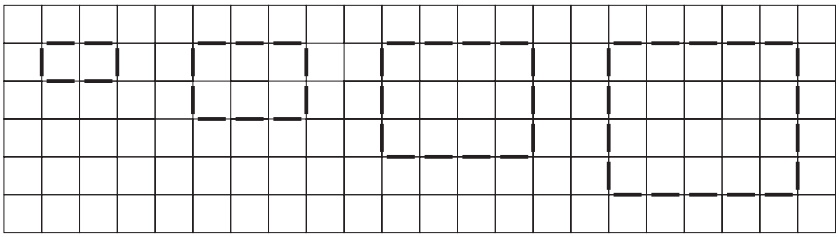
(b) How many matches would be needed for the *n*th shape?

(c) Which shape contains 88 matches?

6. How many matches would the *n*th shape in the pattern below contain?



7. A pattern of rectangles is made from matches:

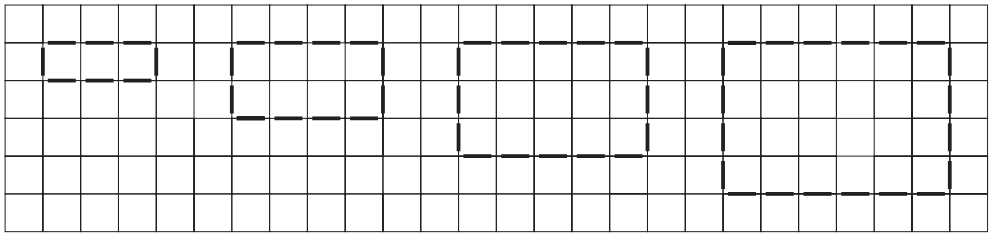


(a) How many matches are needed for the 10th rectangle?

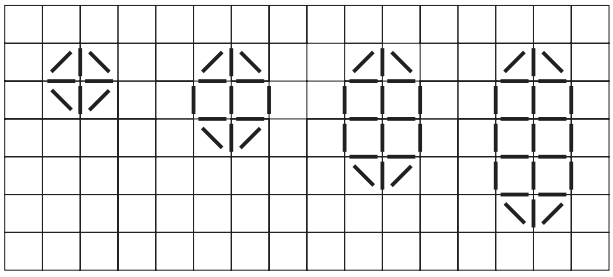
(b) How many matches are needed for the *n*th rectangle?

(c) Which rectangle requires 50 matches?

8. How many matches are needed to make the *n*th shape in the pattern of rectangles below?



9. A pattern of shapes is made from matches:



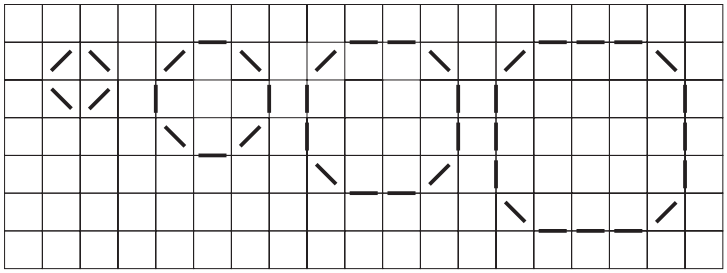
Write down:

(a) the number of matches in each of the 5th and 10th shapes,

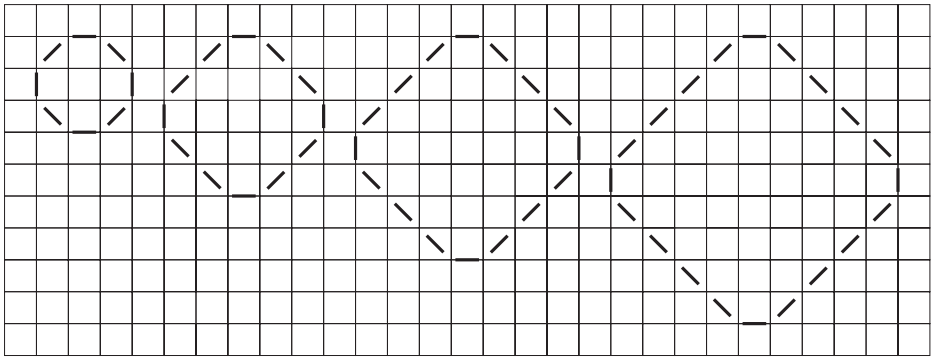
(b) the number of matches in the *n*th shape.

10. How many matches are needed to make the *n*th shape of each of these patterns?

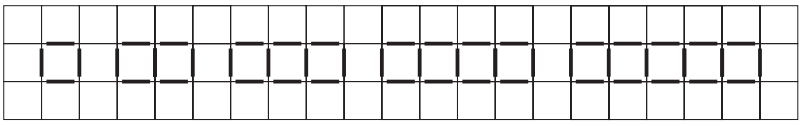
(a)



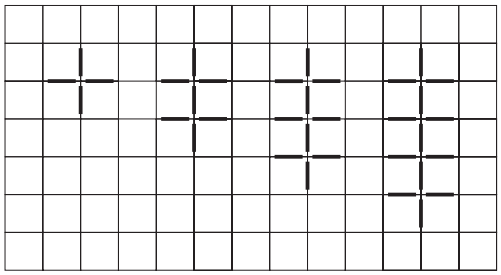
(b)



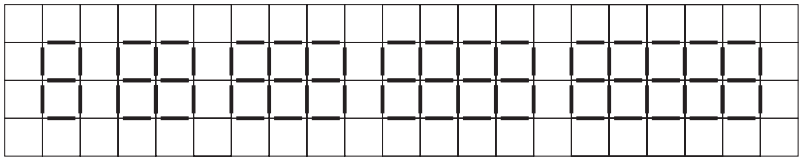
(c)



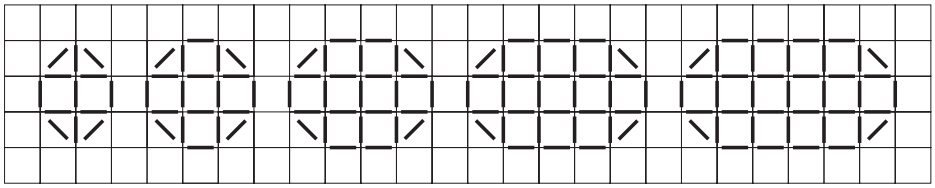
(d)



(e)



(f)



13.4 Two-Dimensional Number Patterns

This section explores 2-dimensional number patterns. One of the most famous of these is *Pascal's triangle.*

Example 1

Here are the first 4 rows of Pascal's triangle.

1

1 1

1 2 1

1 3 3 1

Write down the next 3 rows of the triangle.

Solution

Note that each row starts and ends with a 1.

1

1 1

1 2 1

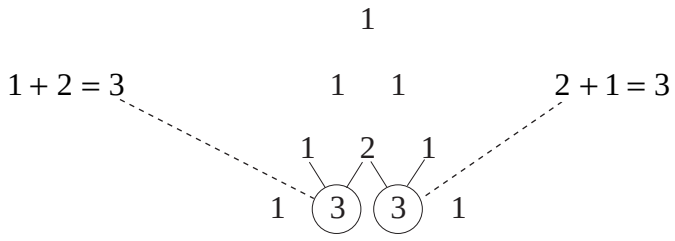
1 3 3 1

1 1

1 1

1 1

The other numbers are found by adding together the two numbers that are diagonally above them in the previous row.



Using this rule the triangle can be completed:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Example 2

What are the next 2 diagrams in the sequence:

1 , 1 1 , 1 1 1 , . . .

2 2 2

4

Solution

Note how the numbers in the next row are obtained by adding together two of the numbers in the row above.

For example, in the 2nd diagram, $1 + 1 = 2$

and in the 3rd diagram, $2 + 2 = 4$.

Using this rule, the sequence can be extended:

1 , 1 1 , 1 1 1 , 1 1 1 1 , 1 1 1 1 1 , . . .

2 2 2 2 2 2 2 2 2 2

4 4 4 4 4 4

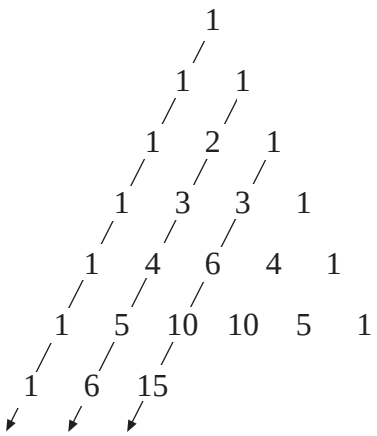
8 8 8

16

Exercises

1. In example 1, the first 7 rows of Pascal's triangle are listed. By adding the next 3 rows, write down the first 10 rows of the triangle.

2. Patterns can be found in the diagonals of Pascal's triangle. Copy the part of the triangle shown here and add the next 4 terms to the three diagonals shown.



3. Write down the next two diagrams in the sequence:

2 , 2 2 , 2 2 2 , . . .

4 4 4

8

4. Write down the next three diagrams in this sequence:

1 , 1 2 , 1 2 3 , . . .

3 3 5

8

5. Write down the next three diagrams in the sequence:

1 , 1 2 , 1 2 1 , 1 2 1 2 , . . .

3 3 3 3 3 3

6 6 6

12

6. Write down the next three diagrams in the sequence:

1 , 1 3 , 1 3 5 , . . .

4 4 8

12

7. Here are the first 5 rows of a triangle of numbers:

2

2 2

2 4 2

2 6 6 2

2 8 12 8 2

(a) Write down the next 3 rows of the triangle.

(b) Explain why the triangle contains no odd numbers.

8. Here are the first three diagrams in a sequence:

3 , 3 3 , 3 3 3 , ...

6 6 6

12

(a) What is the next diagram in the sequence?

(b) What is the largest number in the 5th diagram in the sequence?

(c) What is the largest number in the 10th diagram in the sequence?

(d) For which diagram is the largest number 384?

9. Here is a sequence of number diagrams:

1 , 1 2 , 1 2 3 , ...

2 2 6

12

Write down the next 2 diagrams in this sequence.

10. Write down the next 3 diagrams in this sequence:

1 , 1 2 , 1 2 3 , 1 2 3 4 , 1 2 3 4 5 ,

3 3 5 3 5 7 3 5 7 9

8 8 2 8 2 6

0 0 8

8

What is the largest number that will appear in any of the diagrams of this sequence?