# 10 Probability - Two Events

## 10.1 Recap: Basic Probability for One Event

In this section we revise the use of probabilities for single events, remembering that:

$$\text{Probability of an event} = \frac{\text{number of } \textit{successful} \text{ outcomes}}{\text{number of } \textit{possible} \text{ outcomes}}$$

### Example 1

A tube of sweets contains 10 red sweets, 7 blue sweets, 8 green sweets and 5 orange sweets. If a sweet is taken at random from the tube, what is the probability that it is:

(a) red,

(b) orange,

(c) green *or* red,

(d) *not* blue ?

### Solution

There are 30 sweets in the tube.

(a) There are 10 red sweets in the tube, so

$$\begin{aligned} p\,(\text{red}) &= \frac{10}{30} \\ &= \frac{1}{3} \end{aligned}$$

(b) There are 5 orange sweets in the tube, so

$$\begin{aligned} p\,(\text{orange}) &= \frac{5}{30} \\ &= \frac{1}{6} \end{aligned}$$

(c) There are 8 green sweets and 10 red sweets in the tube, so

$$\begin{aligned} p\,(\text{green or red}) &= \frac{8+10}{30} \\ &= \frac{18}{30} \\ &= \frac{3}{5} \end{aligned}$$

(d) There are 23 sweets that are *not* blue in the tube, so

$$p\,(\text{not blue}) = \frac{23}{30}$$

## Example 2

Nine balls, each marked with a number from 1 to 9, are placed in a bag and one ball is taken out at random. What is the probability that the number on the ball is:

(a) odd,

(b) a multiple of 3,

(c) a 5,

(d) not a 7 ?

## Solution

There are 9 possible outcomes in each case.

(a) There are 5 possible odd numbers, so

$$p\,(\text{odd}) = \frac{5}{9}$$

(b) There are 3 numbers that are multiples of 3, so

$$\begin{aligned} p\,(\text{multiple of 3}) &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}$$

(c) There is only 1 ball numbered 5, so

$$p\,(5) = \frac{1}{9}$$

(d) There are 8 numbers that are *not* 7, so

$$p\,(\text{not 7}) = \frac{8}{9}$$

## Exercises

1. There are 16 girls and 8 boys in the tennis club. One of these is chosen at random to enter a competition. What is the probability that a girl is chosen?

2. A bag contains 8 blue balls, 7 green balls and 5 red balls. A ball is taken at random from the bag. What is the probability that the ball is:
   (a) red,
   (b) blue,
   (c) green,
   (d) yellow?

3. A card is taken at random from a standard 52-card pack of playing cards. What is the probability that it is:
   (a) a seven,
   (b) a heart,
   (c) a red card,
   (d) a red six ?

4. If you roll a fair dice, what is the probability that the number you get is:
   (a) 5
   (b) an odd number,
   (c) a number greater than 1,
   (d) a multiple of 4 ?

5. Ishmail writes a computer program that produces at random one of the digits

   0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

   What is the probability that the program produces:
   (a) an even number,
   (b) a multiple of 4,
   (c) a number less than 7,
   (d) a multiple of 5 ?

6. The police line up 10 people in an identity parade; only one of the people is the criminal. A witness does not recognise the criminal and so chooses a person at random. What is the probability that:
   (a) the criminal is chosen,
   (b) the criminal is *not* chosen ?

7. There are 18 boys and 17 girls in a class. One of these pupils is selected at random to represent the class. What is the probability that the pupil selected is a girl?

8. In Hannah's purse there are three £1 coins, five 10p coins and eight 2p coins. If she takes a coin at random from her purse, what is the probability that it is:

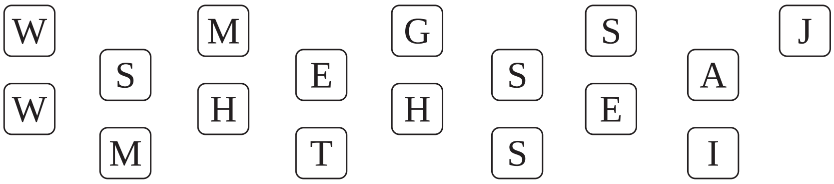
   (a) a £1 coin,

   (b) a 2p coin,

   (c) *not* a £1 coin,

   (d) a £1 coin *or* a 10p coin ?

9. Some of the children in a class write down the first letter of their surname on a card; these cards are shown below:



   (a) One of these cards is taken at random. What is the probability that the letter on it is:

      (i) W,

      (ii) S *or* T,

      (iii) J *or* M,

      (iv) *not* H

      (v) a vowel ?

   (b) Which letter is the most likely to be chosen?

10. Rachel buys a new CD, on which is her favourite track, 8 other tracks she likes and 2 tracks that she does not like. She sets her CD player to play at random. What is the probability that the first track it plays is:

   (a) Rachel's favourite,

   (b) a track that she likes,

   (c) a track that she does *not* like ?

# 10.2 Outcomes with Two Events

When two events take place at the same time, it is important to list *all* the possible outcomes in some way. There are three possible approaches: *systematic listing, using a table* or *using a tree diagram.*

## Example 1

Caitlin and Dave each buy a chocolate bar from a vending machine that sells *Aero, Bounty, Crunchie* and *Dime* bars.

List the possible pairs of bars which Caitlin and Dave can choose.

## Solution

| *Caitlin* | *Dave* |
|---|---|
| A | A |
| A | B |
| A | C |
| A | D |
| B | A |
| B | B |
| B | C |
| B | D |
| C | A |
| C | B |
| C | C |
| C | D |
| D | A |
| D | B |
| D | C |
| D | D |

A = *Aero*
B = *Bounty*
C = *Crunchie*
D = *Dime*

## Example 2

A fair dice is rolled and an unbiased coin is tossed. Draw a table to show the possible outcomes.

### Solution

*Possibilities for coin*

*Possibilities for dice*

| | | *DICE* | | | | | |
|---|---|---|---|---|---|---|---|
| | | *1* | *2* | *3* | *4* | *5* | *6* |
| *COIN* | *H* | H1 | H2 | H3 | H4 | H5 | H6 |
| | *T* | T1 | T2 | T3 | T4 | T5 | T6 |

The table shows that there are 12 possible outcomes.

## Example 3

Draw a table to show all the possible total scores when two fair dice are thrown at the same time.

### Solution

| | | *DICE B* | | | | | |
|---|---|---|---|---|---|---|---|
| | | *1* | *2* | *3* | *4* | *5* | *6* |
| *DICE A* | *1* | 2 | 3 | 4 | 5 | 6 | 7 |
| | *2* | 3 | 4 | 5 | 6 | 7 | 8 |
| | *3* | 4 | 5 | 6 | 7 | 8 | 9 |
| | *4* | 5 | 6 | 7 | 8 | 9 | 10 |
| | *5* | 6 | 7 | 8 | 9 | 10 | 11 |
| | *6* | 7 | 8 | 9 | 10 | 11 | 12 |

The table shows that there are 36 possible outcomes, and gives the total score for each outcome.

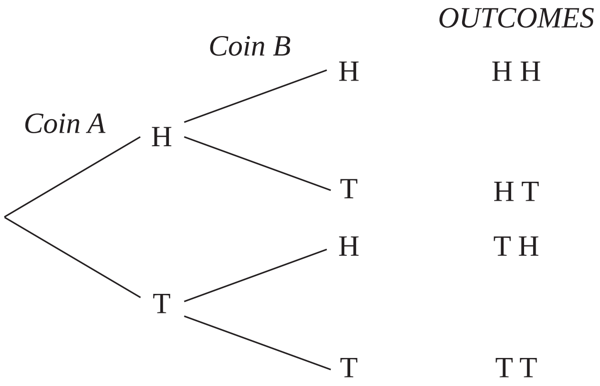
From the table it can be seen that there are 6 outcomes that give a score of 7.

## Example 4

Use a tree diagram to show the possible outcomes when two unbiased coins are tossed.

### Solution

The diagram shows that there are 4 possible outcomes.

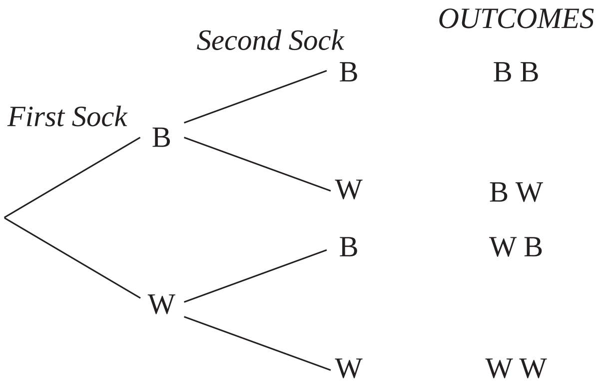


## Example 5

In a drawer there are some white socks and some black socks. Tim takes out one sock and then a second. Draw a tree diagram to show the possible outcomes.

## Solution

There are four possible outcomes, of which two will will produce two socks of the same colour.



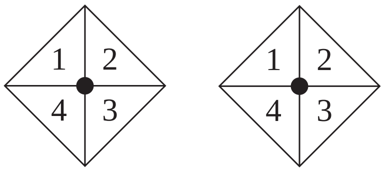
## Exercises

1. Copy and complete the table to show all possible outcomes when 2 fair coins are tossed.

| | | *COIN B* | |
|---|---|---|---|
| | | *H* | *T* |
| *COIN A* | *H* | | |
| | *T* | | |

2. Two spinners are numbered 1 to 4 as shown in the diagram:

   (a) Copy and complete the table below, to show all possible outcomes when they are spun, writing the *total score* for each outcome.



| | | *SPINNER B* | | | |
|---|---|---|---|---|---|
| | | *1* | *2* | *3* | *4* |
| *SPINNER A* | *1* | | | | |
| | *2* | | | | |
| | *3* | | | | |
| | *4* | | | | |

   (b) What is the total number of possible outcomes?

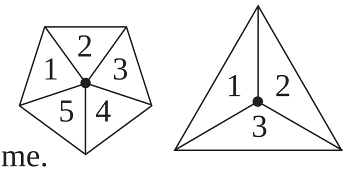
   (c) How many outcomes give a score of 5 ?

3. Two fair dice are renumbered using $-2, -1, 0, 1, 2, 3$ instead of the usual numbers. The two dice are thrown in the normal way.

   (a) Draw a table to show the total score for each of the possible outcomes.

   (b) How many ways are there of scoring 0 ?

4. The two spinners shown in the diagram opposite, are spun at the same time:



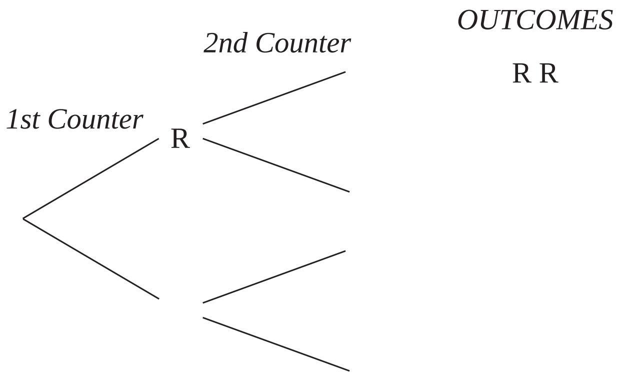
   (a) Draw a table to show all possible outcomes, and the total score for each outcome.

   (b) How many different outcomes are there?

   (c) How many outcomes give a score of 6 ?

5. In a bag there are red and blue counters. Two counters are taken out of the bag at random.

   (a) Copy and complete the tree diagram below, to show all outcomes:



   (b) How many outcomes include a *red* counter?

   (c) How many outcomes include a *blue* counter?

6. (a) Draw a tree diagram to show all possible outcomes when two unbiased coins are tossed.

   (b) Extend your tree diagram to show the possible outcomes when three unbiased coins are tossed.

   (c) How many outcomes are there when three unbiased coins are tossed?

   (d) How many outcomes are there when four unbiased coins are tossed?

7. In a jar there are three different types of sweets, *eclairs, mints* and *toffees*; two sweets are taken at random.

   (a) Draw a tree diagram to show the possible outcomes.

   (b) How many of the outcomes include a *toffee*?

   (c) How many of the outcomes include a *mint* and a *toffee*?

8. A red dice, a blue dice and a green dice are put into a bag; all the dice are fair. One is then taken out and rolled. The colour of the dice and the score shown are recorded.
   (a) How many possible outcomes are there?
   (b) How many outcomes include a 5 ?

9. In a game, two fair dice are rolled and the scores are multiplied together.
   (a) Draw a table to show the possible outcomes and their scores.
   (b) How many ways are there of scoring 12 ?
   (c) How many ways are there of scoring 18 ?

10. A bag contains a mixture of red, green and white balls. Three balls are taken at random from the bag.
    (a) Write down all possible outcomes.
    (b) How many outcomes include a red ball?
    (c) How many outcomes include a red or a white ball?
    (d) How many outcomes include a red and a green ball?

# 10.3 Probability Using Listings

When the outcomes for two events are equally likely, the probabilities of particular outcomes can be found.

## Example 1

Look at the list of chocolate bars which can be chosen by Caitlin and Dave in Example 1 of section 10.2. What is the probability that they both choose the *same* type of chocolate bar?

## **Solution**

There are 16 different outcomes and all are equally likely.

In 4 of these outcomes both Caitlin and Dave choose the same type of bar.

So

$$p\,(\text{same type}) = \frac{4}{16} \quad \text{or} \quad \frac{1}{4}$$

## Example 2

When two unbiased coins are tossed, determine the probability of obtaining:

(a) two heads,

(b) two tails,

(c) a head and a tail.

## Solution

The table shows the possible outcomes:

In this situation there are 4 outcomes that are equally likely.

|  | *H* | *T* |
|---|---|---|
| *H* | H H | H H |
| *T* | T H | T H |

(a) Here 1 of the 4 outcomes gives 2 heads, so

$$p\,(\text{2 heads}) = \frac{1}{4}$$

(b) Here 1 of the 4 outcomes gives 2 tails, so

$$p\,(\text{2 tails}) = \frac{1}{4}$$

(c) Here 2 of the outcomes gives a head and a tail, so

$$p\,(\text{head and a tail}) = \frac{2}{4}$$

$$= \frac{1}{2}$$

## Example 3

Two fair dice are rolled at the same time. What is the probability that the total score is:

(a) 6,

(b) greater than 9,

(c) less than 7 ?

## Solution

The table show the possible outcomes.

There are 36 equally likely scores.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

(a) There are 5 outcomes that give a score of 6, so

$$p\,(6) = \frac{5}{36}$$

(b) There are 6 outcomes that give a score greater than 9, so

$$p\left(\text{greater than } 9\right) = \frac{6}{36}$$
$$= \frac{1}{6}$$

(c) There are 15 outcomes that give scores of less than 7, so

$$p\left(\text{less than } 7\right) = \frac{15}{36}$$
$$= \frac{5}{12}$$

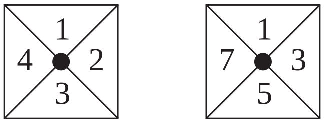
## Exercises

1. Use information from the table in Example 3 to answer this question:

When two fair dice are thrown, what is the probability that the total score is:

(a) 9,
(b) an odd number,
(c) greater than 10,
(d) less than 8 ?

2. The diagram shows two spinners which are both spun.



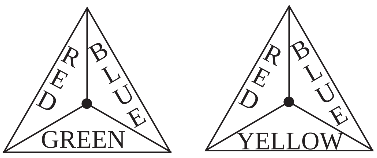
What is the probability that the total score on the two spinners is:

(a) 7,
(b) 6,
(c) greater than 10,
(d) less than 5 ?

3. An unbiased coin is tossed and a fair dice is thrown. Use a table of outcomes to determine the probability of each of the following:

(a) obtaining a head and a 3,
(b) obtaining a tail and an even number,
(c) obtaining a head and a prime number.

4. The two spinners shown in the diagram are both spun.



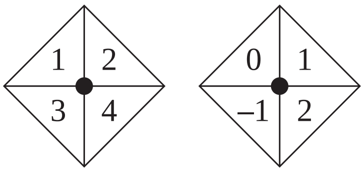
(a) Draw up a table to show the possible outcomes.

(b) What is the probability that both spinners show the same colour?

(c) What is the probability of obtaining a yellow and a red?

(d) What is the probability of obtaining a red and a blue?

5. The diagram shows two spinners that are spun at the same time:



Use a table to determine the probability of obtaining a total score of:

(a) 6 (b) 0 (c) 1 (d) 3

6. For the spinners in question 5, determine the probability of obtaining a total score that is:

(a) an even number,

(b) greater than 1,

(c) less than 1,

(d) less than 6.

7. Two unbiased coins are tossed at the same time. What is the probability of obtaining:

(a) at least one head,

(b) no heads ?

8. Three unbiased coins are tossed at the same time. Use a tree diagram to show the outcomes and determine the probability of obtaining:

(a) 3 heads,

(b) at least 1 head,

(c) at least 2 heads.

9. Two fair dice are rolled and the scores on each dice are multiplied together to give a total score. What is the probability of getting a total score:

(a) of 12,

(b) of 20,

(c) greater than 25,

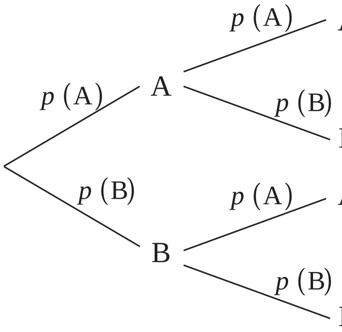
(d) less than 30,

(e) that is an even number ?

10. If 4 unbiased coins are tossed at the same time, what is the probability of obtaining the same number of heads as tails?

# 10.4 Multiplication Law for Independent Events

Probabilities can be assigned to tree diagrams, and then multiplication can be used to determine the probabilities for combined events.



| *OUTCOMES* | *PROBABILITIES* |
|---|---|
| A A | $p(\text{A})\times p(\text{A})$ |
| A B | $p(\text{A})\times p(\text{B})$ |
| B A | $p(\text{B})\times p(\text{A})$ |
| B B | $p(\text{B})\times p(\text{B})$ |

Note: Here we have an experiment with two possible outcomes, A and B, and the experiment is repeated once. It is assumed that the probability of either A or B remains the *same* when the experiment is repeated; in this case, we say that A and B are *independent* events.

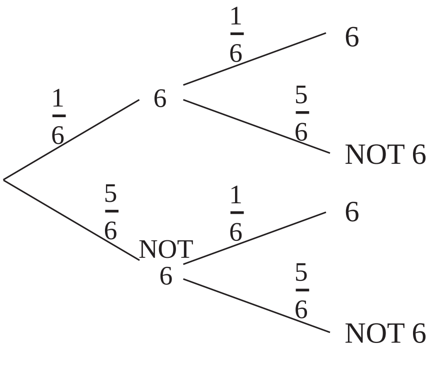
## Example 1

Two fair dice are rolled. Use a tree diagram to determine the probability of obtaining:

(a) 2 sixes, (b) 1 six, (c) no sixes.

## Solution

The tree diagram is shown below:



| *OUTCOMES* | *PROBABILITIES* |
|---|---|
| 6, 6 | $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$ |
| 6, NOT 6 | $\frac{1}{6}\times\frac{5}{6}=\frac{5}{36}$ |
| NOT 6, 6 | $\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}$ |
| NOT 6, NOT 6 | $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$ |
| | total $=\frac{36}{36}=1$ |

(a) $p(2\text{ sixes}) = \frac{1}{36}$

(b) $p(1\text{ six}) = \frac{5}{36}+\frac{5}{36} = \frac{10}{36} = \frac{5}{18}$

(c) $p(\text{no sixes}) = \frac{25}{36}$

*Note that these probabilities add up to 1. This will always be so when the probabilities are added from the outcome of the tree diagram.*
*This is a very useful means of checking your working.*

## Example 2

A bag contains 4 red balls and 3 green balls. A ball is taken out at random, and then put back; a second ball is then taken from the bag. What is the probability that:

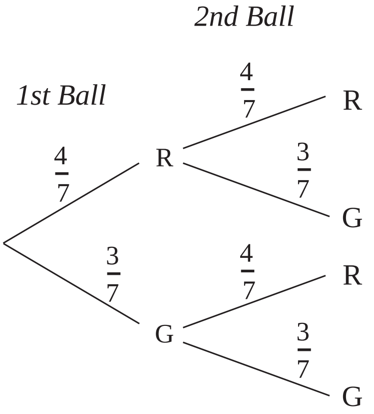
(a) both balls are the same colour,

(b) at least one of the balls is green,

(c) the balls are of different colours?

## Solution

Use a tree diagram:



| OUTCOMES | PROBABILITIES |
|---|---|
| R R | $\frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$ |
| R G | $\frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$ |
| G R | $\frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$ |
| G G | $\frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$ |
| | total $= \frac{49}{49} = 1$ |

(a) $$\begin{aligned} p(\text{both the same}) &= p(\text{R R or G G}) \\ &= p(\text{R R}) + p(\text{G G}) \\ &= \frac{16}{49} + \frac{9}{49} \\ &= \frac{25}{49} \end{aligned}$$

(b) $p(\text{at least one green ball})$

$$\begin{aligned} &= p(\text{G G or G R or R G}) \\ &= p(\text{G G}) + p(\text{G R}) + p(\text{R G}) \\ &= \frac{9}{49} + \frac{12}{49} + \frac{12}{49} \\ &= \frac{33}{49} \end{aligned}$$

or

$$\begin{aligned} &= 1 - p(\text{R R}) \\ &= 1 - \frac{16}{49} \\ &= \frac{33}{49} \end{aligned}$$

(c)

$$\begin{aligned} p(\text{both different colours}) &= p(\text{R G or G R}) \\ &= p(\text{R G}) + p(\text{G R}) \\ &= \frac{12}{49} + \frac{12}{49} \\ &= \frac{24}{49} \end{aligned}$$

Note: In probability questions of this type, 'or' means adding the probabilities.

## Example 3

On her way to work, Sylvia drives through three sets of traffic lights. The probability of each set of lights being green is 0.3. What is the probability that they are *all* green?

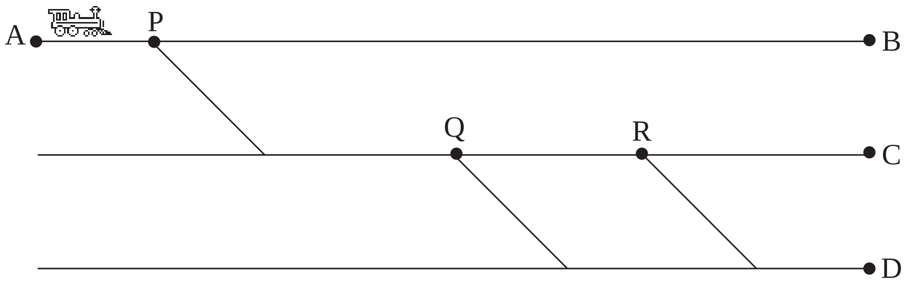
## Solution

$$\begin{aligned} p(\text{all green}) &= p(\text{1st green and 2nd green and 3rd green}) \\ &= p(\text{1st green}) \times p(\text{2nd green}) \times p(\text{3rd green}) \\ &= 0.3 \times 0.3 \times 0.3 \quad [\text{or } 0.3^3] \\ &= 0.027 \end{aligned}$$

Note: In probability questions of this type, 'and' means multiplying the probabilities.

*Remember* A tree diagram is drawn when it will *help* you to analyse a problem; so if it will help, draw one. On the other hand, if you are able to solve a problem *without* one (see Example 3 above), then do so.

## Example 4



The diagram shows a model railway track. At each of the junctions P, Q and R, the probability of a train going straight ahead is $\frac{2}{3}$ and the probability of it branching to the right is $\frac{1}{3}$.

A train starts at point A.

(a) What is the probability that it reaches point C?

(b) What is the probability that it reaches point D ?

## Solution

(a) $p(\text{right and straight and straight}) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}$

$$= \frac{4}{27}$$

(b) $p((\text{right and right}) \text{ or } (\text{right and straight and right}))$

$$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$$

$$= \frac{1}{9} + \frac{2}{27}$$

$$= \frac{5}{27}$$

## Exercises

1. A bag contains 3 red balls and 2 blue balls. A ball is taken at random from the bag and then put back. A second ball is then taken out of the bag.

   What is the probability that:

   (a) both balls are red,

   (b) both balls are the same colour,

   (c) at least one of the balls is red ?

2. Repeat question 1 for a bag with 7 red balls and 3 blue balls.

3. Two fair dice are rolled at the same time. Use a tree diagram to determine the probability of obtaining:

   (a) two even numbers,

   (b) at least one even number,

   (c) no even numbers.

4. Two fair dice are rolled at the same time. Use a tree diagram to determine the probability of obtaining:

   (a) two multiples of 3,

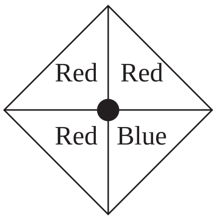
   (b) exactly one multiple of 3,

   (c) less than two multiples of 3.

5. A coin has been weighted, so that the probability of getting a head is $\frac{2}{5}$ and the probability of getting a tail is $\frac{3}{5}$; the coin is thrown twice. Determine the probability of obtaining:

(a) 2 heads, (b) no heads, (c) at least one head.

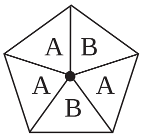
6. The spinner shown in the diagram is spun twice. Use a tree diagram to determine the probability of obtaining:



(a) 2 reds, (b) at least one red,

(c) no reds.

7. The spinner in the diagram is spun twice. Determine the probability of obtaining:



(a) at least one A, (b) at least one B,

(c) two As, (d) two Bs.

8. The spinner in question 6 is spun 3 times. Use a tree diagram to determine the probability of obtaining:

(a) 3 reds, (b) 2 reds, (c) at least 1 red.

9. A bag contains 1 red ball, 2 green balls and 4 yellow balls. A ball is taken from the bag at random. The ball is then put back, and a second ball is taken at random from the bag.

What is the probability that:

(a) both balls are the same colour,

(b) no yellow balls are taken out,

(c) at least one yellow ball is taken out?

10. Each of 10 balls is marked with a different number from 1 to 10. One ball is taken at random and then replaced. A second ball is then taken at random. Determine the probability that:

(a) both balls taken are marked with the number 5,

(b) both balls taken have even numbers,

(c) both balls taken have numbers which are multiples of 3,

(d) at least one of the balls taken has a number greater than 2.

11. On his way to work, Paul has to pass through 2 sets of traffic lights. The probability that the first set of lights is green is 0.5, and the probability that the second set of lights is green is 0.4.

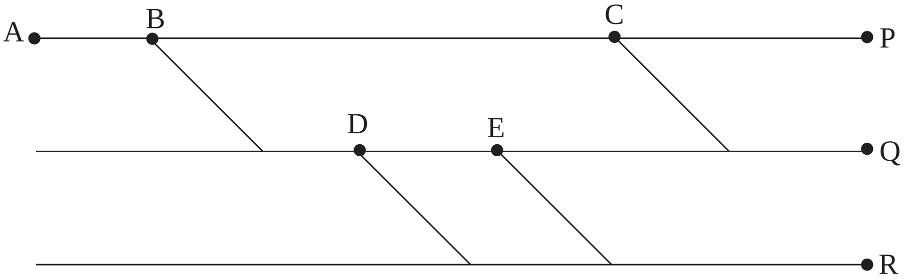
What is the probability that *both* sets of lights are green?

12. On her way to the theatre, Sheila passes through 3 sets of traffic lights. The probability that each set of lights is green is $\frac{1}{3}$.

    (a) What is the probability that *none* of the lights is green?

    (b) What is the probability that *two sets* of lights are green and the other set is not green?

13.

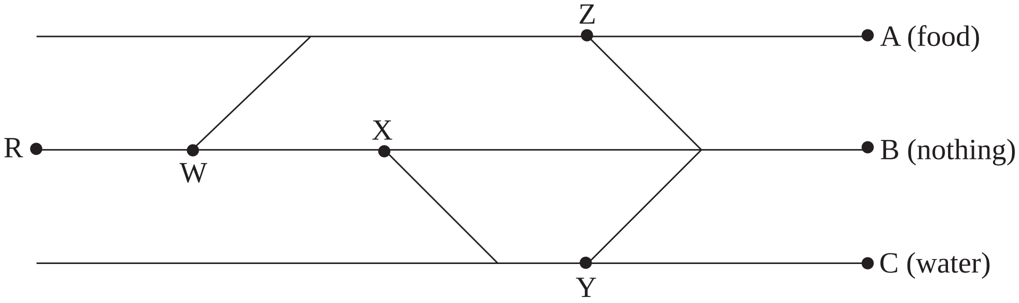


The diagram shows a section of a railway track. At each of the junctions B, C, D and E, the probability of going straight on is $\frac{3}{4}$.
The train starts at A.

(a) What is the probability that it reaches P?

(b) What is the probability that it reaches Q?

14.



A rat leaves position R and starts walking towards B. If it reaches B it gets nothing, if it reaches A it gets food and if it reaches C it gets water.

At each of the junctions W, X, Y and Z, the probability of going straight on is 0.6 and the probability of branching off is 0.4.

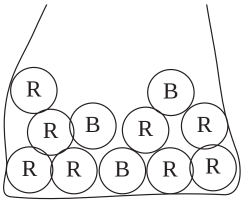
(a) What is the probability that the rat gets *food*?

(b) What is the probability that the rat gets *water*?

(c) What is the probability that it gets *nothing*?

15. When two fair dice are thrown, what is the probability that the score on the second dice is higher than the score on the first dice?

# 10.5 Conditional Probability



In some situations where events are repeated, the probabilities will change after the first event. For example, consider a bag containing 8 red balls and 3 blue balls.

The probability that a ball taken at random is red is $\frac{8}{11}$.

If a second ball is taken out *without the first ball being replaced*, then:

EITHER the first ball was *red*, so the probability that the second ball is red is $\frac{7}{10}$, since there are 3 blue balls but only 7 red balls left.

OR the first ball was *blue*, so the probability that the second is red is $\frac{8}{10} = \frac{4}{5}$, since there are 8 red balls but only 2 blue balls left.

Tree diagrams are very useful for this type of problems.

## Example 1

A bag contains 7 yellow balls and 5 red balls. One ball is taken from the bag at random, and is not replaced. A second ball is then taken from the bag.

Determine the probability that:

(a) both balls are red,

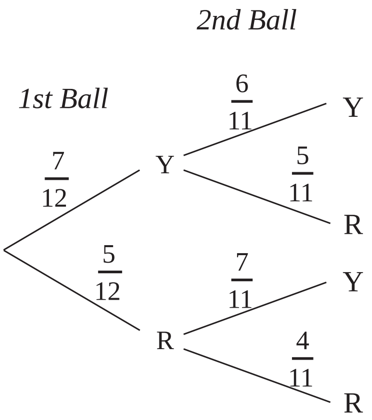
(b) both balls are the same colour,

(c) the balls are different colours,

(d) at least one ball is yellow.

## **Solution**

The tree diagram below shows the probabilities and outcomes:



| *OUTCOMES* | *PROBABILITIES* |
|---|---|
| Y Y | $\frac{7}{12} \times \frac{6}{11} = \frac{42}{132}$ |
| Y R | $\frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$ |
| R Y | $\frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$ |
| R R | $\frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$ |
| | total $= \frac{132}{132} = 1$ |

(a) $$p(\text{both red}) = \frac{20}{132} = \frac{5}{33}$$

(b) $$p(\text{both the same colour}) = p(\text{Y Y}) + p(\text{R R}) = \frac{42}{132} + \frac{20}{132} = \frac{62}{132} = \frac{31}{66}$$

(c) $p(\text{different colours})$

$$= 1 - p(\text{same colour}) = 1 - \frac{31}{66} = \frac{35}{66}$$

or

$$p(\text{Y R}) + p(\text{R Y}) = \frac{35}{132} + \frac{35}{132} = \frac{70}{132} = \frac{35}{66}$$

(d) $p(\text{at least one yellow})$

$$= \frac{42}{132} + \frac{35}{132} + \frac{35}{132} = \frac{112}{132} = \frac{28}{33}$$

or

$$= 1 - p(\text{R R}) = 1 - \frac{20}{132} = \frac{112}{132} = \frac{28}{33}$$

## Example 2

There are 4 boys and 5 girls who are hoping to be selected for a school quiz team. Two of them are selected at random to be in the team.

Determine the probability that:

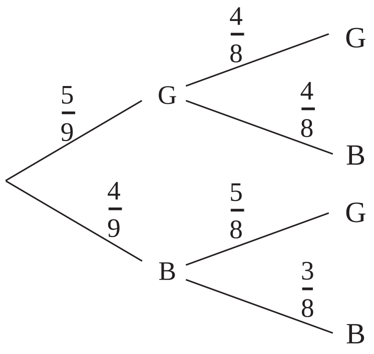
(a) 2 boys are chosen,

(b) at least 1 girl is chosen,

(c) 1 girl and 1 boy are chosen.

## Solution

The tree diagram below shows the outcomes and the probabilities:



| *OUTCOMES* | *PROBABILITIES* |
|---|---|
| G G | $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72} = \frac{5}{18}$ |
| G B | $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72} = \frac{5}{18}$ |
| B G | $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72} = \frac{5}{18}$ |
| B B | $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{3}{18}$ |
| | total $= \frac{18}{18} = 1$ |

(a) $$p(2 \text{ boys}) = \frac{3}{18} = \frac{1}{6}$$

(b) $$p(\text{at least 1 girl}) = \frac{5}{18} + \frac{5}{18} + \frac{5}{18} = \frac{15}{18} = \frac{5}{6}$$

(c) $$p(1 \text{ boy and 1 girl}) = \frac{5}{18} + \frac{5}{18} = \frac{10}{18} = \frac{5}{9}$$

Note: The questions in Examples 1 and 2 could have been answered *without* the use of tree diagrams, but a tree diagram helps greatly with the analysis of the problem; the same is true for the next example.

## Example 3

The probability that Ravi does his homework is $\frac{1}{10}$ if he goes out with his friends and $\frac{3}{5}$ of he does not go out with his friends. The probability that Ravi goes out with his friends is $\frac{3}{4}$. What is the probability that Ravi does his homework?

## Solution

*Solution 1*

$$p\big((\text{goes out and does homework}) \text{ or } (\text{does not go out and does homework})\big)$$

$$=$$

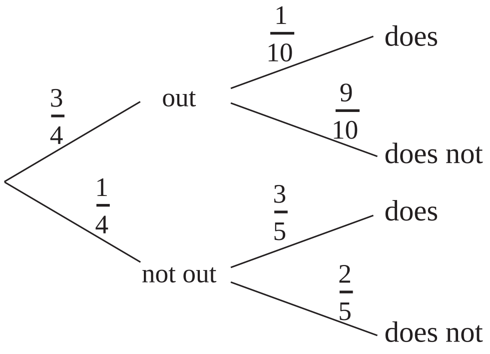
$$p(\text{goes out}) \times p(\text{does homework}) + p(\text{does not go out}) \times p(\text{does homework})$$

$$= \quad \frac{3}{4} \times \frac{1}{10} \qquad + \qquad \frac{1}{4} \times \frac{3}{5}$$

$$= \frac{3}{40} + \frac{3}{20}$$

$$= \frac{9}{40}$$

*Solution 2*



| *OUTCOMES* | *PROBABILITIES* |
|---|---|
| O D | $\frac{3}{4} \times \frac{1}{10} = \frac{3}{40}$ |
| O D' | $\frac{3}{4} \times \frac{9}{10} = \frac{27}{40}$ |
| O' D | $\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$ |
| O' D' | $\frac{1}{4} \times \frac{2}{5} = \frac{2}{20} = \frac{1}{10}$ |

Note: O' means *does not* go out, and D' means *does not* do homework.

$$p(\text{does homework}) = \frac{3}{40} + \frac{3}{20}$$

$$= \frac{9}{40}$$

## Exercises

1. A bag contains 3 pink balls and 2 blue balls. One ball is taken out at random and not replaced. A second ball is then taken out.

   Determine the probability that:

   (a) both balls are pink,

   (b) both balls are the same colour,

   (c) at least one ball is blue.

2. In Tim's drawer there are 6 black socks and 5 white socks. He takes out two socks at random. What is the probability that he has taken two socks of the same colour?

3. In a tennis club there are 5 boys and 3 girls in a training squad. Two are chosen at random to represent the club.

   Determine the probability that they are:

   (a) both boys,

   (b) both girls,

   (c) a boy and a girl.

4. Tara has five 10p coins and four 20p coins in her purse. She takes out two coins at random. What is the probability that she takes out at least 30p?

5. There are 8 footballs in a store cupboard; one is yellow and the others are white. A pupil takes 2 footballs out of the cupboard at random. What is the probability that one of them is the yellow ball?

6. The probability of Jeremy passing a maths exam is $\frac{2}{3}$ if he revises and $\frac{1}{3}$ if he does not revise. The probability that he revises is $\frac{1}{4}$. What is the probability of Jeremy passing the maths exam?

7. The probability of Jenny getting to work on time is 0.8 if she gets up before 7 a.m. and 0.4 if she does not get up before 7 a.m. The probability that Jenny gets up before 7 a.m. is 0.7. What is the probability that Jenny is *late* for work?

8. Ian is an inept mountaineer who tends to fall from rock faces. The probability that he falls is 0.2 if the weather is dry but rises to 0.5 if it is wet. The probability of wet weather is 0.3. Determine the probability that Ian falls.

9. A bag contains 7 blue counters, 5 green counters, 2 black counters and 1 white counter. 3 counters are taken at random from the bag, without replacement. What is the probability that they are all the same colour?

10. Peter and Jane play a game in which they each in turn take a counter at random from a bag containing 7 red counters and 3 yellow counters. The winner is the first to get a red counter. Jane goes first. By drawing a tree diagram, determine the probability that Peter wins the game.