10 Arithmetic: Fractions

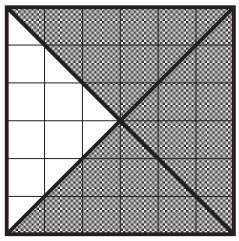
10.1 Fractions

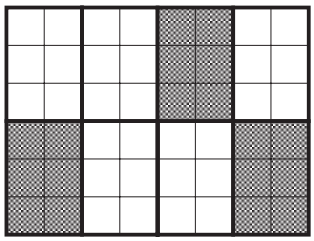
This unit deals with the important topic of fractions. These are numbers of the form $\frac{a}{b}$ when a and b are whole numbers and $b \neq 0$.

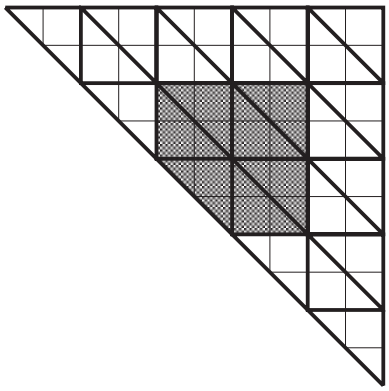
We first see what is meant by a fraction when related to geometrical shapes.

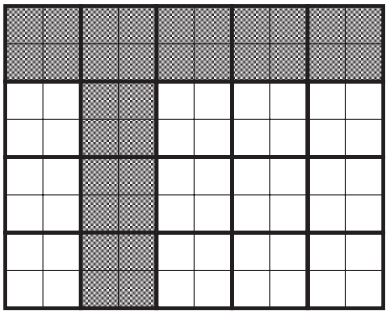
Example

What fraction of each shape is shaded?

(a) 

(b) 

(c) 

(d) 

Solution

(a) $\frac{3}{4}$

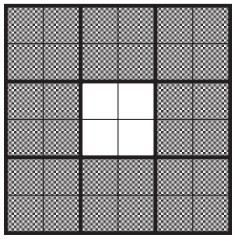
(b) $\frac{3}{8}$

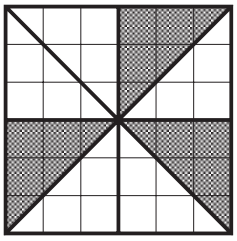
(c) $\frac{7}{25}$ (as there are 7 shaded triangles and 25 in total)

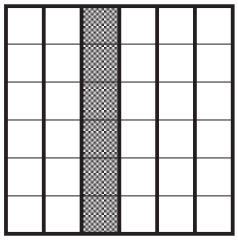
(d) $\frac{8}{20}$, but this is the same as $\frac{4}{10}$ or $\frac{2}{5}$.

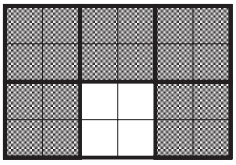
Exercises

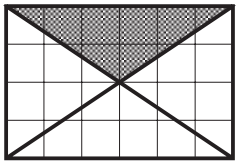
1. What fraction of each of the following shapes is shaded?

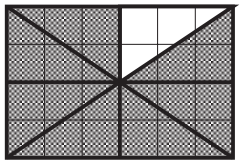
(a) 

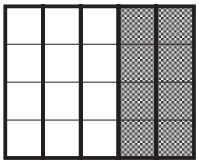
(b) 

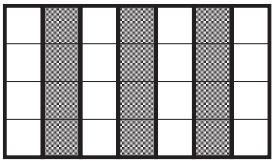
(c) 

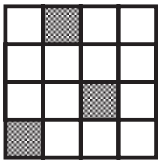
(d) 

(e) 

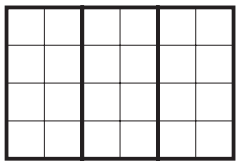
(f) 

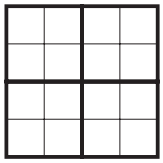
(g) 

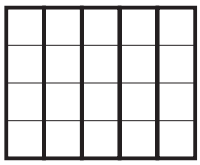
(h) 

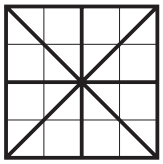
(i) 

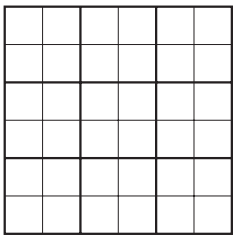
2. Copy each of these shapes and shade the fraction stated.

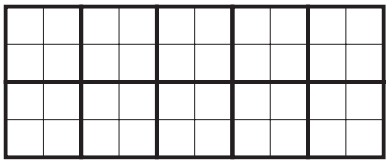
(a)  $\frac{1}{3}$

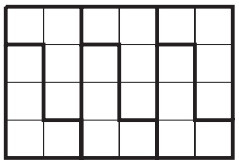
(b)  $\frac{3}{4}$

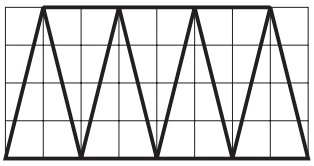
(c)  $\frac{3}{5}$

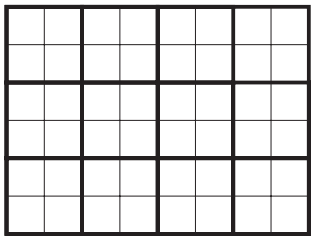
(d)  $\frac{5}{8}$

(e)  $\frac{7}{9}$

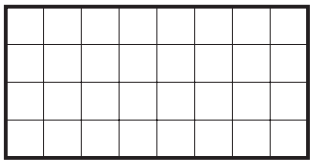
(f)  $\frac{3}{10}$

(g)  $\frac{5}{6}$

(h)  $\frac{5}{7}$

(i)  $\frac{5}{12}$

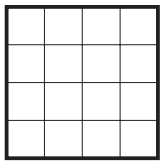
3. (a) On a copy of this rectangle, draw lines to divide it into 8 equal parts.

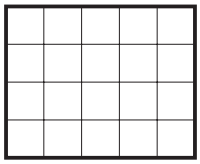


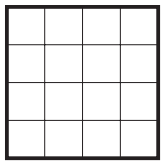
(b) Shade $\frac{1}{8}$ of the rectangle.

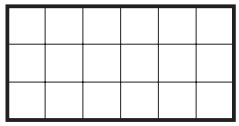
(c) What fraction has *not* been shaded?

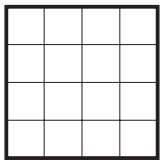
4. Copy each shape and shade the fraction stated.

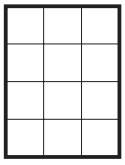
(a)  $\frac{1}{4}$

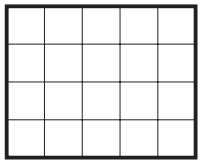
(b)  $\frac{4}{5}$

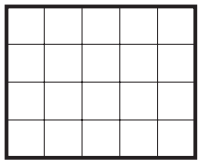
(c)  $\frac{7}{8}$

(d)  $\frac{2}{3}$

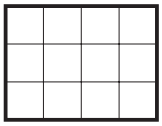
(e)  $\frac{7}{16}$

(f)  $\frac{9}{12}$

(g)  $\frac{7}{10}$

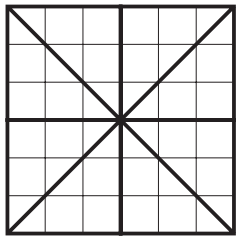
(h)  $\frac{13}{20}$

5. On separate diagrams, shade the stated fraction of this shape.



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$

6. (a) Copy this shape.



(b) Shade $\frac{3}{8}$ of the shape.

(c) Shade another $\frac{2}{8}$ of the shape.

(d) What is the total fraction now shaded?

(e) How much is left unshaded?

7. Sarah shades $\frac{3}{7}$ of a shape. What fraction of the shape is left unshaded?

8. A cake is divided into 12 equal parts. John eats $\frac{3}{12}$ of the cake and Kate eats another $\frac{1}{12}$. What fraction of the cake is left?

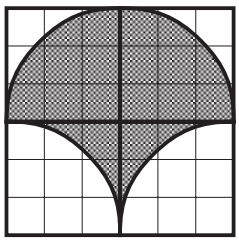
9. A car park contains 20 spaces. There are 17 cars parked in the car park.
   (a) What fraction of the car park is full?
   (b) What fraction of the car park is empty?

10. Ali eats $\frac{3}{10}$ of the sweets in a packet.

    Tariq eats another $\frac{4}{10}$ of the sweets.

    (a) What fraction of the sweets has been eaten?
    (b) What fraction of the sweets is left?

11. Draw as many ways as you can of shading $\frac{1}{2}$ of a shape.

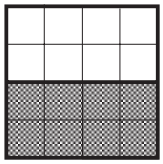


Be as imaginative as you can.

Here is an idea to get you thinking.

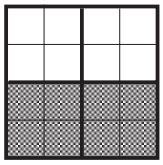
10.2 Equivalent Fractions

It is easy to see from the diagram opposite that

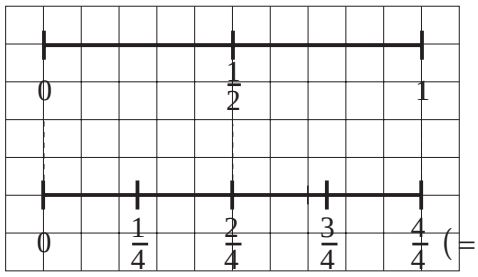


$$\frac{1}{2} \text{ and } \frac{2}{4}$$

are *equivalent fractions,* i.e. they both have the same value.

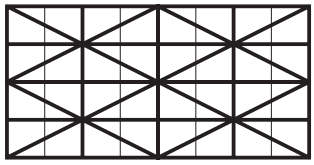


Another way of looking at this is to note that the fractions are at the same place on a number line.



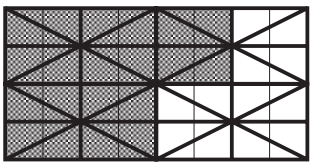
Example

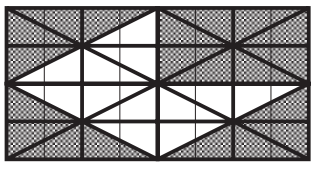
Use the diagram below to shade $\frac{5}{8}$ of the rectangle in different ways.



Solution

Here are two possible answers.

(a) 

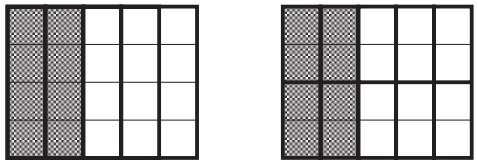
(b) 

In fact, shading any 20 of the small triangles (out of 32) gives the fraction $\frac{5}{8}$

(which is equivalent to $\frac{10}{16}$ or $\frac{20}{32}$, etc.)

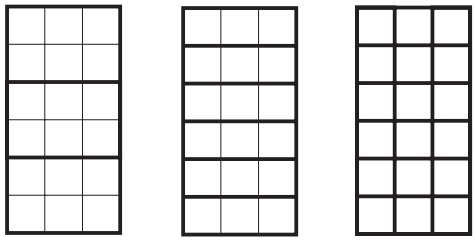
Exercises

1. The diagrams show that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$.

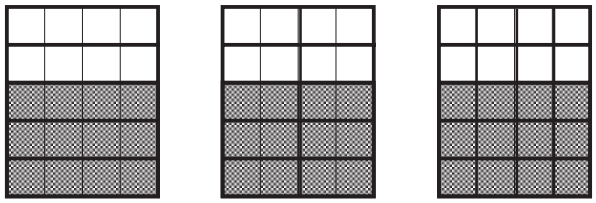


Draw a diagram to show that $\frac{2}{5}$ is also equivalent to $\frac{8}{20}$.

2. Use the diagrams below to show that $\frac{2}{3} = \frac{4}{6} = \frac{12}{18}$.



3. Write down the equivalent fractions shown by these diagrams.



4. Use the diagram on the next page to fill in the missing numbers.

(a) $\frac{1}{2} = \frac{?}{4} = \frac{?}{6} = \frac{?}{8} = \frac{?}{10} = \frac{?}{12} = \frac{?}{14} = \frac{?}{16}$

(b) $\frac{1}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12} = \frac{?}{15}$

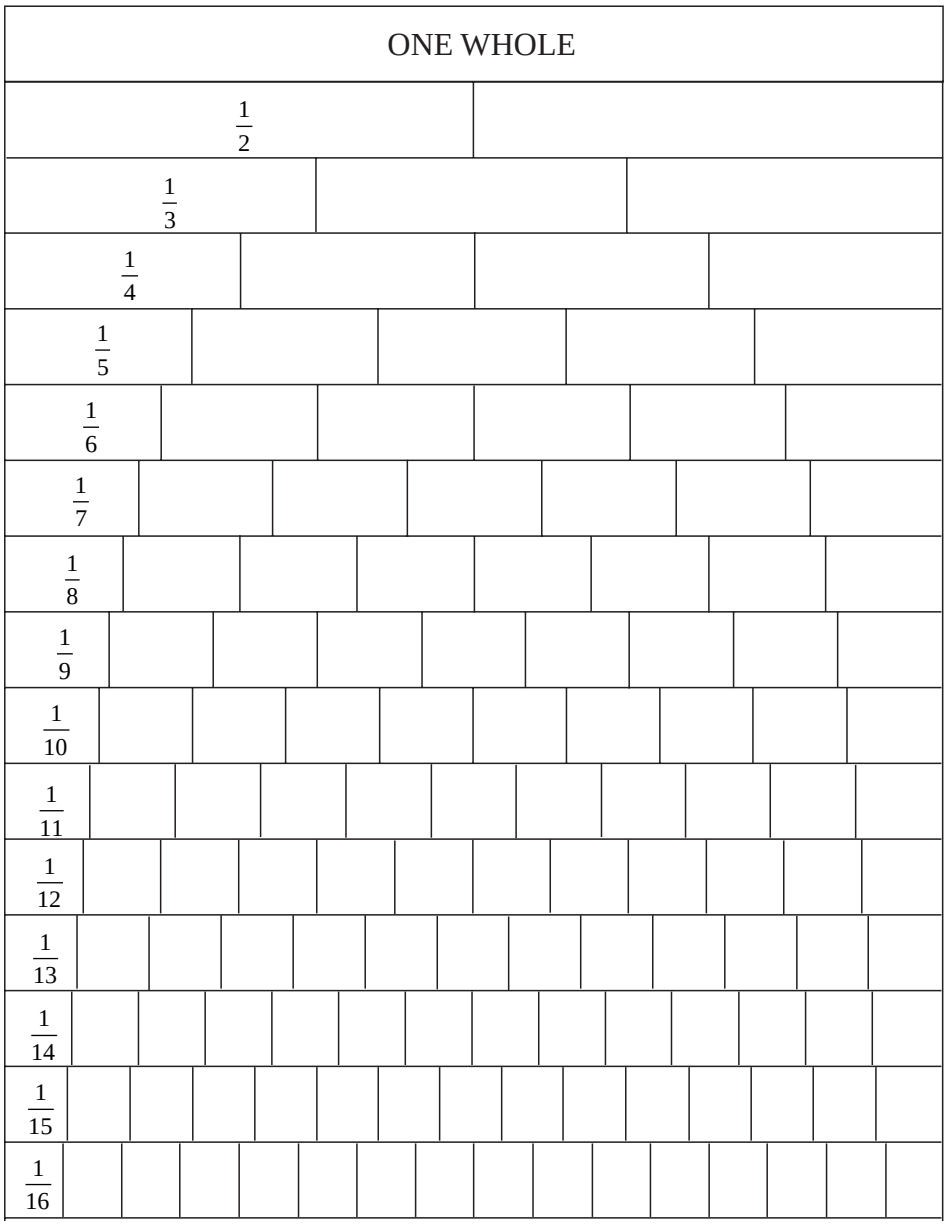
(c) $\frac{1}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16}$

(d) $\frac{1}{5} = \frac{?}{10} = \frac{?}{15}$

(e) $\frac{1}{6} = \frac{?}{12}$

(f) $\frac{1}{7} = \frac{?}{14}$

(g) $\frac{1}{8} = \frac{?}{16}$



5. Write down the missing numbers.

(a) $\frac{3}{4} = \frac{?}{8}$ (b) $\frac{2}{5} = \frac{?}{10}$ (c) $\frac{5}{7} = \frac{?}{14}$

(d) $\frac{3}{4} = \frac{?}{12}$ (e) $\frac{2}{5} = \frac{?}{15}$ (f) $\frac{2}{3} = \frac{?}{9}$

(g) $\frac{3}{8} = \frac{?}{16}$ (h) $\frac{2}{7} = \frac{?}{14}$ (i) $\frac{2}{3} = \frac{?}{12}$

(j) $\frac{3}{4} = \frac{?}{20}$ (k) $\frac{3}{5} = \frac{?}{20}$ (l) $\frac{5}{7} = \frac{?}{21}$

6. Write out each of these pairs of fractions, inserting either a $<$ or a $>$ into each one to make it correct.

(a) $\frac{1}{2} \quad \frac{1}{3}$ (b) $\frac{1}{4} \quad \frac{1}{5}$ (c) $\frac{1}{6} \quad \frac{1}{7}$

(d) $\frac{1}{10} \quad \frac{1}{9}$ (e) $\frac{1}{2} \quad \frac{2}{3}$ (f) $\frac{3}{4} \quad \frac{2}{3}$

(g) $\frac{2}{5} \quad \frac{1}{2}$ (h) $\frac{7}{10} \quad \frac{7}{8}$ (i) $\frac{5}{7} \quad \frac{3}{5}$

(j) $\frac{5}{6} \quad \frac{5}{7}$ (k) $\frac{2}{3} \quad \frac{5}{7}$ (l) $\frac{4}{5} \quad \frac{5}{6}$

7. Write down the missing numbers.

(a) $\frac{15}{30} = \frac{?}{2}$ (b) $\frac{6}{9} = \frac{?}{3}$ (c) $\frac{9}{12} = \frac{?}{4}$

(d) $\frac{3}{12} = \frac{?}{4}$ (e) $\frac{8}{18} = \frac{?}{9}$ (f) $\frac{16}{40} = \frac{?}{5}$

(g) $\frac{30}{50} = \frac{?}{5}$ (h) $\frac{14}{21} = \frac{?}{3}$ (i) $\frac{16}{24} = \frac{?}{3}$

(j) $\frac{17}{51} = \frac{?}{3}$ (k) $\frac{144}{200} = \frac{?}{25}$ (l) $\frac{132}{216} = \frac{?}{18}$

8. Write each set of fractions in *increasing* order.

(a) $\frac{1}{7}, \quad \frac{1}{9}, \quad \frac{1}{3}, \quad \frac{1}{10}, \quad \frac{1}{4}$

(b) $\frac{2}{3}, \quad \frac{3}{5}, \quad \frac{4}{7}, \quad \frac{2}{9}$

(c) $\frac{3}{5}, \quad \frac{2}{7}, \quad \frac{5}{6}, \quad \frac{4}{9}$

(d) $\frac{2}{5}, \quad \frac{3}{5}, \quad \frac{2}{7}, \quad \frac{3}{7}, \quad \frac{5}{7}$

(e) $\frac{3}{7}, \quad \frac{1}{9}, \quad \frac{5}{7}, \quad \frac{5}{9}, \quad \frac{7}{9}$

9. Write each fraction in its simplest form.

(a) $\frac{8}{24}$ (b) $\frac{15}{20}$ (c) $\frac{20}{25}$

(d) $\frac{9}{13}$ (e) $\frac{21}{28}$ (f) $\frac{40}{64}$

(g) $\frac{9}{36}$ (h) $\frac{80}{200}$ (i) $\frac{132}{150}$

10. State whether each of the following is *true* or *false*. If *false*, explain why.

(a) $\frac{3}{7} > \frac{3}{5}$ (b) $\frac{3}{8} = \frac{36}{88}$ (c) $\frac{11}{44} = \frac{1}{4}$

(d) $\frac{5}{8} > \frac{1}{2}$ (e) $\frac{3}{8} > \frac{1}{2}$ (f) $\frac{1}{6} < \frac{1}{7}$

(g) $\frac{8}{9} > \frac{7}{8}$ (h) $\frac{9}{10} < \frac{10}{11}$ (i) $\frac{44}{99} = \frac{4}{11}$

10.3 Fractions of Quantities

Now we start to use fractions in a practical way.

Example

(a) Find $\frac{1}{5}$ of £30.

(b) Find $\frac{4}{5}$ of £30

Solution

You can, of course, do this practically, but it is much easier to work out

(a) $\frac{1}{5} \times £30 = £30 \div 5$

$= £\frac{30}{5}$

$= £6$

(b) $\frac{1}{5} \times £30 = £6$

$\frac{4}{5} \times £30 = £4 \times 6$

$= £24$

Exercises

1. Find:

(a) $\frac{1}{2}$ of 12	(b) $\frac{1}{4}$ of 8	(c) $\frac{1}{5}$ of 15
(d) $\frac{1}{3}$ of 12	(e) $\frac{1}{5}$ of 30	(f) $\frac{1}{4}$ of 40
(g) $\frac{1}{7}$ of 14	(h) $\frac{1}{8}$ of 64	(i) $\frac{1}{8}$ of 40
(j) $\frac{1}{3}$ of 24	(k) $\frac{1}{4}$ of 32	(l) $\frac{1}{9}$ of 36

2. Find:

(a) $\frac{3}{4}$ of 24	(b) $\frac{4}{5}$ of 20	(c) $\frac{3}{7}$ of 14
(d) $\frac{2}{9}$ of 18	(e) $\frac{5}{6}$ of 30	(f) $\frac{4}{7}$ of 28
(g) $\frac{3}{5}$ of 15	(h) $\frac{7}{9}$ of 45	(i) $\frac{3}{8}$ of 64
(j) $\frac{5}{9}$ of 36	(k) $\frac{3}{5}$ of 45	(l) $\frac{7}{8}$ of 56

3. In a test there are 30 marks. Nasir gets $\frac{3}{5}$ of the marks. How many marks does he get?

4. In a school $\frac{1}{2}$ of the pupils are girls. There are 382 pupils in the school. How many girls are there in the school?

5. In a class there are 32 pupils. Of these, $\frac{3}{8}$ come to school by bus. How many pupils come to school by bus?

6. In a school, $\frac{3}{10}$ of the pupils have pets. There are 510 pupils in the school.
   (a) How many of them have pets?
   (b) How many of them do not have pets?

7. There are 20 houses in a street and $\frac{3}{4}$ of them have satellite TV. How many of these houses do not have satellite TV?

8. Rachel has 360 stamps in her stamp collection. $\frac{5}{8}$ of these stamps are foreign. How many foreign stamps has she got?

9. Ben and Chris sell their old toys at a car boot sale. They get £45. They agree that Ben will have $\frac{2}{5}$ of the money and Chris the rest. How much do they each get?

10. In a school there are 550 pupils. If $\frac{3}{50}$ of the pupils are left-handed, how many left-handed pupils are there in the school?

10.4 Mixed Numbers and Vulgar (Improper) Fractions

So far we have worked with fractions of the form $\frac{a}{b}$ where $a < b$,

e.g. $\frac{3}{4}, \frac{2}{7}, \frac{5}{6}, \ldots$

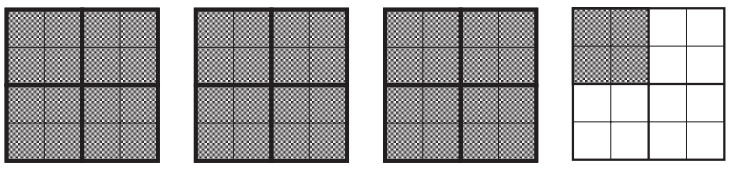
We also need to work with what are sometimes called *vulgar* or *improper* fractions, e.g. $\frac{5}{4}, \frac{7}{2}$, which are of the form $\frac{a}{b}$ when a and b are whole numbers and $a > b$.

Example

Show that the numbers $\frac{13}{4}$ and $3\frac{1}{4}$ are equivalent.

Solution

You can illustrate $3\frac{1}{4}$ as opposite.



If you count the quarters, you can see that there are $4 + 4 + 4 + 1 = 13$,

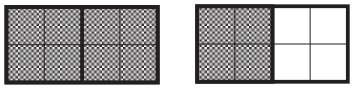
i.e. the number is $\frac{13}{4}$.

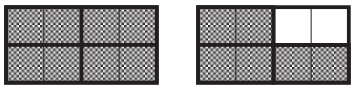
(You can see that $3\frac{1}{4} = \frac{(3 \times 4) + 1}{4} = \frac{13}{4}$.)

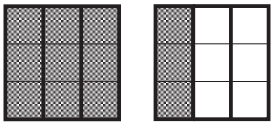
Note that $3\frac{1}{4}$ is called a *mixed* number.

Exercises

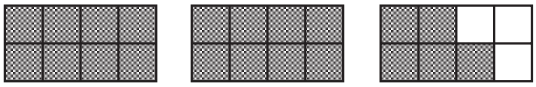
1. Write each of the numbers represented by these diagrams, in two different ways.

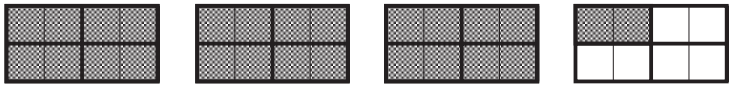
(a) 

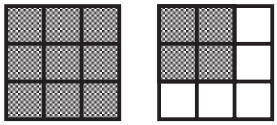
(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

2. Draw diagrams for these mixed numbers.

(a) $1\frac{1}{5}$ (b) $2\frac{1}{4}$ (c) $2\frac{2}{3}$

Write each number as an improper fraction.

3. Draw diagrams to show these improper fractions.

(a) $\frac{7}{2}$ (b) $\frac{8}{3}$ (c) $\frac{18}{5}$

Write each improper fraction as a mixed number.

4. Convert these improper fractions to mixed numbers.

(a) $\frac{9}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$

(d) $\frac{12}{5}$ (e) $\frac{18}{5}$ (f) $\frac{9}{7}$

(g) $\frac{11}{9}$ (h) $\frac{9}{4}$ (i) $\frac{8}{5}$

(j) $\frac{22}{9}$ (k) $\frac{6}{5}$ (l) $\frac{14}{5}$

(m) $\frac{13}{7}$ (n) $\frac{19}{7}$ (o) $\frac{20}{9}$

5. Convert these mixed numbers to improper fractions.

(a) $1\frac{3}{5}$ (b) $4\frac{1}{2}$ (c) $2\frac{1}{4}$

(d) $6\frac{1}{2}$ (e) $7\frac{1}{3}$ (f) $5\frac{2}{3}$

(g) $8\frac{3}{7}$ (h) $4\frac{2}{5}$ (i) $7\frac{1}{5}$

(j) $3\frac{5}{9}$ (k) $4\frac{3}{7}$ (l) $3\frac{4}{5}$

(m) $6\frac{1}{9}$ (n) $7\frac{2}{3}$ (o) $4\frac{7}{8}$

6. Write these fractions in order of increasing size.

$$6\frac{1}{2}, \quad \frac{18}{5}, \quad 3\frac{1}{4}, \quad 5\frac{1}{3}, \quad \frac{17}{3}$$

7. Write out each of these pairs of fractions, inserting either $<$, $>$ or $=$ into each one to make it correct.

(a) $2\frac{3}{5}$ $\frac{13}{5}$ (b) $3\frac{4}{7}$ $\frac{26}{7}$ (c) $3\frac{7}{8}$ 4

(d) $6\frac{1}{2}$ $\frac{22}{3}$ (e) $4\frac{1}{4}$ $\frac{19}{4}$ (f) $\frac{3}{2}$ $\frac{2}{3}$

(g) $\frac{2}{5}$ $\frac{1}{2}$ (h) $\frac{7}{10}$ $\frac{7}{8}$ (i) $\frac{5}{7}$ $\frac{3}{5}$

(j) $\frac{5}{6}$ $\frac{5}{7}$ (k) $\frac{2}{3}$ $\frac{5}{6}$ (l) $\frac{4}{5}$ $\frac{5}{6}$

8. Explain why $3\frac{5}{8} = \frac{58}{16}$.

9. In an office there are $2\frac{1}{2}$ packets of paper. There are 500 sheets of paper in each full packet. How many sheets of paper are there in the office?

10. A young child is 44 months old. Find the age of the baby in years as a mixed number in the simplest form.