1 Mathematical Diagrams

1.1 Mileage Charts

In this section we look at mileage charts.

Example 1

Distances in the table below are given in miles.

	BARNSTAPLE	BRISTOL	EXETER	PENZANCE	PLYMOUTH
BRISTOL	100				
EXETER	55	84			
PENZANCE	108	194	110		
PLYMOUTH	67	125	44	77	
TAUNTON	50	51	34	144	75

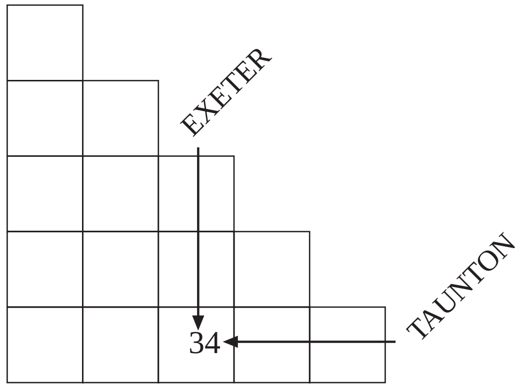
Using the table, answer the following questions:

(a) How far is it from Taunton to Exeter?

(b) Jerry travels from Barnstaple to Exeter, then from Exeter to Plymouth, and finally from Plymouth back to Barnstaple.
How far does he travel altogether?

Solution

(a) 34 miles (see table and diagram opposite).

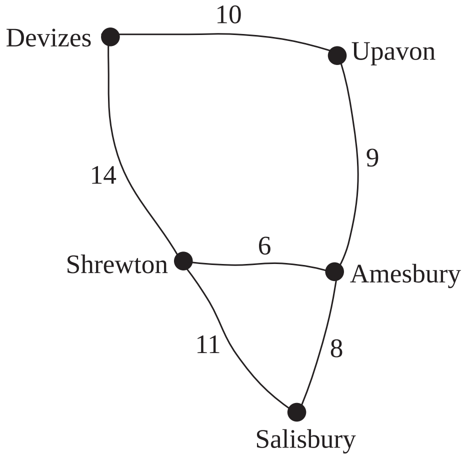


(b) Barnstaple to Exeter: 55 miles
Exeter to Plymouth: 44 miles
Plymouth to Barnstaple: 67 miles

$$\text{Total distance} = 55 + 44 + 67$$
$$= 166 \text{ miles}$$

Example 2

The network diagram opposite shows the distances, in miles, between some towns.



Copy and complete the following mileage chart to show the shortest distances between these towns:

	AMESBURY	DEVIZES	SALISBURY	SHREWTON
DEVIZES				
SALISBURY				
SHREWTON				
UPAVON				

Solution

The direct distances can be completed first:

	AMESBURY	DEVIZES	SALISBURY	SHREWTON
DEVIZES				
SALISBURY	8			
SHREWTON	6	14	11	
UPAVON	9	10		

the shortest route from Amesbury to Devizes is via Upavon, a total of 19 miles;

the shortest route from Devizes to Salisbury is via Shrewton, a total of 25 miles;

the shortest route from Salisbury to Upavon is via Amesbury, a total of 17 miles;

the shortest route from Shrewton to Upavon is via Amesbury, a total of 15 miles.

With this information, the table can now be completed, as shown opposite.

	AMESBURY	DEVIZES	SALISBURY	SHREWTON
DEVIZES	19			
SALISBURY	8	25		
SHREWTON	6	14	11	
UPAVON	9	10	17	15

Exercises

1. Use the table opposite, where the distances are given in miles, to find out how far it is from:

   (a) Leeds to Lincoln,

   (b) Hull to York,

   (c) Leeds to Manchester,

   (d) Sheffield to Leeds,

   (e) Manchester to York.

	HULL	LEEDS	LINCOLN	MANCHESTER	SHEFFIELD
LEEDS	60				
LINCOLN	47	72			
MANCHESTER	97	44	85		
SHEFFIELD	66	36	47	39	
YORK	38	24	80	71	57

2. Ross travels from Leeds to Manchester, then from Manchester to Sheffield and finally from Sheffield back to Leeds. Use the table in question 1 to calculate the total distance he travels.

3. Hannah drives from Bristol to Exeter, continues to Plymouth, on to Barnstaple and from there back to Bristol. Use the table in Example 1 to calculate the total distance she drives.

4. The table opposite gives the distances in kilometres between some towns in northern France.

   What is the distance between:

   (a) Alençon and Paris,

   (b) Reims and Orleans,

   (c) Rouen and Calais,

   (d) Paris and Reims,

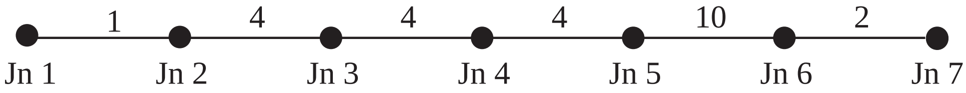
   (e) Le Mans and Rouen?

	ALENÇON	CALAIS	LE MANS	ORLEANS	PARIS	REIMS
CALAIS	365					
LE MANS	49	419				
ORLEANS	167	433	138			
PARIS	220	293	204	135		
REIMS	355	278	340	270	146	
ROUEN	146	215	199	210	140	284

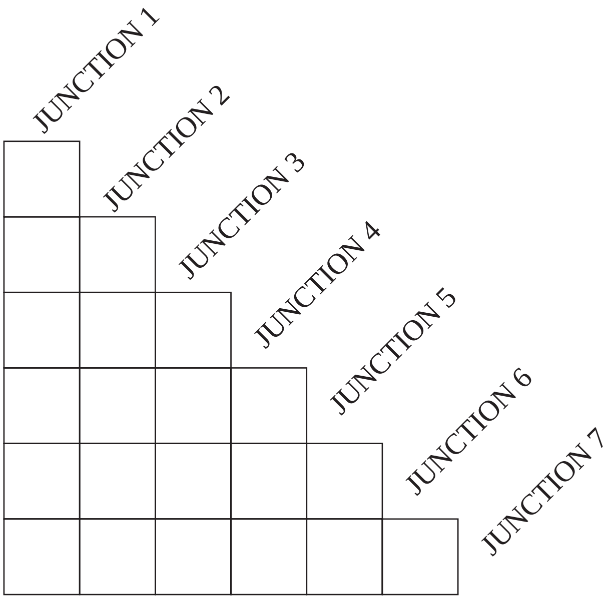
5. Debbie drives from Calais to Paris and back while she is on holiday. Use the table in question 4 to calculate how far she travels altogether on this journey.

6. Laura travels from Calais to Paris, on to Alençon and then to Rouen before returning to Paris. Use the table in question 4 to calculate how far she travels altogether.

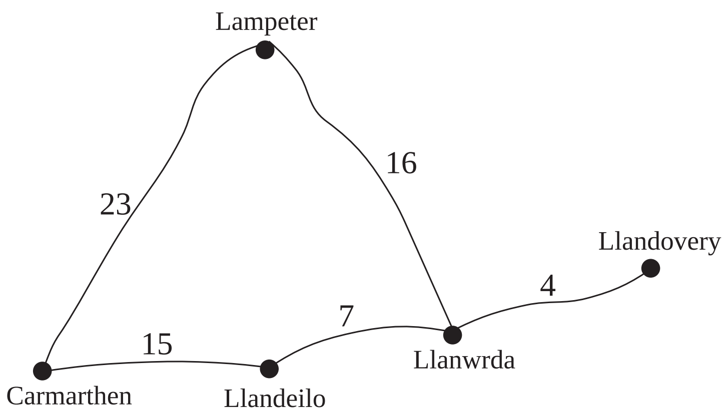
7. The diagram below shows the distances, in miles, between some junctions on the M2 motorway:



Copy and complete the chart below to show the shortest distances between junctions:



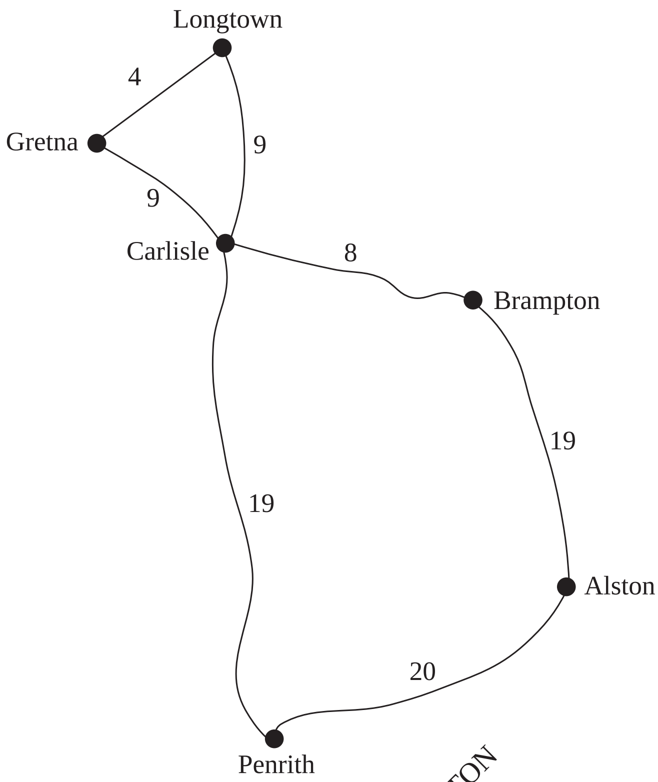
8. The following network diagram shows the distances, in miles, between some towns in Wales:



Use the information in the diagram to complete a copy of the table on the next page, giving the shortest distances between the towns.

	CARMARTHEN	LAMPETER	LLANDEILO	LLANDOVERY
LAMPETER				
LLANDEILO				
LLANDOVERY				
LLANWRDA				

9. The network diagram below shows the distances, in miles, by road between some towns close to the Scottish border:



Use information from the diagram to complete a copy of the table opposite, giving the shortest distances between the towns.

	ALSTON	BRAMPTON	CARLISLE	GRETNA	LONGTOWN
BRAMPTON					
CARLISLE					
GRETNA					
LONGTOWN					
PENRITH					

10. The diagram below shows stations on the GNER railway:

Some distances, in miles, are shown in the table opposite.

(a) Copy the table and fill in the missing distances.

(b) What distance is travelled in a return journey between London and York?

	LONDON	STEVENAGE	PETERBOROUGH	DONCASTER	YORK
STEVENAGE	21				
PETERBOROUGH	68	47			
DONCASTER	188	167	120		
YORK				45	

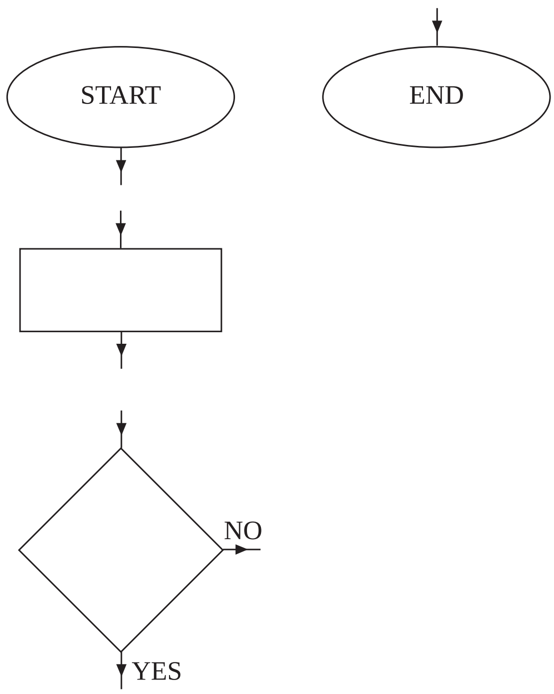
1.2 Using Flow Charts to Plan Practical Tasks

A flow chart can be used to organise the instructions for carrying out a task. Boxes of different shapes are used for particular operations:

START or *END POINTS*

INSTRUCTION BOXES

DECISION BOXES

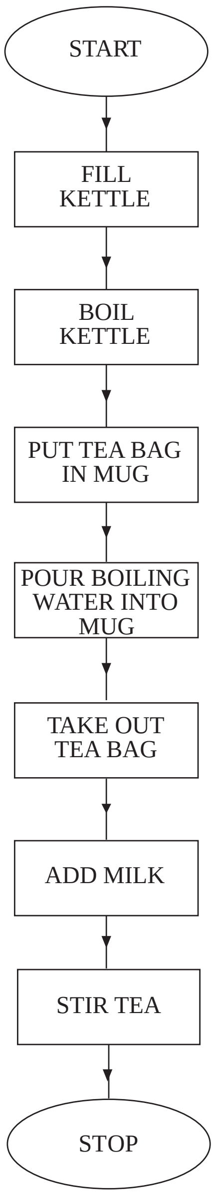


Each box contains only *one* instruction.

Example 1

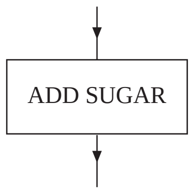
Draw a flow chart to give the instructions for making a mug of tea.

Solution



The flow chart gives the instructions for making a mug of tea without sugar.

A mug of tea *with* sugar, needs this extra instruction box:



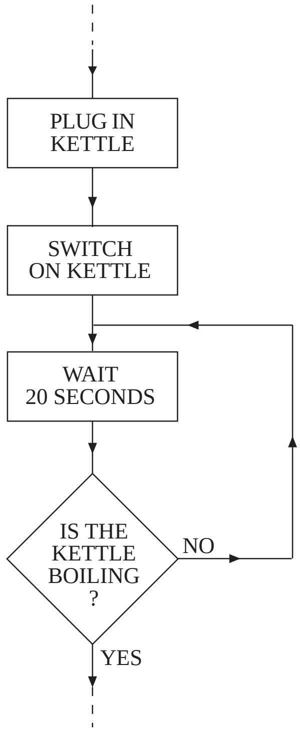
Note that the instruction box PUT TEA BAG IN MUG can go earlier in the flow chart, for example, immediately after the START box. The extra box ADD SUGAR can also go in various positions: for example, before or after the PUT TEA BAG IN MUG box, or before or after the ADD MILK box.

Example 2

The instruction BOIL KETTLE in Example 1 can be broken down into separate stages.

Draw a flow chart to show this.

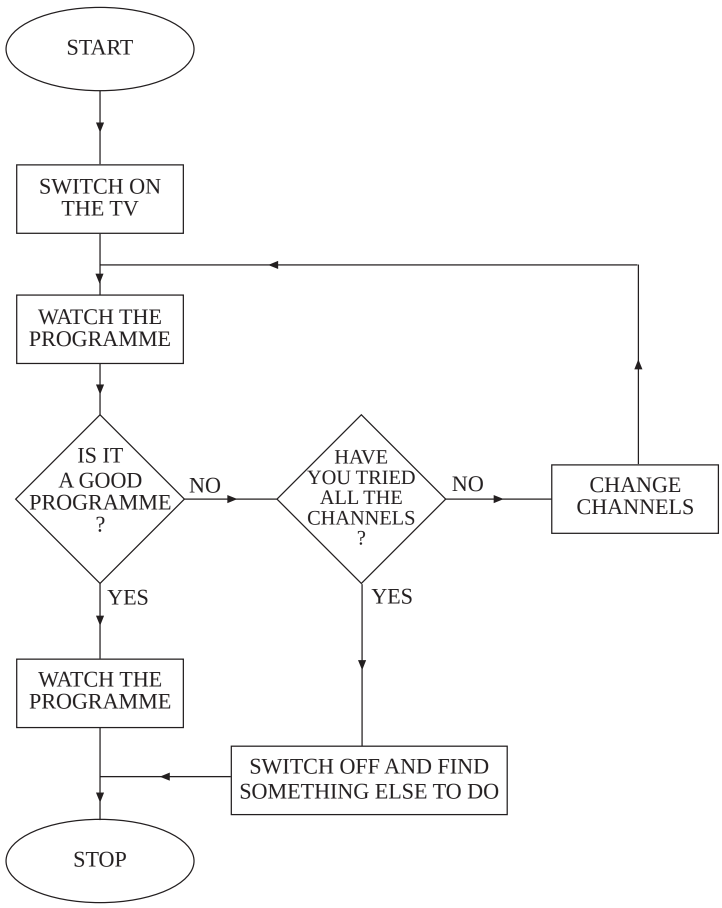
Solution



Example 3

Draw a flow chart showing how to find a programme you would like to watch on television.

Solution



Exercises

1. Draw a flow chart showing how to prepare a drink of blackcurrant squash in a glass.

2. Draw a flow chart for each of the following:
   (a) making a cup of coffee with milk and sugar,
   (b) buying a can of drink from a vending machine,
   (c) making a telephone call from a pay phone,
   (d) shutting down a computer.

3. Draw a flow chart that describes how to cross a road. You should include decision boxes in your flow chart.

4. Imagine you are driving along a road. You see a 30 mph speed limit sign and a speed camera. Draw a flow chart that you, as a sensible driver, would be advised to follow.

5. Jerry needs to work out 4.72×11.61 using a calculator.
   (a) Draw a flow chart to show how to carry out this calculation on a calculator.
   (b) Redraw the flow chart to include all the following processes:
      (i) *estimating* the answer to 4.72×11.61,
      (ii) *calculating* the answer to 4.72×11.61
      (iii) comparing the answer with the estimate to decide whether the calculator answer is reasonable.

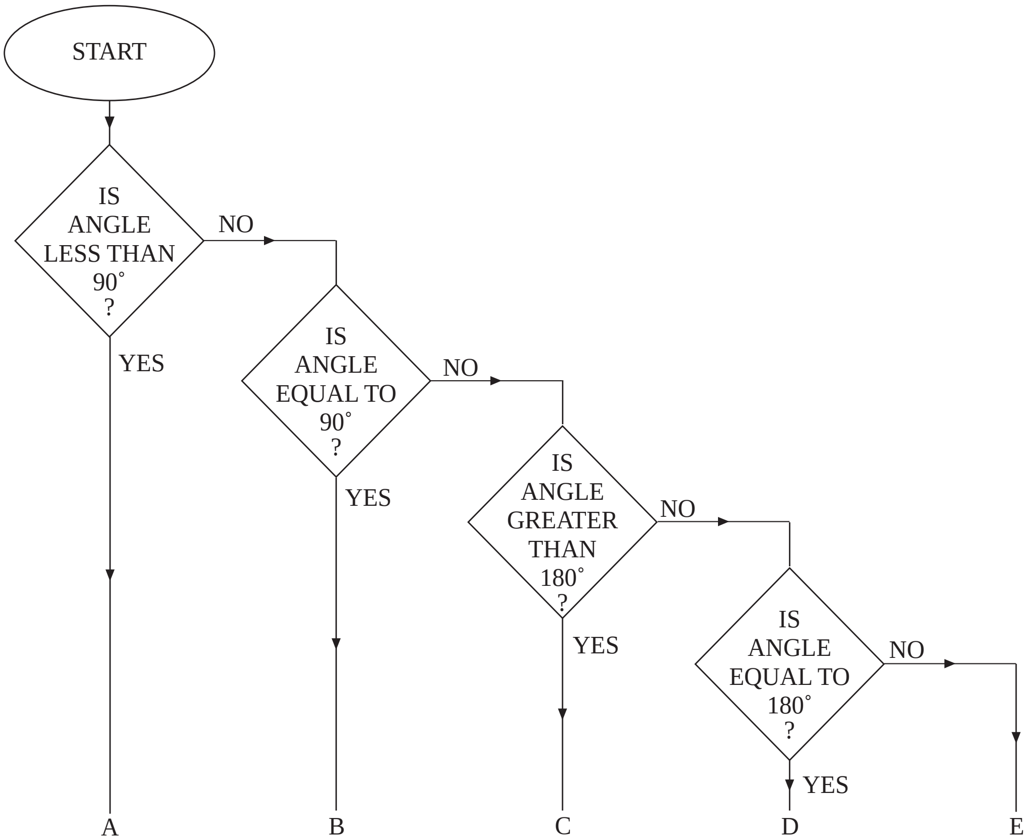
6. You are playing *Snakes and Ladders.*
   (a) Draw a flow chart to describe how to move your counter for one go.
   (b) Describe how you would change your chart when you have an extra turn after throwing a six.

1.3 Using a Flow Chart for Classification

Flow charts can also be used to sort or classify things.

Example 1

This flow chart can be used to classify angles between $0°$ and $360°$. What comes out at the end points A, B, C, D and E ?

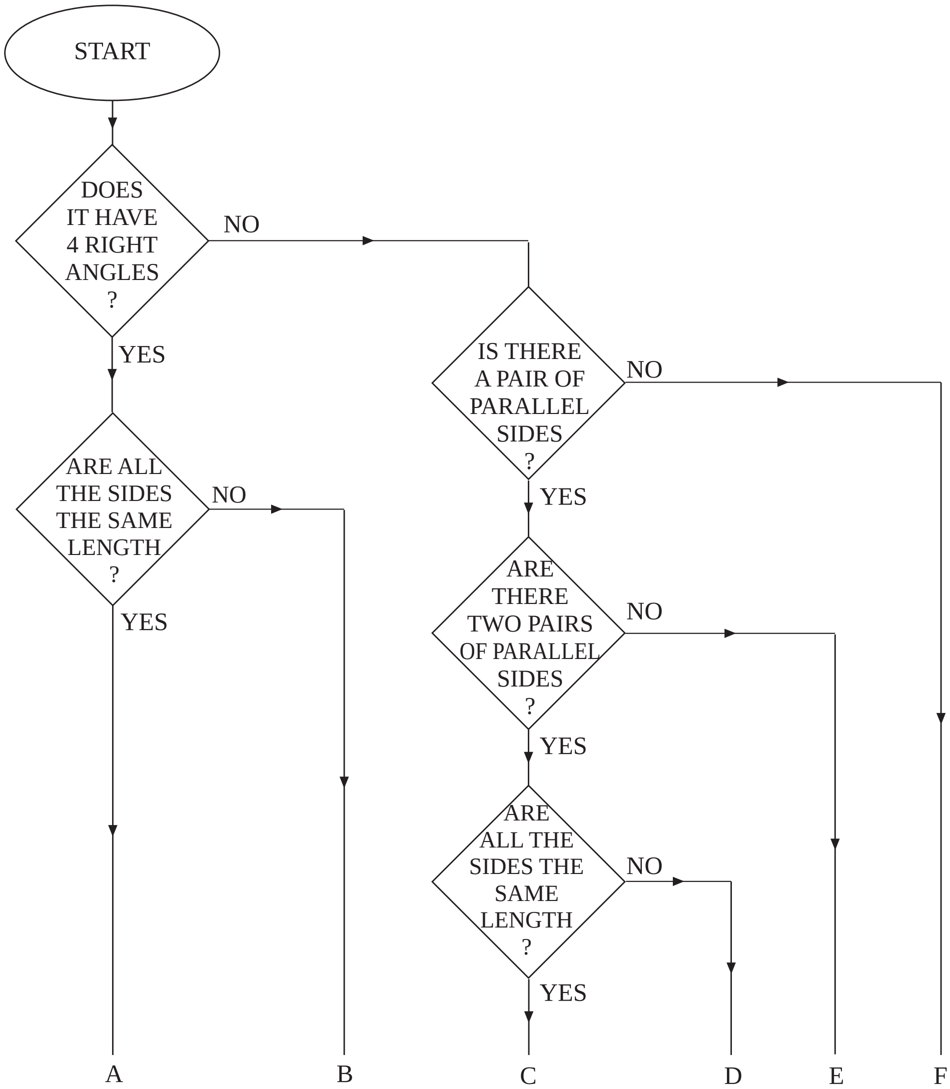


Solution

A	Angles less than $90°$	ACUTE ANGLES
B	Angles equal to $90°$	RIGHT ANGLES
C	Angles greater than $180°$	REFLEX ANGLES
D	Angles equal to $180°$	ANGLES ON STRAIGHT LINES
E	Angles greater than $90°$ but less than $180°$	OBTUSE ANGLES

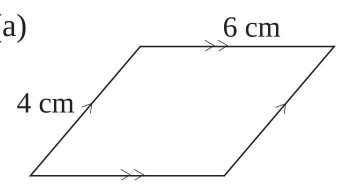
Example 2

The flow chart below can be used for sorting quadrilaterals:

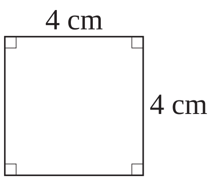


Where would each of these shapes come out?

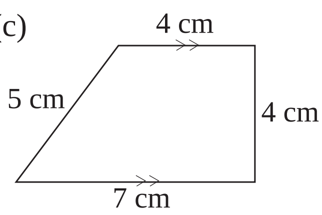
(a)



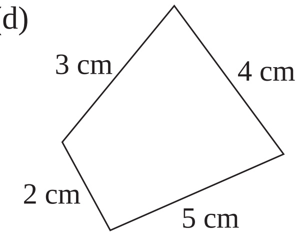
(b)



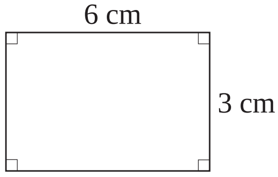
(c)



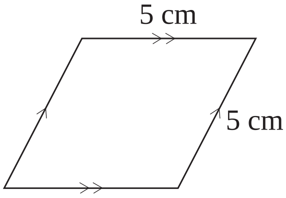
(d)



(e)



(f)



Solution

(a) A parallelogram; comes out at D.

(b) A square; comes out at A.

(c) A trapezium; comes out at E.

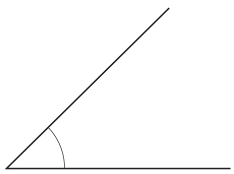
(d) Quadrilateral with no special properties; comes out at F.

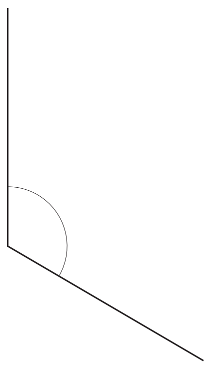
(e) A rectangle; comes out at B.

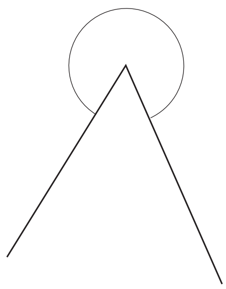
(f) A rhombus; comes out at C.

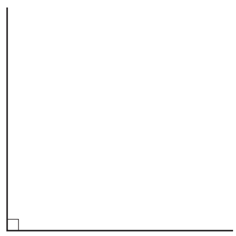
Exercises

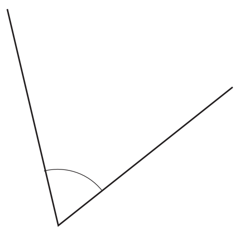
1. The angles below are classified using the flow chart in Example 1. Where does each angle come out of the flow chart?

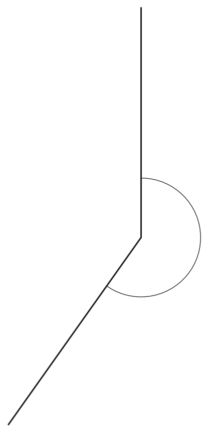
(a)


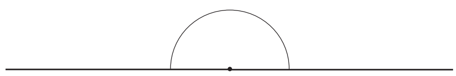
(b)


(c)


(d)


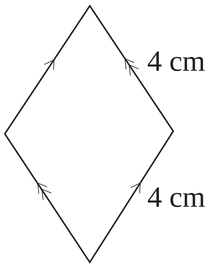
(e)


(f)


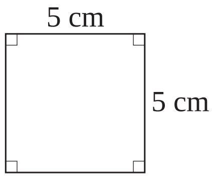
(g)


2. The quadrilaterals below can be classified using the flow chart in Example 2. Where does each quadrilateral come out of the flow chart?

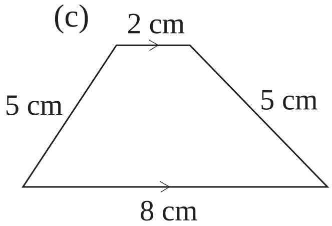
(a)



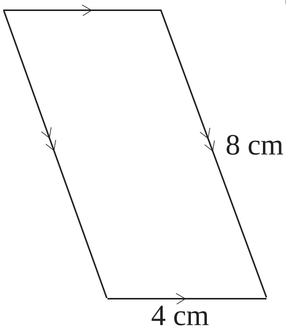
(b)



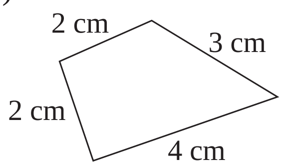
(c)



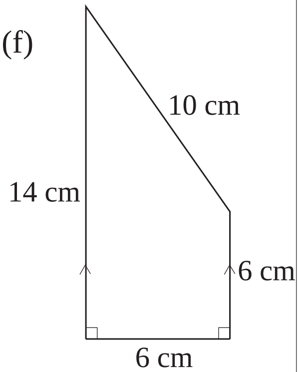
(d)



(e)



(f)



3. Draw a flow chart that will sort trees, into those that shed their leaves during winter, and those that do not.

4. Draw a flow chart that will sort triangles into *equilateral* (all sides of equal length), *isosceles* (two sides of equal length) or *scalene* (all sides of different lengths).

5. Draw a flow chart that will sort polygons into the following categories:

Triangles

Quadrilaterals

Pentagons

Hexagons

Heptagons

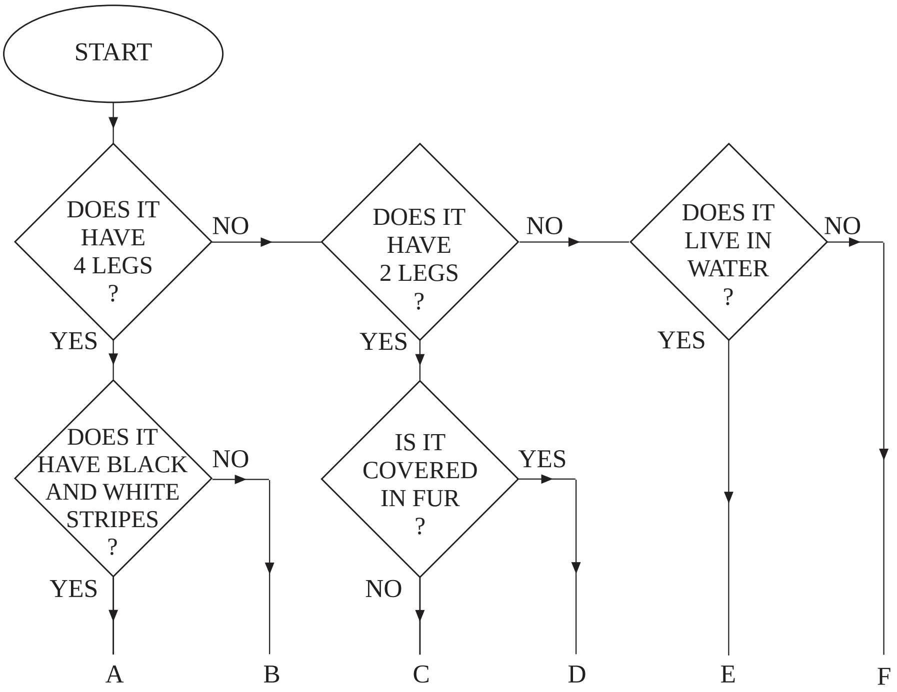
Octagons

Polygons with more than 8 sides

6. The flow chart on the following page can be used to sort animals.

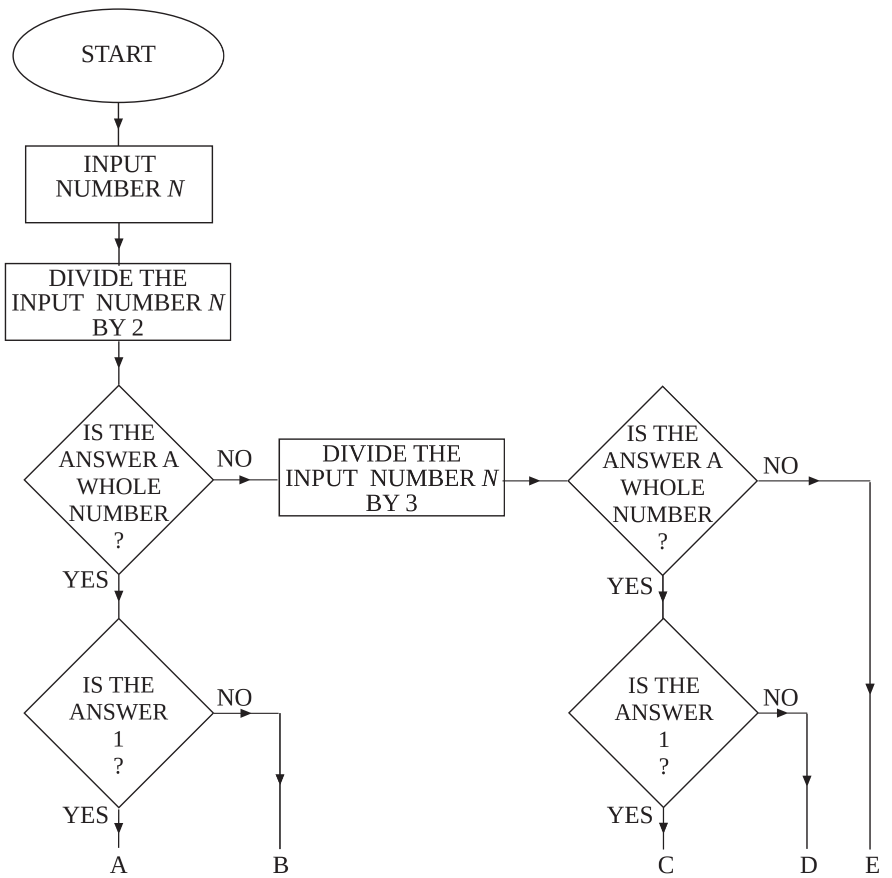
(a) Where would each of these animals come out of the flow chart?

Monkey	*Bird*	*Giraffe*
Horse	*Centipede*	*Elephant*
Zebra	*Pig*	*Kangaroo*
Fish	*Dolphin*	*Cat*



(b) Name one more animal that would come out of the flow chart at each of the end points A, B, C, D and E.

7. The following flow chart will sort numbers. The numbers 1 to 10 are put into this flow chart. Where do they each come out?



8. Draw a flow chart that will classify numbers as *odd* or *even.*

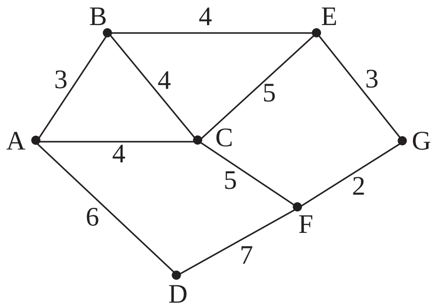
9. Draw a flow chart that will test numbers up to 20 to see if they are prime.

1.4 Networks

In this section we consider networks. Problems can be solved by finding the shortest or quickest route through a network, which can represent a system of roads, pipelines, cables, or anything else that connects different points or places.

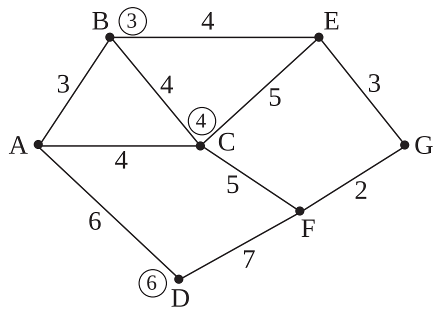
Example 1

Find the shortest route from A to G, using the distances shown on the network opposite:

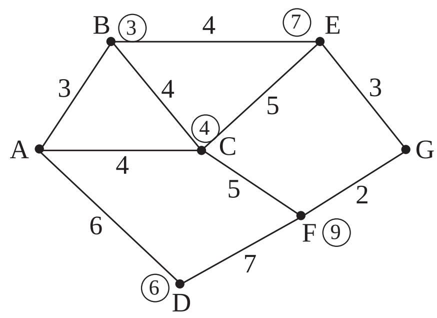


Solution

First find the shortest distances from A to the points B, C and D; these are shown in circles on this diagram:



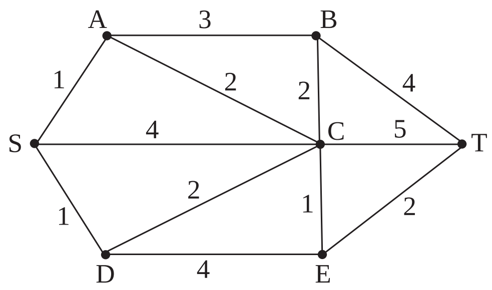
Now work out the shortest distances from A to E and F and put these in circles on the diagram:



It is now possible to see that the shortest route from A to G is 10, using the route, A B E G.

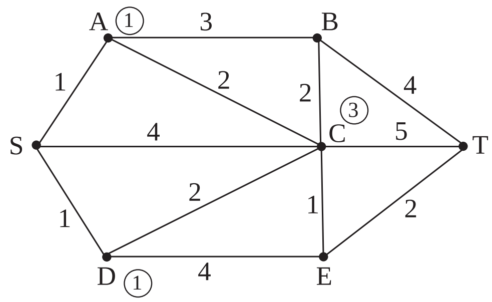
Example 2

Find the shortest route from S to T through this network:

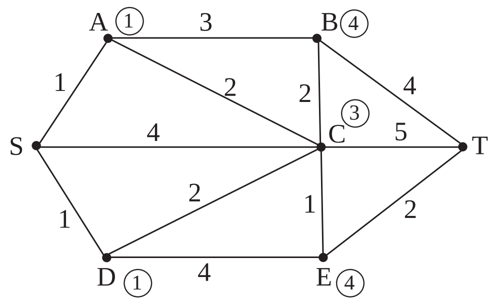


Solution

The diagram opposite shows the shortest routes from S to A, C and D:



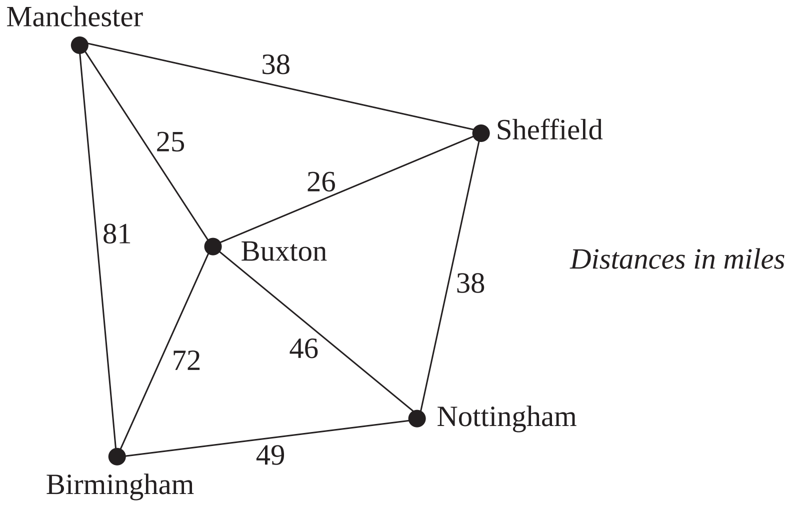
The shortest routes from S to B and E are now added to the diagram:



We can see that the shortest route from S to T is 6, using either route S D C E T or route S A C E T.

Exercises

1. The network diagram below shows the distances between some towns and cities:

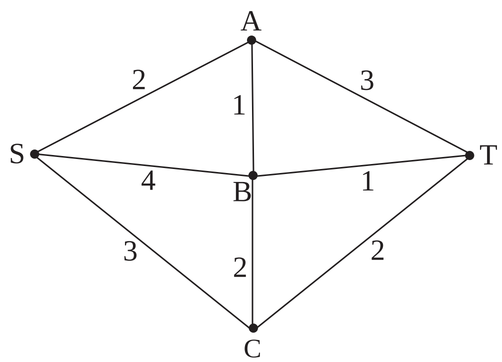


Find the shortest distance and route between:

(a) Manchester and Nottingham,

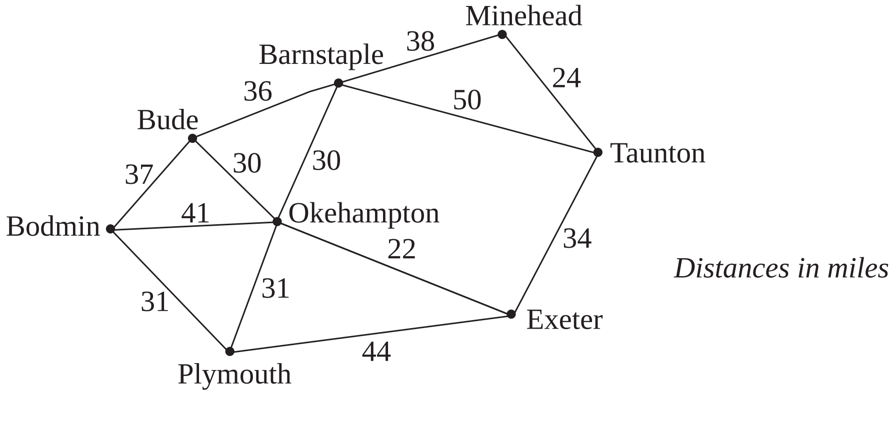
(b) Sheffield and Birmingham.

2. The diagram below shows the shortest journey times, in hours, between 5 towns, A, B, C, S and T:



Find the shortest journey time from S to T and state the route.

3. The network diagram below shows the distances between some towns and cities in the south west of England:

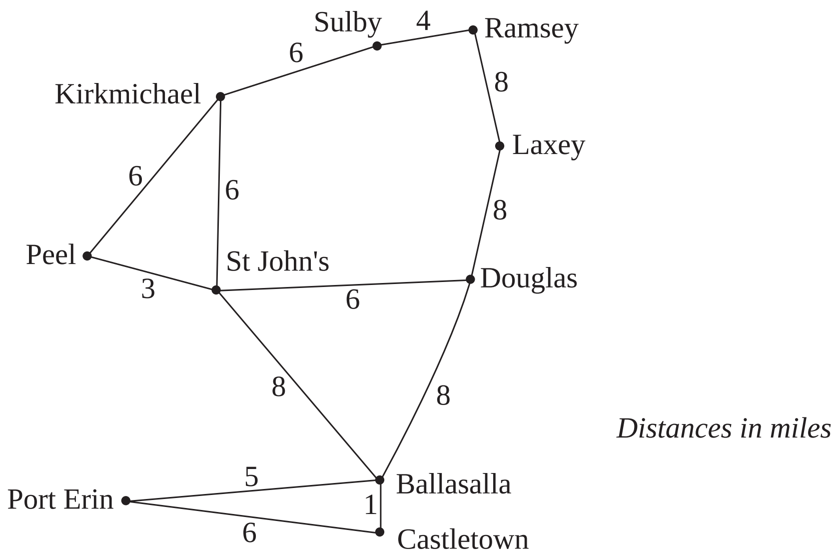


Find the shortest distance and route between:

(a) Bodmin and Exeter,

(b) Plymouth and Minehead.

4. The network diagram below shows some of the places on the Isle of Man, and the distances between them:



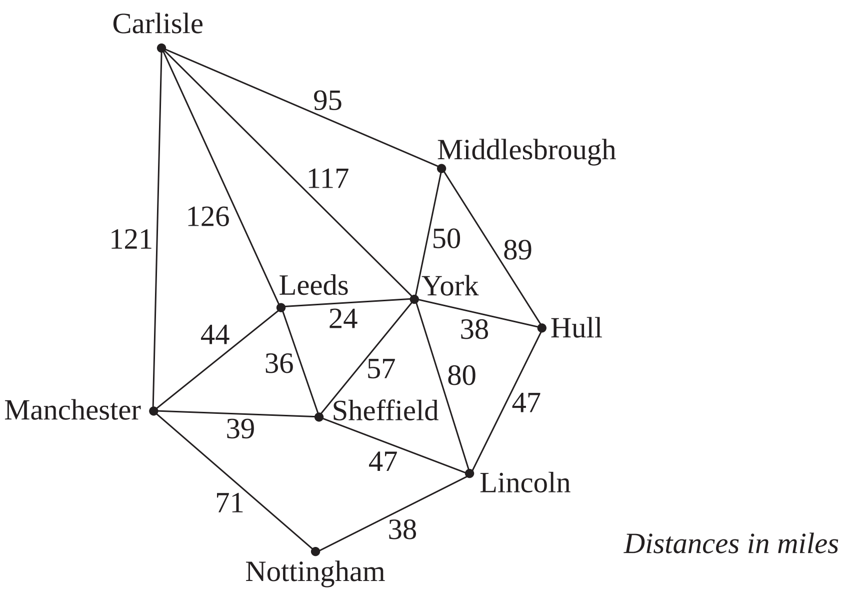
Which is the shortest route and what is the distance from:

(a) Douglas to Sulby,

(b) Ramsey to Port Erin,

(c) Peel to Laxey?

5. Use the network diagram below to find both the shortest route and the distance from Carlisle to Lincoln.



1.5 Critical Path Analysis

When you are planning to carry out a task, critical path analysis can be used to help you find the most efficient way to do it; this works by showing how activities need to be scheduled.

For example, when making your breakfast, you can boil the kettle *and* cook your toast at the same time. You do not have to wait until you have boiled the kettle before you start to make your toast, whereas you *do* have to boil the kettle before you can make a cup of tea.

Example 1

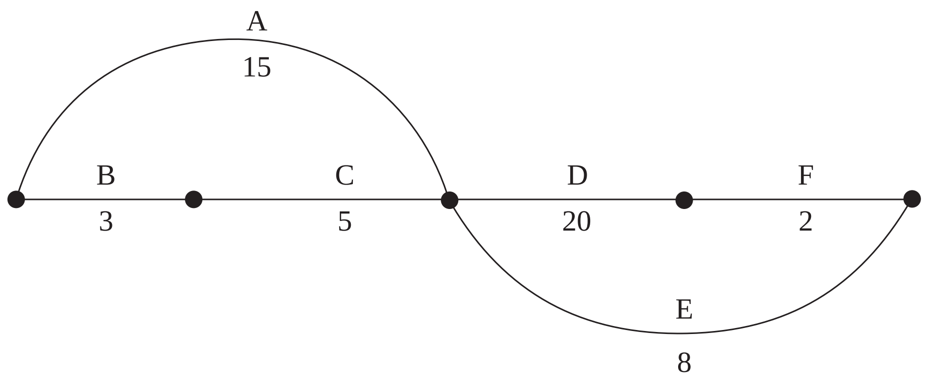
Veronica is going to make a cake. She has six tasks to do, which are listed below:

Activity		*Time needed in minutes*	*Preceded by*
A	Warm oven	15	
B	Weigh ingredients	3	
C	Mix ingredients	5	Weigh ingredients
D	Bake cake	20	Mix ingredients
E	Wash up mixing bowl and utensils	8	Mix ingredients
F	Wash up cake tin	2	Bake cake

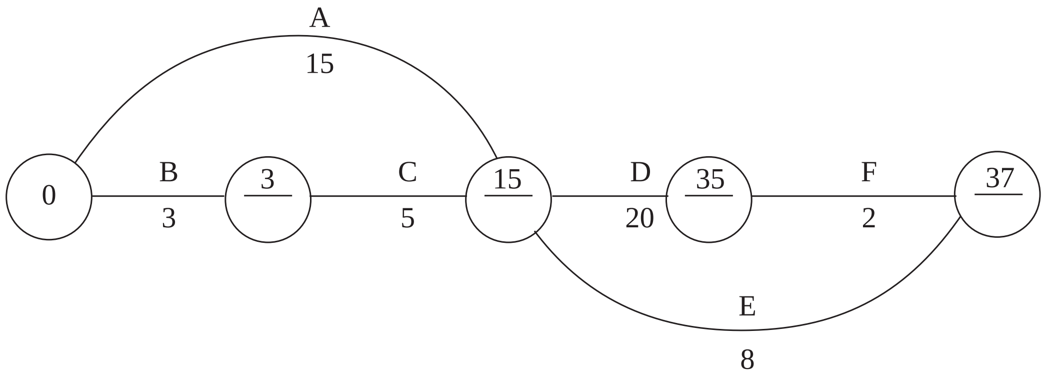
Draw an activity network and find the shortest time to make the cake.

Solution

The first step is to draw an *activity network*, which is a way of showing the data concerning the tasks that have to be completed, how long each one takes, and the order in which they must be undertaken. The activity network for making Veronica's cake is shown below:

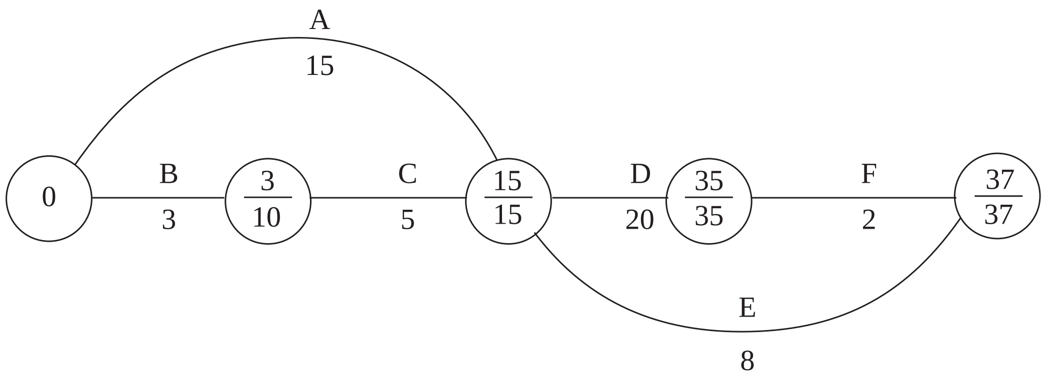


We move through the network from *left to right,* calculating the earliest start time for each activity. For example, D (bake cake) cannot begin until A, B and C have all happened, i.e. the oven is warm and the cake ingredients have been weighed out and mixed. The oven requires at least 15 minutes to warm, so the earliest time that D and E can start is 15 minutes after warming the oven. In turn, this means that F cannot start until 35 minutes have elapsed.



At this stage, we can see that the shortest time to complete the task of baking the cake is 37 minutes.

We then work through the network from *right to left,* calculating the latest start time for each activity. For example, E could begin 29 minutes after the start, and still finish at the same time as F. However, D must begin no later than 15 minutes after the start, for the whole activity to be completed in 37 minutes.



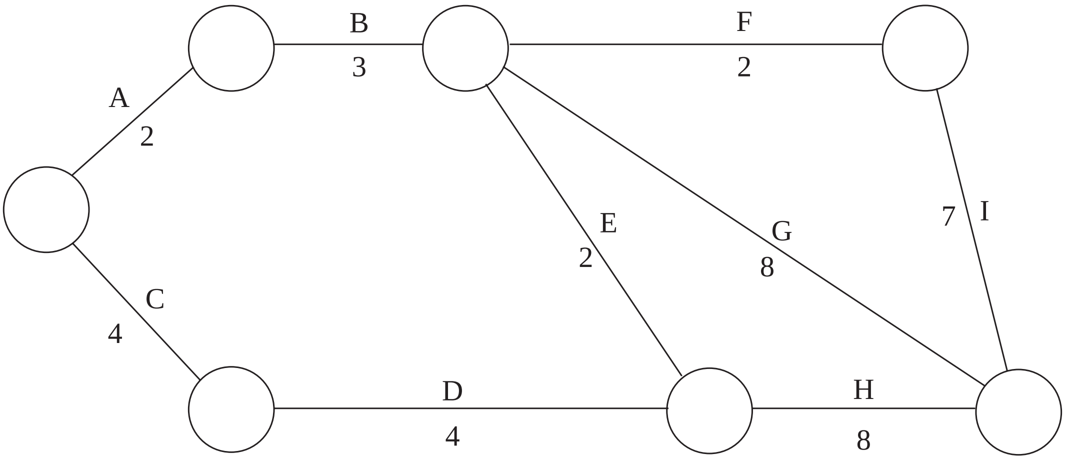
The *critical path* is A D F as these tasks must be completed on time or the whole project will be delayed.

For activity E there is a *float time* of 14 minutes. We use this term because there is a 22 minute period in which the activity must take place, but the task itself takes only 8 minutes. C also has a float time, but this time of 7 minutes duration.

There is *no float time* for tasks on the critical path.

Example 2

On the following activity network the numbers show the time, in minutes, for each task. Complete the earliest and latest finishing times for each task, identify the critical path, and state the shortest completion time for the whole activity.

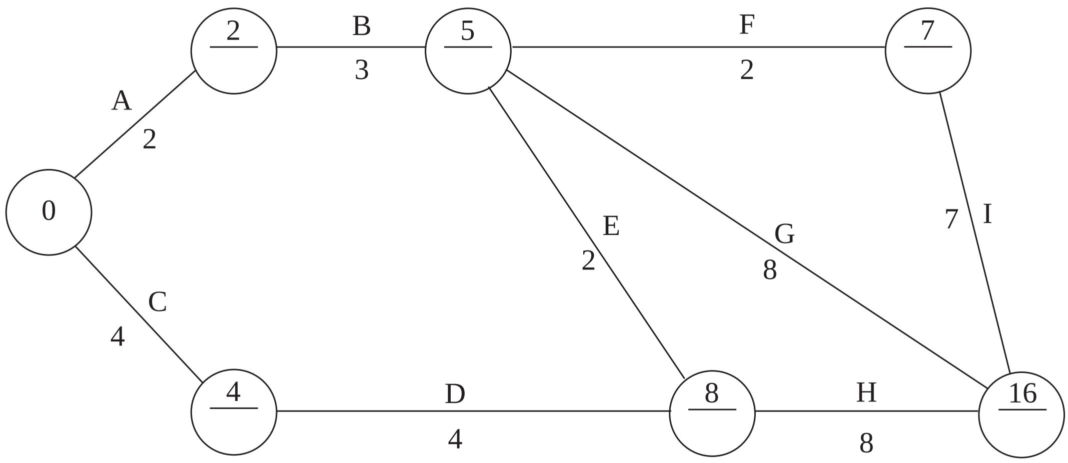


Solution

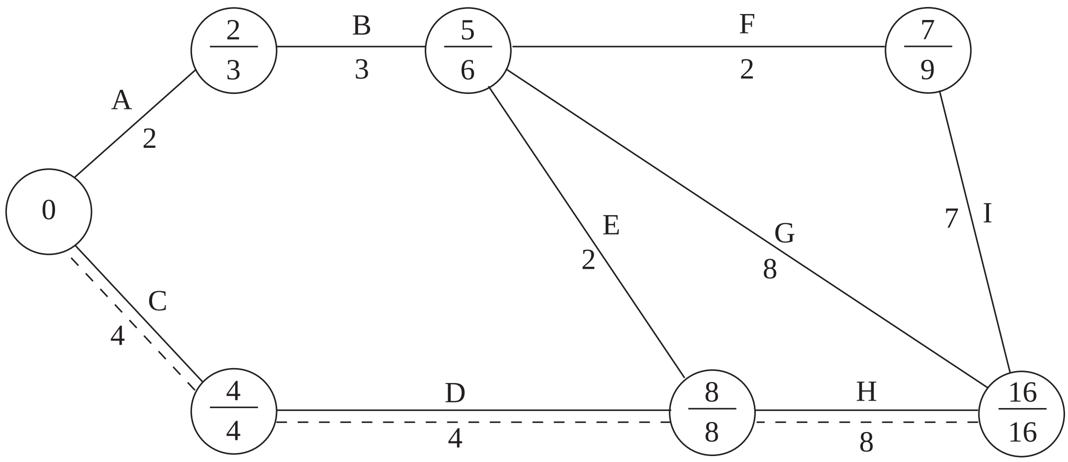
The layout of the activity network, from left to right, tells us the following information about the order in which the tasks must be carried out:

Task	*Time* (minutes)	*Preceded by*
A	2	
B	3	A
C	4	
D	4	C
E	2	A, B
F	2	A, B
G	8	A, B
H	8	A, B, C, D, E
I	7	A, B, F

We first calculate the earliest start time for each activity by working from *left to right* through the activity network:



We now work back from *right to left*, calculating the latest start time for each activity:



So the critical path is C D H, because for this route the earliest start times match the latest start times.

The shortest completion time is 16 minutes.

The diagram shows that there are float times for some activities; for example, 2 minutes for activity I, and 1 minute for both activities F and B.

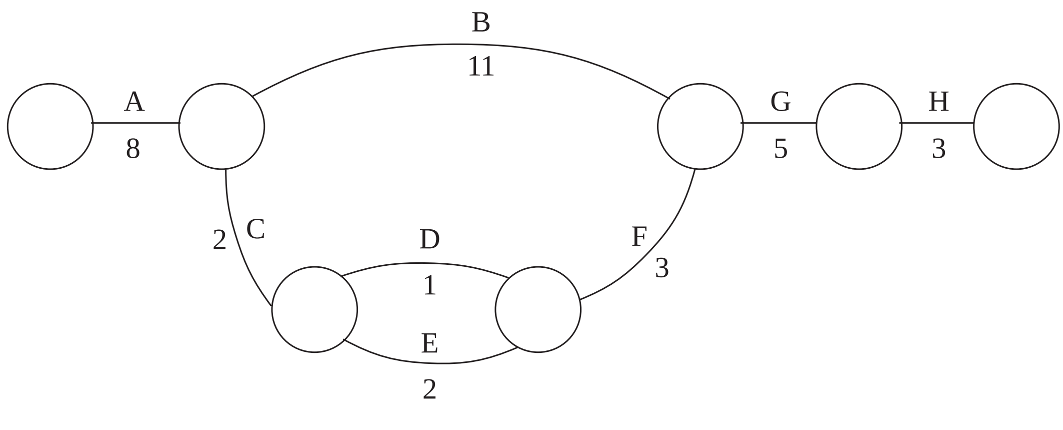
Example 3

The table lists the tasks needed to completely refurbish a kitchen; the times are given in days. Find both the critical path, and the shortest completion time.

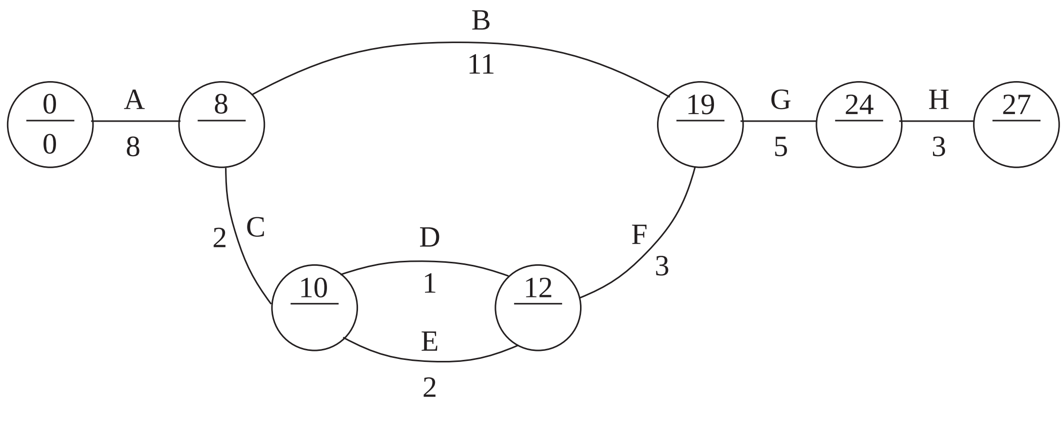
	Task	*Time needed*	*Preceded by*
A	Design kitchen	8	
B	Make kitchen units	11	A
C	Remove old units	2	A
D	Fit new power points	1	C
E	Fit new plumbing	2	C
F	Paint and decorate	3	D, E
G	Fit new units	5	B, F
H	Fix wall tiles	3	G

Solution

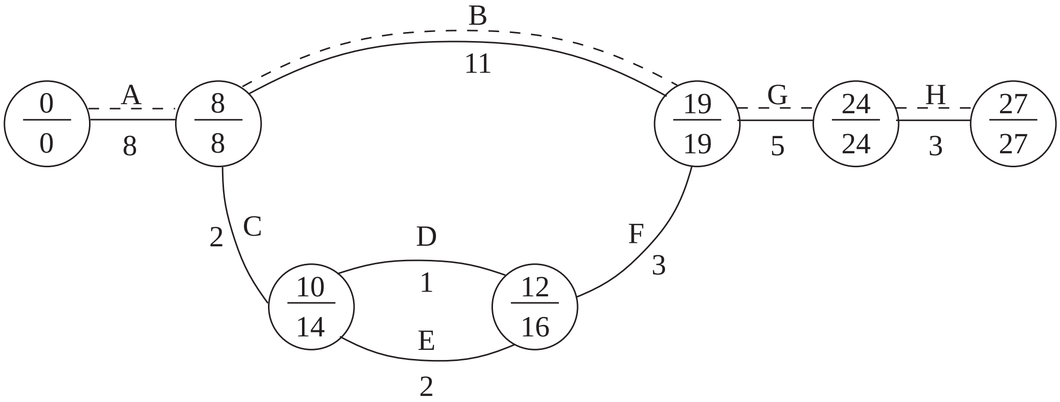
The activity network is shown below:



Now move from left to right through the network to find the earliest start times:



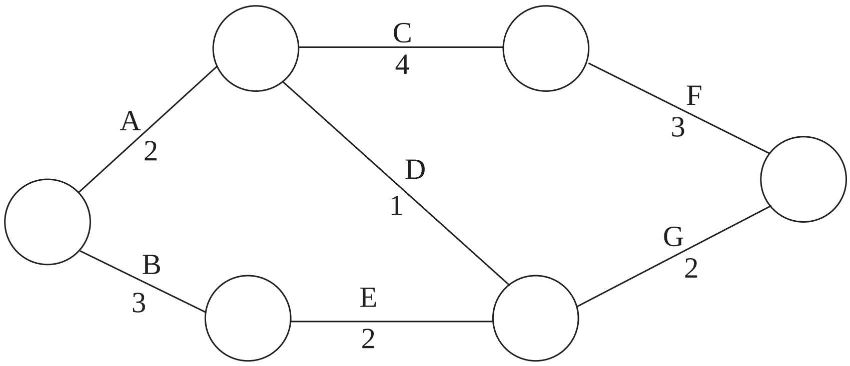
Now work back through the network, putting in the latest start times:



The critical path is A B G H, as shown in the diagram, and the shortest completion time is 27 days.

Exercises

1. The activity diagram below shows the time, in minutes, for different parts of a process. Find the critical path and the shortest possible completion time.

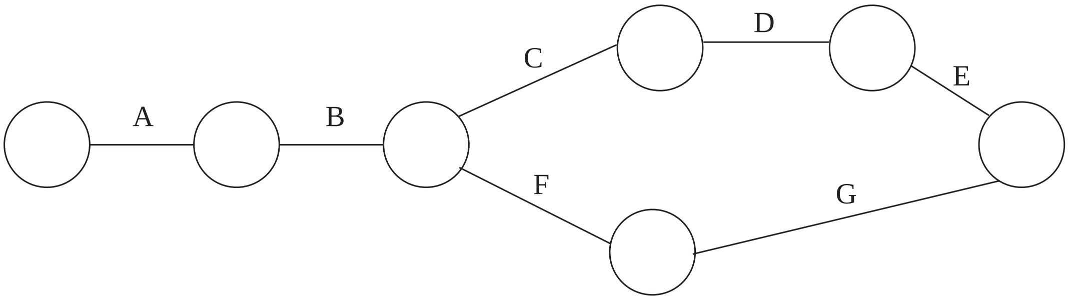


2. Jamil and Halim are making a model garage for their brother's birthday present.

   There are a number of tasks that must be completed to make the garage; these are listed in the table opposite:

	Task	*Time needed (hours)*
A	Design the garage	1
B	Buy materials	2
C	Cut out wooden panels	2
D	Glue panels in place	1
E	Paint garage	2
F	Make cars	3
G	Paint cars	3

The activity network is shown below:

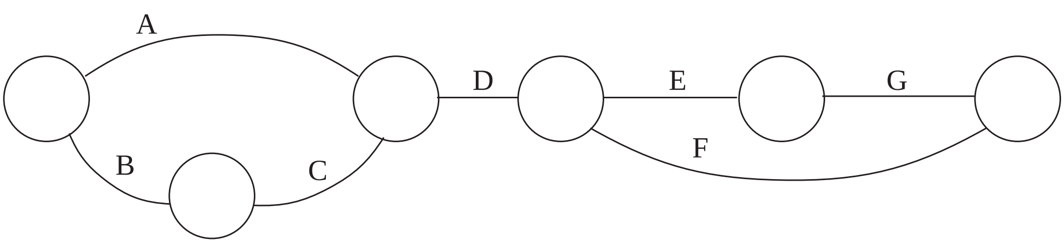


Find the critical path and the shortest possible completion time.

3. The following instructions must be carried out to put up a dome tent:

	Task	*Time (mins)*	*Preceded by*
A	Peg down inner tent	3	
B	Assemble poles	1	
C	Fit poles in flysheet	4	B
D	Erect flysheet	4	A, B, C
E	Peg down flysheet	2	A, B, C, D
F	Hang inner tent from poles	2	A, B, C, D
G	Attach guy ropes	4	A, B, C, D, E

The activity network is shown below:



Find both the critical path, and the shortest time needed to put up the tent.

4. You are going to prepare a meal of chicken and potato pie, peas and gravy. You have to carry out the tasks listed below:

	Task	*Time (mins)*	*Preceded by*
A	Peel potatoes	10	
B	Heat oven	5	
C	Make pie	12	A
D	Cook pie	40	B, C
E	Make gravy	5	B, C
F	Cook peas	8	B, C
G	Cook potatoes	25	A
H	Lay table	2	

(a) Draw an activity network in line with the information given in the table.

(b) Find both the critical path and the shortest time needed to prepare the meal.

(c) Which tasks have a float time? State the float times for these activities.

5. The building of a house is broken down into the tasks listed in the table below. Draw a network diagram and use it to find the critical path and the shortest possible construction time.

	Task	*Time (days)*	*Preceded by*
A	Order materials	5	
B	Lay drains	7	A
C	Lay foundations	7	B
D	Erect blockwork	11	C
E	Roofing work	5	D
F	Install floors	4	E
G	Plumbing and heating	10	F
H	Electrical installation	6	F
I	Install windows	5	D
J	Plastering	6	G, H, I
K	Decoration	5	J
L	Install fittings	3	K
M	Clear site	2	L
N	Lay paths	2	D