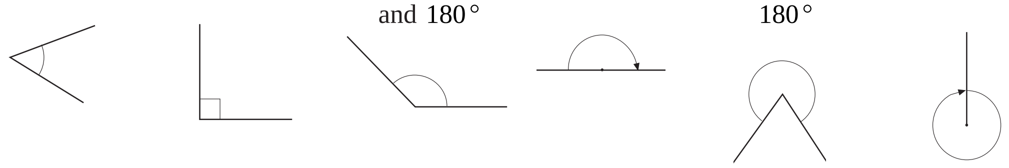
11 Angles, Bearings and Maps

11.1 Angle Measures

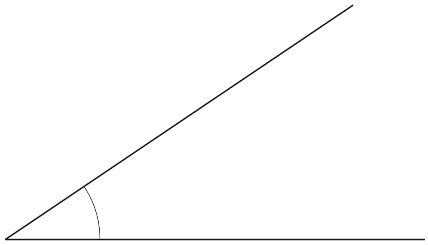
In this section we review measuring angles, and the different types of angles.

Acute angle	*Right angle*	*Obtuse angle*	*Straight line*	*Reflex angle*	*Complete turn*
less than $90°$	$= 90°$	between $90°$ and $180°$	$= 180°$	greater than $180°$	$= 360°$



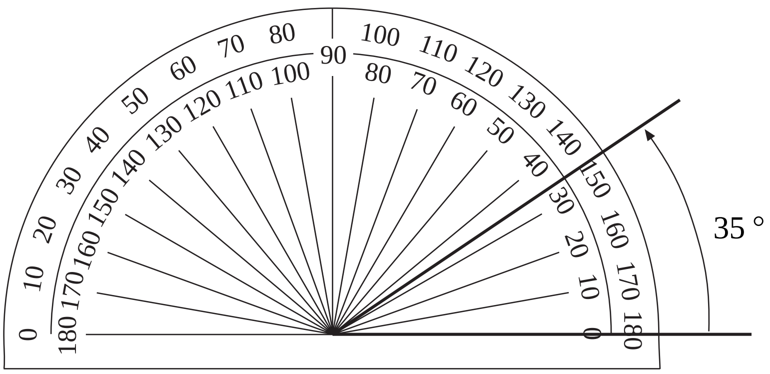
Example 1

Measure the angle in the diagram.



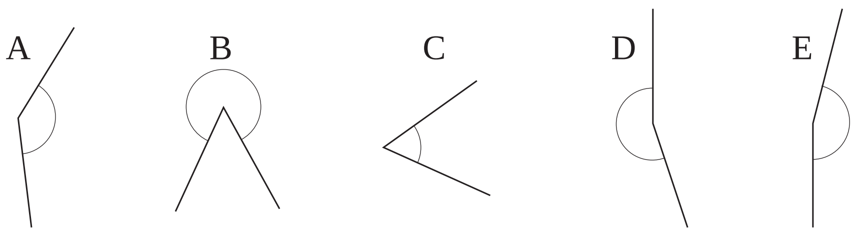
Solution

Using a protractor, the angle can be measured as $35°$.



Example 2

State whether each of the angles below is *acute, obtuse* or *reflex.*



Solution

A *Obtuse* as it is between $90°$ and $180°$.

B *Reflex* as it is greater than $180°$.

C *Acute* as it is less than $90°$

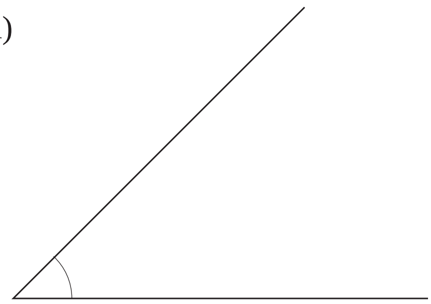
D *Reflex* as it is greater than $180°$.

E *Obtuse* as it is between $90°$ and $180°$.

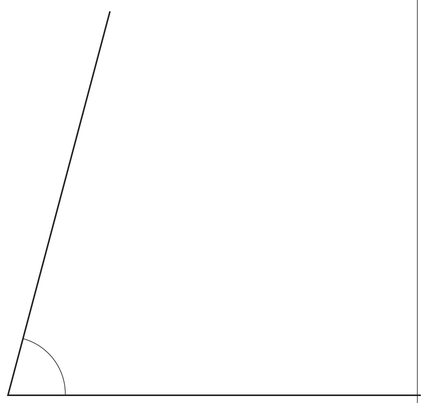
Exercises

1. Measure the following angles:

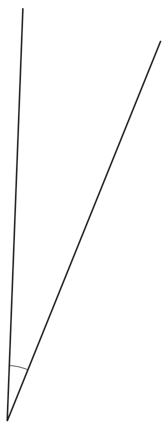
(a)



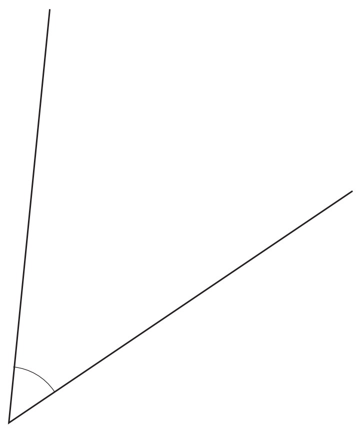
(b)



(c)

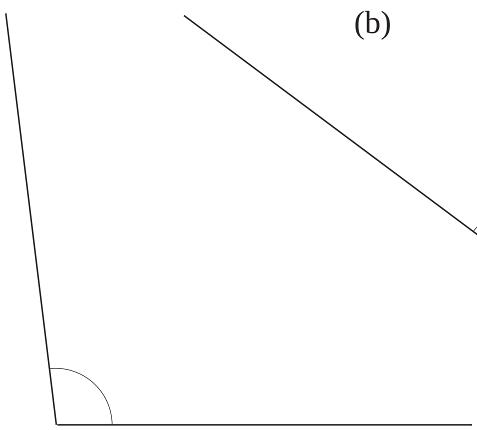


(d)

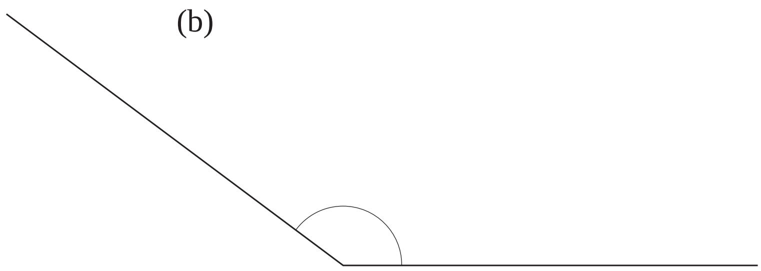


2. Measure the following angles:

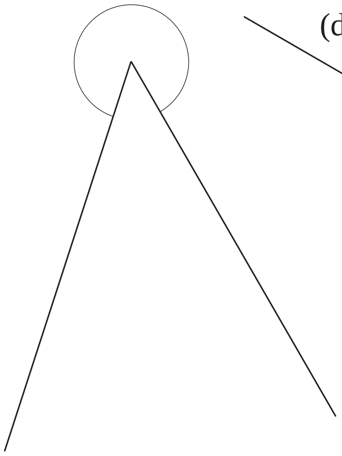
(a)



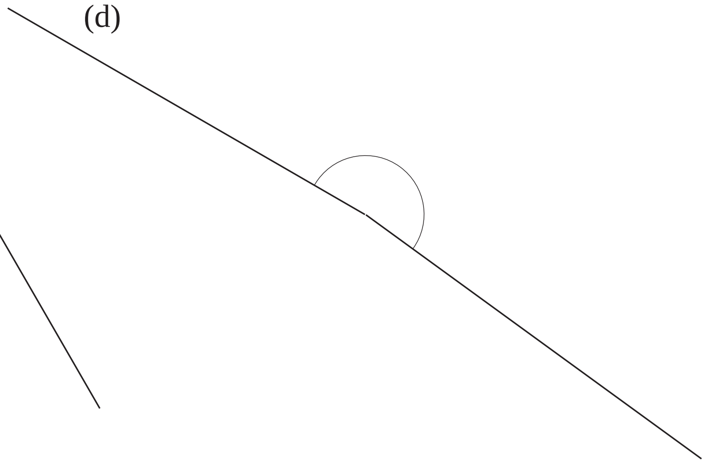
(b)



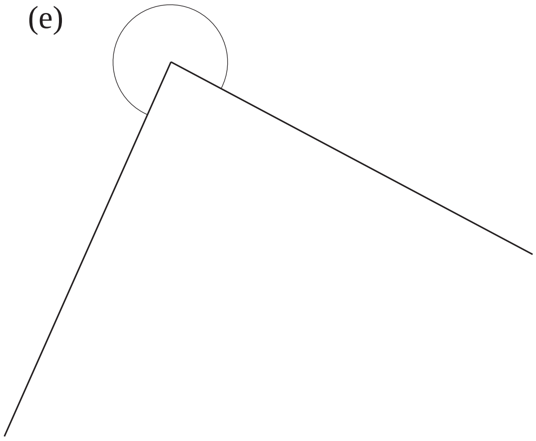
(c)



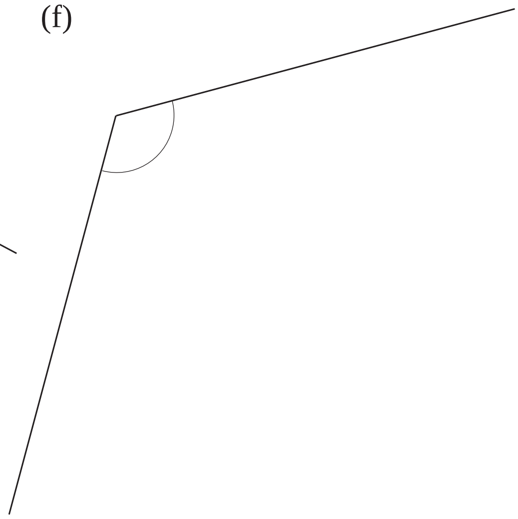
(d)



(e)

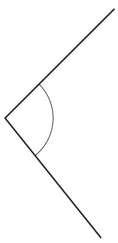


(f)

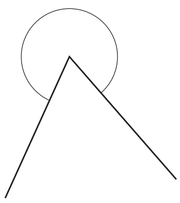


3. State whether each of the following angles is *acute, obtuse* or *reflex.*

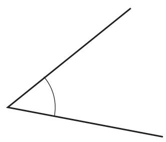
(a)



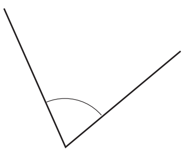
(b)



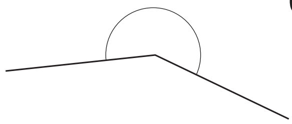
(c)



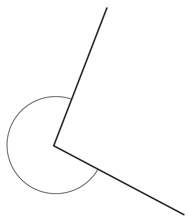
(d)



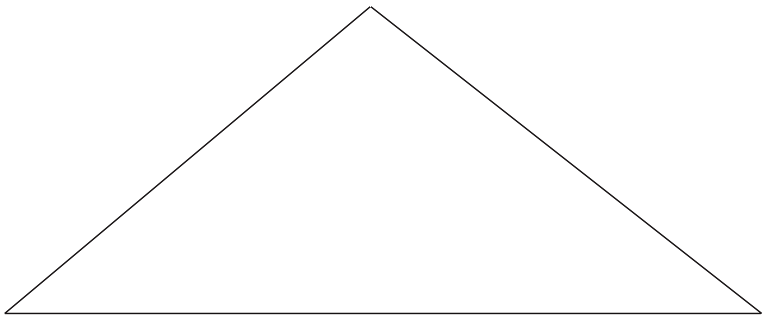
(e)



(f)



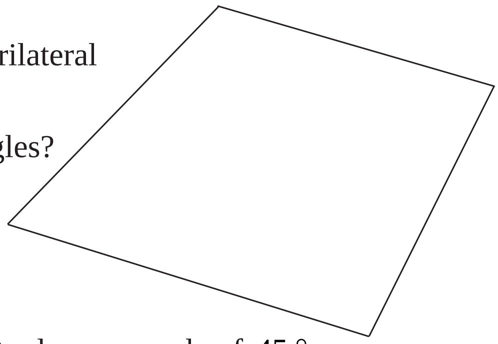
4. (a) Measure the angles in the triangle below:



(b) What is the sum of the three angles?

5. (a) Measure the angles in the quadrilateral opposite:

(b) What is the sum of the four angles?



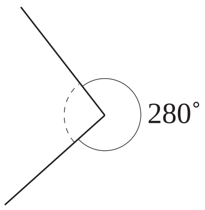
6. (a) Without using a protractor, try to draw an angle of $45°$.

(b) Measure your angle to see how accurate you were.

7. (a) Draw the angle shown in the diagram.

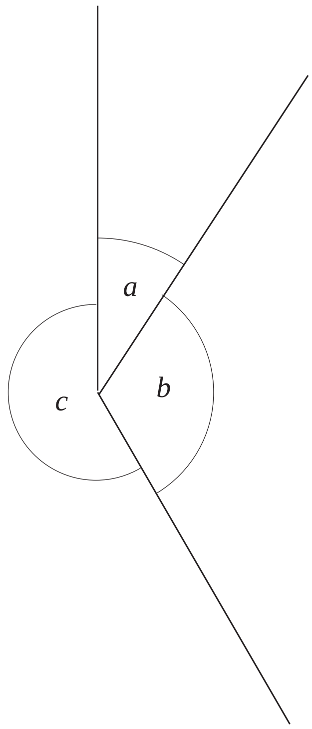
(b) Measure the acute angle that you also draw.

(c) Check that the two angles add up to $360°$.



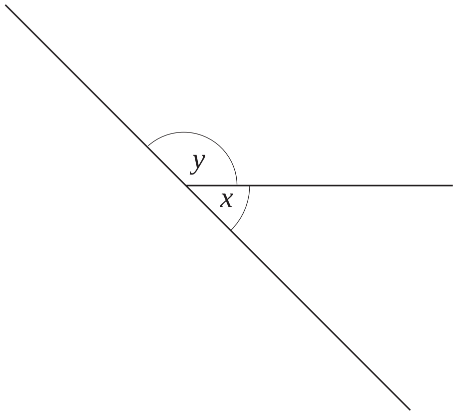
8. (a) Measure the three angles marked in the diagram.

(b) Check that they add up to $360°$.



9. (a) Measure the two angles in the diagram.

   (b) Check that they add up to $180\,°$.



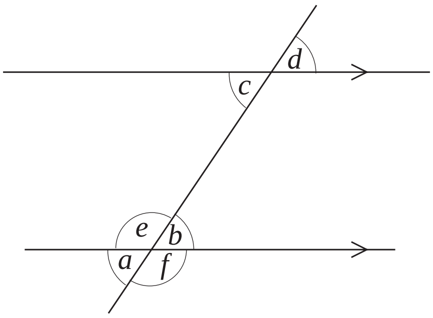
10. (a) Without using a protractor, try to draw an angle of $300\,°$.

    (b) Check your answer by measuring the angle with a protractor.

11.2 Parallel and Intersecting Lines

When a line *intersects* (or crosses) a pair of parallel lines, there are some simple rules that can be used to calculate unknown angles.

The arrows on the lines indicate that they are parallel.



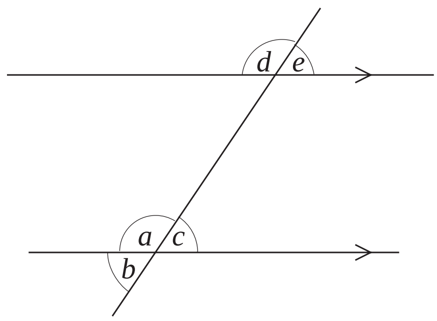
$a = b$ (and $c = d$, and $e = f$)	These are called *vertically opposite* angles.
$a = c$ (and $b = d$)	These are called *corresponding* angles.
$b = c$	These are called *alternate* angles.

$a + e = 180\,°$, because adjacent angles on a straight line add up to $180\,°$.
These are called *supplementary* angles.

Note also, that $c + e = 180\,°$ (*allied* or *supplementary* angles)

Example 1

In the diagram opposite, find the unknown angles if $a = 150°$.



Solution

To find b:

$$\begin{aligned} a + b &= 180° && \text{(angles on a straight line, supplementary angles)} \\ 150° + b &= 180° \\ b &= 30° \end{aligned}$$

To find c:

$$\begin{aligned} c &= b && \text{(vertically opposite angles } \textit{or} \text{ angles on a straight line)} \\ c &= 30° \end{aligned}$$

To find d:

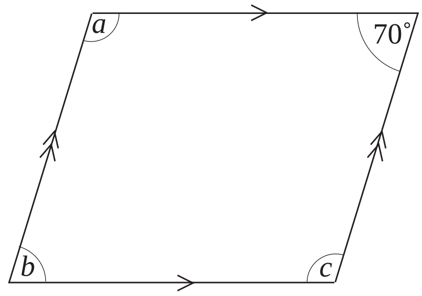
$$\begin{aligned} d &= a && \text{(corresponding angles)} \\ d &= 150° \end{aligned}$$

To find e:

$$\begin{aligned} e &= c && \text{(corresponding angles)} \\ e &= 30° \end{aligned}$$

Example 2

Find the size of the unknown angles in the parallelogram shown in this diagram:



Solution

To find a:

$$\begin{aligned} a + 70° &= 180° && \text{(allied } \textit{or} \text{ supplementary angles)} \\ a &= 110° \end{aligned}$$

To find b:

$$\begin{aligned} b + a &= 180° && \text{(allied } \textit{or} \text{ supplementary angles)} \\ b + 110° &= 180° \\ b &= 70° \end{aligned}$$

To find c:

$$c + 70° = 180° \qquad \text{(allied } or \text{ supplementary angles)}$$
$$c = 110°$$

or

$$c = 360° - (a + b + 70°) \qquad \text{(angle sum of a quadrilateral)}$$
$$= 360° - 250°$$
$$= 110°$$

or

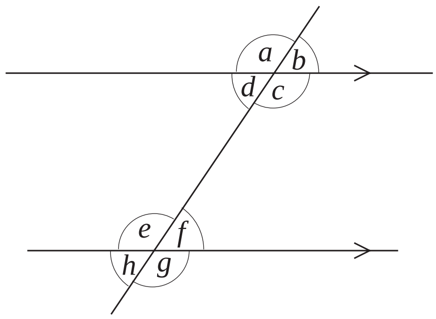
$$c = a \qquad \text{(opposite angles of a parallelogram are equal)}$$

Exercises

1. Which angles in the diagram are the same size as:

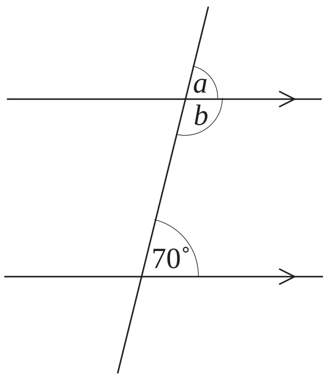
   (a) a,

   (b) b ?

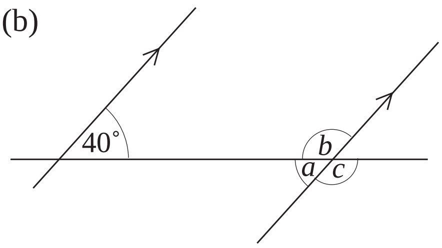


2. Find the size of each of the angles marked with letters in the diagrams below, giving reasons for your answers:

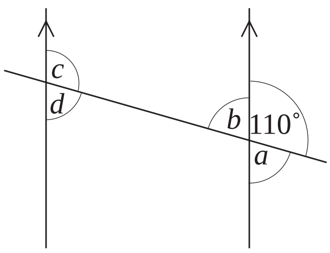
   (a)



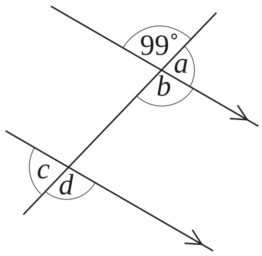
   (b)



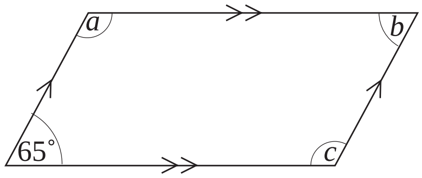
   (c)



   (d)



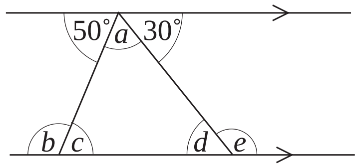
3. Find the size of the three unknown angles in the parallelogram opposite:



4. One angle in a parallelogram measures $36°$. What is the size of each of the other three angles?

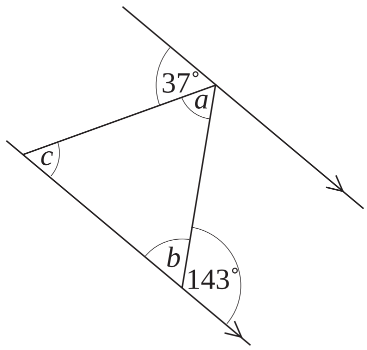
5. One angle in a rhombus measures $133°$. What is the size of each of the other three angles?

6. Find the sizes of the unknown angles marked with letters in the diagram:

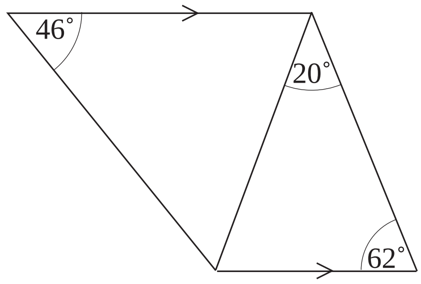


7. (a) In the diagram opposite, find the sizes of the angles marked in the triangle. Give reasons for your answers.

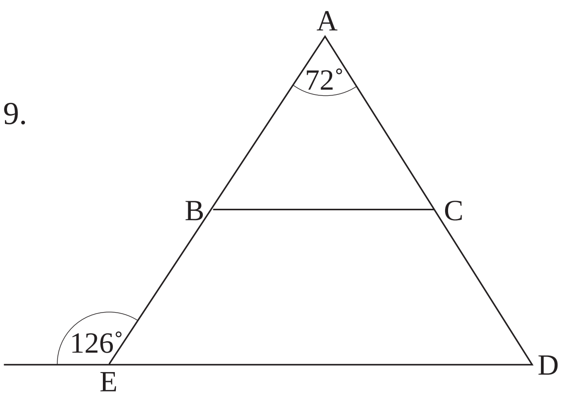
(b) What special name is given to the triangle in the diagram?



8. The diagram shows a bicycle frame. Find the sizes of the unknown angles *a*, *b* and *c*.



9.



BCDE is a trapezium.

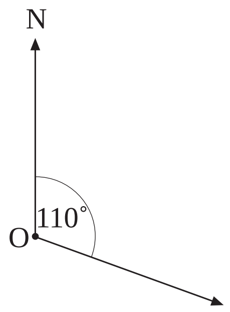
(a) Find the sizes of all the unknown angles, giving reasons for your answers.

(b) What is the special name given to this type of trapezium?

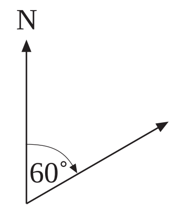
11.3 Bearings

Bearings are a measure of direction, with north taken as a reference. If you are travelling north, your bearing is 000 °.

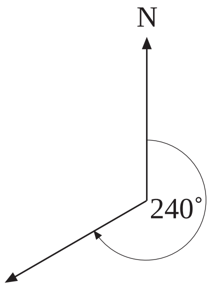
If you walk from O in the direction shown in the diagram, you are walking on a bearing of 110 °.



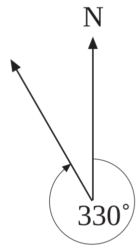
Bearings are always measured *clockwise from north*, and are given as three figures, for example:



Bearing 060 °



Bearing 240 °



Bearing 330 °

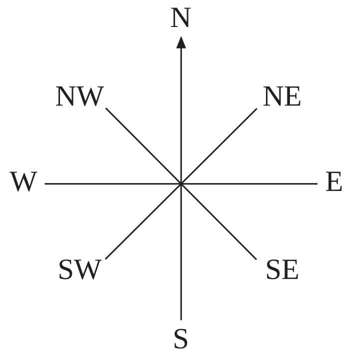
Example 1

On what bearing is a ship sailing if it is heading:

(a) E, (b) S,

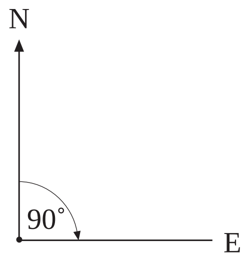
(c) W, (d) SE,

(e) NW ?



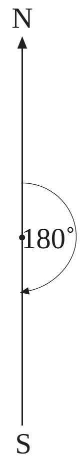
Solution

(a)



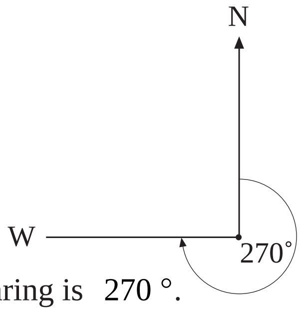
Bearing is 090 °.

(b)



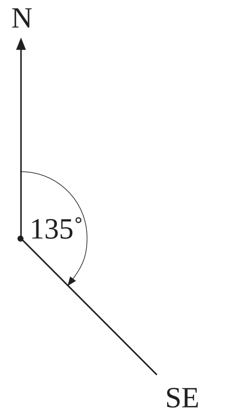
Bearing is 180 °.

(c)



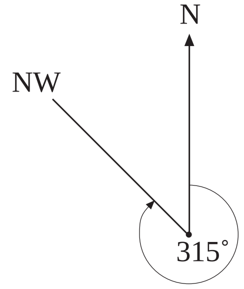
Bearing is 270 °.

(d)



Bearing is $135°$

(e)



Bearing is $315°$

Example 2

A ship sails from A to B on a bearing of $060°$. On what bearing must it sail if it is to return from B to A?

Solution

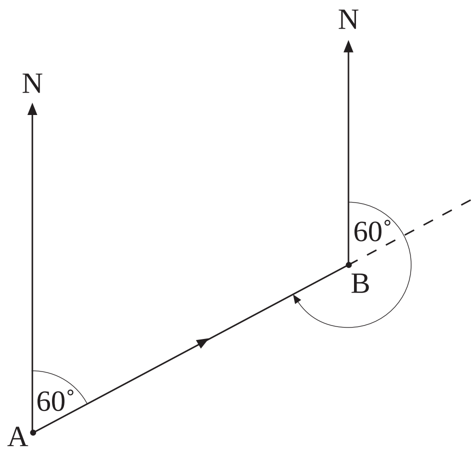
The diagram shows the journey from A to B.

Extending the line of the journey allows an angle of $60°$ to be marked at B.

Bearing of A from B $= 60° + 180°$

$= 240°$

and this is called a *back bearing* or a *reciprocal bearing.*



Exercises

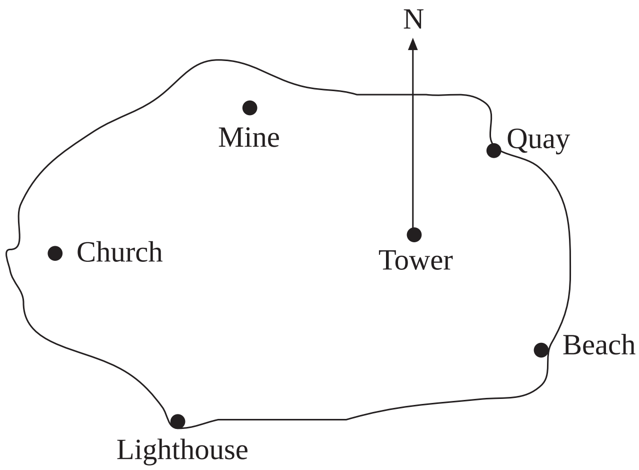
1. What angle do you turn through if you turn clockwise from:

   (a) N to S,
   (b) E to W,
   (c) N to NE,
   (d) N to SW,
   (e) W to NW ?

2. Copy and complete the table:

Direction	*Bearing*
N	
NE	
W	
SW	

3. The map of an island is shown below:

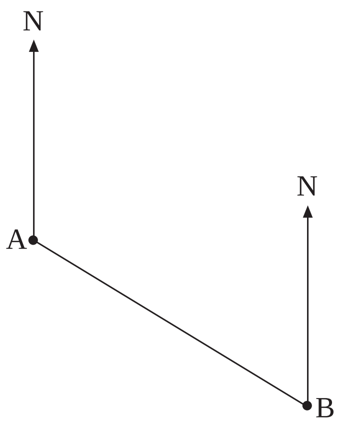


What is the bearing from the tower, of each place shown on the map?

4. The diagram shows the positions of two ships, A and B.

(a) What is the bearing of ship A from ship B ?

(b) What is the bearing of ship B from ship A ?



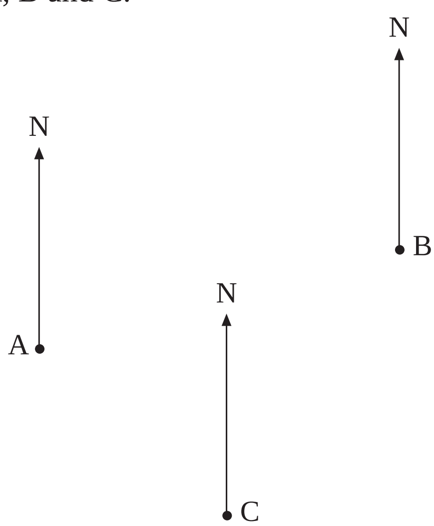
5. The diagram shows 3 places, A, B and C.
Find the bearing of:

(a) A from C,

(b) B from A,

(c) C from B,

(d) B from C.

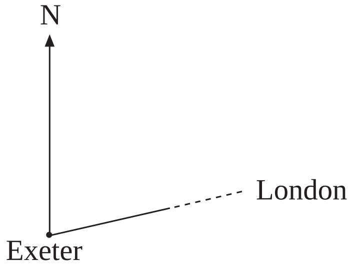


6. An aeroplane flies from Newquay to Birmingham on a bearing of $044°$. On what bearing should the pilot fly, to return to Newquay from Birmingham?

7. On four separate occasions, a plane leaves Exeter airport to fly to a different destination. The bearings of these destinations from Exeter airport are given below.

Destination	*Bearing*
London	077 °
Glasgow	356 °
Leeds	036 °
Guernsey	162 °

Copy and complete the diagram to show the direction in which the plane flies to each destination.



8. A ship sails NW from a port to take supplies to an oil rig. On what bearing must it sail to return from the oil rig to the port?

9. If A is north of B, C is southeast of B and on a bearing of 160 ° from A, find the bearing of:

(a) A from B,

(b) A from C,

(c) C from B,

(d) B from C.

10. If A is on a bearing of 300 ° from O, O is NE of B, and the bearing of B from A is 210 °, find the bearing of:

(a) A from B,

(b) O from A,

(c) O from B.

11.4 Scale Drawings

Using bearings, scale drawings can be constructed to solve problems

Example 1

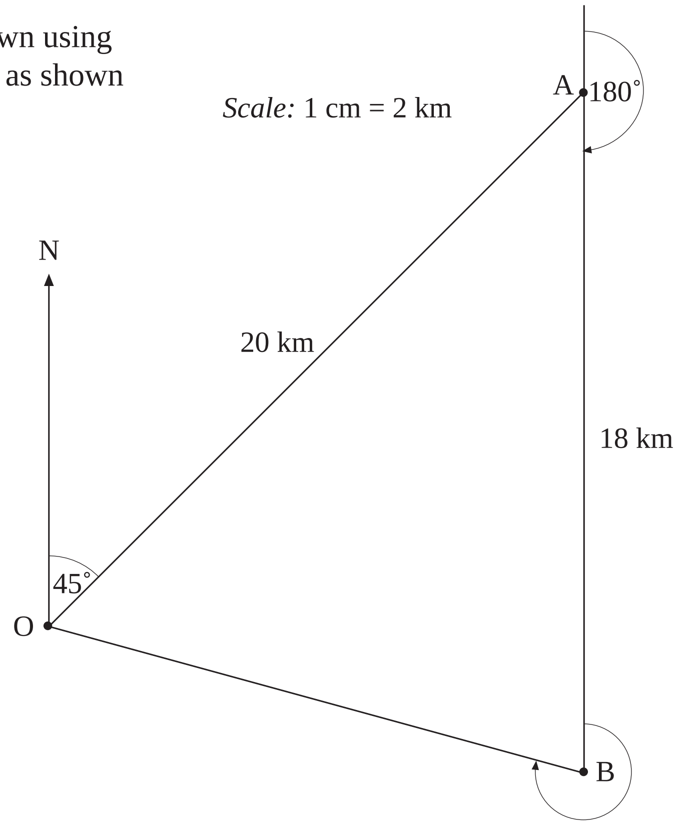
A ship sails 20 km NE, then 18 km S, and then stops.

(a) How far is it from its starting point when it stops?

(b) On what bearing must it sail to return to its starting point?

Solution

The path of the ship can be drawn using a scale of 1 cm for every 2 km, as shown in the diagram.



(a) The distance BO can be measured on the diagram as 7.3 cm which represents an actual distance of 14.6 km.

(b) The bearing of O from B can be measured as 285 °.

Note: Remember to always put the scale on the diagram.

Example 2

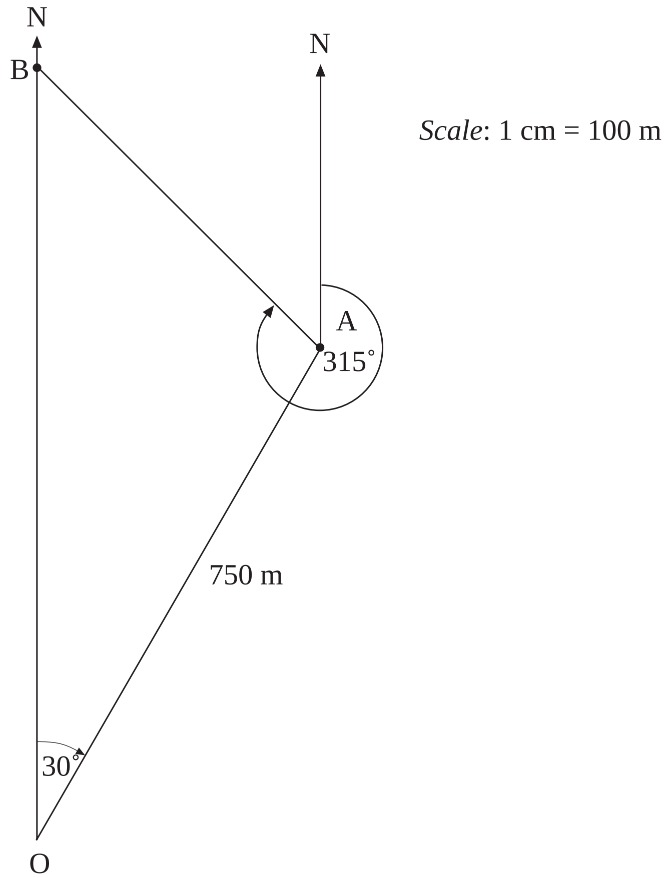
A man walks 750 m on a bearing of 030 °. He then walks on a bearing of 315 ° until he is due north of his starting point, and stops.

(a) How far does he walk on the bearing of 315 ° ?

(b) How far is he from his starting point when he stops?

Solution

A scale drawing can be produced, using a scale of 1 cm to 100 m.



(a) The distance AB can be measured as 5.4 cm, which represents an actual distance of 530 m.

(b) The distance OB can be measured as 10.2 cm, representing an actual distance of 1020 m.

Exercises

1. A girl walks 80 m north and then 200 m east.

    (a) How far is she from her starting position?

    (b) On what bearing should she walk to get back to her starting position?

2. Andrew walks 300 m NW and then walks 500 m south and then stops.

    (a) How far is he from his starting position when he stops?

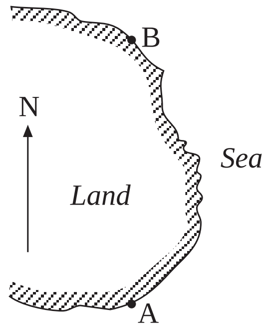
    (b) On what bearing could he have walked to go directly from his starting position to where he stopped?

3. An aeroplane flies 400 km on a bearing of $055°$ It then flies on a bearing of $300°$, until it is due north of its starting position. How far is the aeroplane from its starting position?

4. A captain wants to sail his ship from port A to port B, but the journey cannot be made directly. Port B is 50 km north of A.

   The ship sails 20 km on a bearing of $075°$.

   It then sails 20 km on a bearing of $335°$ and then drops anchor.



   (a) How far is the ship from port B when it drops anchor?

   (b) On what bearing should the captain sail the ship to arrive at port B?

5. Julie intended to walk 200 m on a bearing of $240°$. Her compass did not work properly, so she actually walked 200 m on a bearing of $225°$. What distance and on what bearing should she walk to get to the place she intended to reach?

6. A hot air balloon is blown 5 km NW. The wind then changes direction and the balloon is blown a further 6 km on a bearing of $300°$ before landing. How far is the balloon from its starting point when it lands?

7. Robin and Jane set off walking at the same time. When they start, Robin is 6 km NW of Jane. Jane walks 3 km on a bearing of $350°$ and Robin walks 4 km on a bearing of $020°$. How far apart are they now?

8. An aeroplane flies 200 km on a bearing of $335°$. It then flies 100 km on a bearing of $170°$ and 400 km on $280°$, and then lands.

   (a) How far is the aeroplane from its starting point when it lands?

   (b) On what bearing could it have flown to complete its journey directly?

9. Brian is sailing on a bearing of $135°$. After his boat has travelled 20 km, he realises that he is 1 km north of the port that he wanted to reach.

   (a) On what bearing *should* he have sailed?

   (b) How far from his starting point is the port that he wanted to reach?

10. A pilot knows that to fly to another airport he needs to fly 500 km on a bearing of $200°$. When he has flown 400 km, he realises that he is 150 km from the airport.

(a) On what bearing has the pilot been flying?

(b) On what bearing should he fly to reach the airport?

(Note that there are two answers.)

11. Four planes take off from Exeter airport, each one flying on a different bearing to another UK airport. The bearings and the distances from Exeter to these airports are given in the table below.

Destination	*Bearing*	*Distance*
London	077 °	255 km
Glasgow	356 °	575 km
Leeds	036 °	390 km
Guernsey	162 °	150 km

Using a scale of 1 cm to represent 50 km, draw a map showing the positions of the five airports.