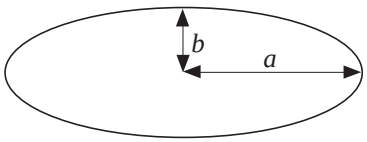
2 Formulae

2.1 Using Formulae

1. 

The area of an ellipse is given by

$$A = \pi ab$$

where a and b are given lengths (as shown).

Find the area of an ellipse when

(a) $a = 4,\ b = 2$

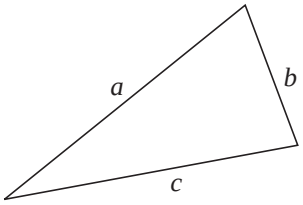
(b) $a = 2,\ b = 4$

(c) $a = b = 3$

2. The perimeter length of a triangle is given by

$$p = a + b + c$$

where a, b and c are the lengths of the three sides of the triangle.



Find the perimeter length when

(a) $a = 1,\ b = 2$ and $c = 3$

(b) $a = 4,\ b = 3$ and $c = 2$

(c) $a = b = c = 3$

If $a = 4,\ b = 3$ and $p = 11$, what is the length c?

3. Euler's formula for the vertices of a shape states that

$$v = e - f + 2$$

where e is the number of edges and f is the number of faces.

Determine v when

(a) $e = 9,\ f = 5$ (b) $e = 6,\ f = 4$

If $v = 8$ and $e = 12$, determine f.

4. Find the value of the function f where $x = 2$ and $y = 3$ and when

(a) $f = x + y$ (b) $f = 4x - 2y$

(c) $f = x^2 + y^2$ (d) $f = \dfrac{x + y}{10}$

(e) $f = \dfrac{x + 2}{y + 1}$ (f) $f = xy - 4$

(g) $f = 2x^2 - y^2$ (h) $f = \dfrac{2x + 1}{y}$

(i) $f = x^2\ y^2$ (j) $f = (x + y)^2$

5. Repeat Question 4 with $x = 3$ and $y = 4$.

6. Find the value of the functions

$$f = \sqrt[3]{xyz} \quad \text{and} \quad g = \frac{x + y + z}{3}$$

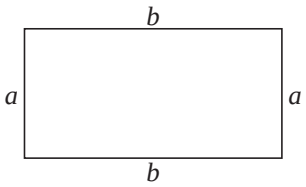
when

(a) $x = y = z = 2$ (b) $x = 1,\ y = 2,\ z = 3$ (c) $x = 2,\ y = 3,\ z = 4$

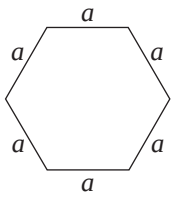
2.2 Construct and Use Simple Formulae

1. Find a formula for the perimeter, P, of each of the shapes below.

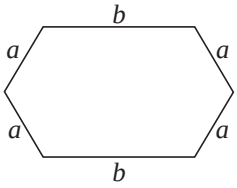
(a)



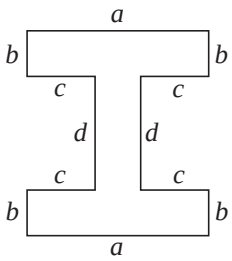
(b)



(c)



(d)



In each of the following cases, use the formula to find the value of P:

(a) when $a = 5$ cm, $b = 10$ cm;

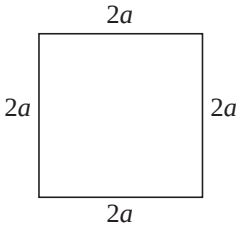
(b) when $a = 5$ cm;

(c) when $a = 5$ cm, $b = 10$ cm;

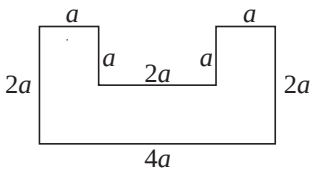
(d) when $a = 8$ cm, $b = 2$ cm, $c = 3$ cm, $d = 5$ cm.

2. Find the area, A, and perimeter length, P, for each of these shapes.

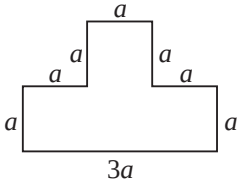
(a)



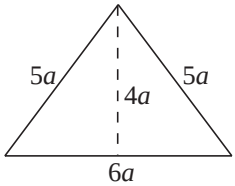
(b)



(c)



(d)



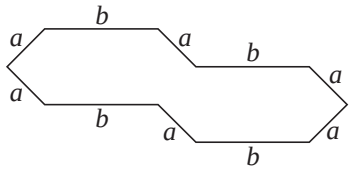
In each case, find A and P when $a = 2$ cm.

3. Petrol costs 58p per litre. What is the cost, in pounds, of x litres?

   Use your formula to find the cost of

   (a) 10 litres (b) 50 litres

4. A piece of wire is bent into the shape shown.



   Find the total length of the wire used.

   Simplify your answer.
   (All dimensions are in centimetres.)

   If the total length of the wire is 24 cm and $a = 2$ cm, what is the length of b?

5. A train operating company calculates its fares using the formula 12p per mile. If you travel a distance x miles, what is the formula for the cost in

   (a) pence (b) pounds

   Use your answer to (b) to find the cost when you travel

   (i) 10 miles (ii) 100 miles (iii) 200 miles (iv) 1000 miles

6. A removals firm charges a fixed cost of £100 plus a payment of £2 for every mile travelled by its van. If the van travels x miles, write down a formula for the total cost.

   Find the actual costs for journeys of

   (a) 50 miles (b) 100 miles (c) 200 miles

2.3 Revision of Negative Numbers

1. *Without a calculator*, answer the following questions.

   (a) $5 - 2$ (b) $-5 + 6$ (c) $-7 + 4$

   (d) $12 - 14$ (e) $-10 + (-5)$ (f) $-10 - (-5)$

   (g) $-15 - 5$ (h) $16 - (-8)$ (i) $4 \times (-2)$

   (j) $(-4) \times (-2)$ (k) $(-8) \times 3$ (l) $(-10) \times (-5)$

   (m) $14 \div (-7)$ (n) $(-10) \div 2$ (o) $(-10) \div (-2)$

   (p) $20 \div (-5)$ (q) $(-15) \div (-3)$ (r) $(-16) \div 4$

2. Evaluate, *without using a calculator*:

   (a) $(-2)^2$ (b) $(-1) \times 1$ (c) $(-1) \times (-1)$

   (d) $(-4)^2$ (e) $3^2 + (-4)^2$ (f) $(-6)^2 + (-8)^2$

   (g) $(-1)^2 \div (-2)^2$ (h) $(-2)^2 \times (-3)^2$

(i) $4 \times (-5) + \big((-100) \div (-4)\big)$ (j) $(-20) \div (-5) + (-2)^2$

(k) $\sqrt{(-3)^2 + (-4)^2}$ (l) $\big[(-3) \times (-4)\big] \div (-2)$

(m) $(-2)^3$ (n) $(-1)^2 \times (-1)$ (o) $(-5)^3$

3. The outside temperature was monitored every 4 hours for one day.

Here is the recorded information.

Time	**Temperature** °C
00.00	–11
04.00	–7
08.00	–1
12.00	5
16.00	6
20.00	0
24.00	–5

(a) What is the difference between the lowest and highest temperatures?

(b) What is the difference between the temperature at

(i) 04.00 and 08.00 (ii) 04.00 and 12.00

(iii) 16.00 and 24.00 (iv) 08.00 and 20.00

(v) 00.00 and 24.00 (vi) 20.00 and 24.00?

2.4 Substitution into Formulae

1. The speed of a bike, v metres per second, is given by the formula

$$v = u + ft$$

when u is its initial speed (in m/s), f its acceleration (in m/s^2) and t, the time in seconds.

Determine v when

(a) $u = 0,\ f = 5$ and $t = 10$ (b) $u = 20,\ f = 2$ and $t = 5$

(c) $u = 20,\ f = 0$ and $t = 5$ (d) $u = 40,\ f = -5$ and $t = 5$

(e) $u = 40,\ f = -5$ and $t = 8$

In each case, briefly describe the motion of the bike.

2. The Fahrenheit scale, (F) and the Celsius scale (C) are related by the formula

$$F = \frac{9}{5}C + 32$$

(a) Give the following temperatures in Fahrenheit.

(i) Normal body temperature: 37 °C

(ii) Boiling point of water: 100 °C

(b) Give the following temperatures in degrees Celsius.

(i) Freezing point of water: 32 °F

(ii) Singapore's average daily temperature: 86 °F

3. If $x = 3,\ y = 4$ and $z = 7,$ find the values of the following expressions:

(a) $5yz$ (b) xyz (c) $\frac{12}{xy}$

(d) $\frac{xy}{18}$ (e) $2x + 3y$ (f) $x - 5y + 2z$

(g) $xy + yz$ (h) $x^2 + y^2$ (i) $2z^2 + y$

(j) $y^2 + x^3$ (k) xy^2 (l) $4x^2y^2$

4. If $a = 3,\ b = 2$ and $c = -1,$ find the value of each of the following.

(a) $a^3 + b^3 + c^3 - 2abc$ (b) $(2a + b - c)(4b - 3c)$

(c) $(a - b)^2 - (b - c)^2$ (d) $\frac{a}{b} + \frac{b}{c} - \frac{c}{a}$

(e) $\frac{a+1}{2} - \frac{b+c}{4} + \frac{c-a}{3}$ (f) $a^b - c^a + b^a$

(g) $2a - 3b^2 + 3abc^2$ (h) $a^2 + 3b^3 - 4c^5$

(i) $\frac{a+b}{c} - \frac{ab-c}{b}$ (j) $\frac{3a-b}{b-c} + \frac{a+c}{b-a}$

(k) $\frac{2c^2 - 3a}{bc - a} - \frac{4b}{3a}$ (l) $\frac{a^2 - b^2}{c^2} - \frac{a^3 - c}{(c - 3b)}$

5. Find the value of $x^3 + 2xy^2 + y^3$ when $x = 2$ and $y = -1.$

6. Find the value of $\frac{x+1}{x-1} + \frac{2x-1}{2x+1}$ when $x = -2.$

7. Find the value of $2ab + 3bc^2$ when $a = 0,\ b = 5$ and $c = -2.$

8. The distance travelled, s metres, by a car is given by

$$s = ut + \frac{1}{2} ft^2$$

Here u is the car's initial speed (in m/s), t the time (in seconds) and f the acceleration (in m/s^2).

(a) Find s when

(i) $u = 0,\ t = 10,\ f = 5$ (ii) $u = 20,\ t = 5,\ f = 6$

(iii) $u = 50,\ t = 4,\ f = -5$ (iv) $u = 60,\ t = 10,\ f = -2$

(b) If the car travels 400 metres in 5 seconds with initial speed of 40 m/s, what is its acceleration?

2.5 More Complex Formulae

1. It is given that $v^2 = u^2 + 2as$. Find the values of u when $v = 0.8,\ a = 0.05$ and $s = 2.8$.

2. It is given that $y = \dfrac{18 - 5x}{2y}$. Find

(a) the values of y if $x = -6.4$

(b) x if $y = 2\dfrac{1}{2}$

3. If $S = \dfrac{n}{2}\left[2a + (n-1)d\right]$, find

(a) the value of S when $n = 10,\ a = -2$ and $d = \dfrac{1}{2}$

(b) a when $S = 440,\ n = 10$ and $d = 5$.

4. The arithmetic mean, A, geometric mean, G, and harmonic mean, H, of three numbers, are given by the formulae

$$A = \frac{a+b+c}{3},\ G = (abc)^{\frac{1}{3}},\ H = \frac{3}{\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)}$$

Find A, G and H for the following sets of numbers.

(a) $a = 2,\ b = 3,\ c = 4$ (b) $a = 1,\ b = 3,\ c = 5$

(c) $a = b = c = 3$ (d) $a = 2.5,\ \mathrm{b} = 3,\ c = 3.5$

What do you notice about the values of A, G and H?

5. Find z, given by each of the following formulae, for the given values of x and y.

(a) $\dfrac{1}{z} = \sqrt{\dfrac{1}{x^2} + \dfrac{1}{y^2}},\quad x = 2,\ y = 3$

(b) $z^2 = x^2 + y^2,\quad x = 3,\ y = -4$

(c) $\dfrac{1}{z} = \dfrac{x+y}{x^2+y^2}, \quad x = -2, \ y = 4$

(d) $\dfrac{x}{z} = \dfrac{x}{y} + 1, \quad x = 5, \ y = 2$

(e) $\dfrac{1}{z^2} = \dfrac{1}{x^2} + \dfrac{1}{y}, \quad x = 4, \ y = 3$

6. The formula $v^2 = u^2 + 2fs$ connects the initial, (u), and final, (v), speeds of a car, with its acceleration, (f), and distance travelled, (s).

Find v (in m/s) when

(a) $u = 0, \ f = 10 \text{ m/s}^2, \ s = 100 \text{ m}$

(b) $u = 20 \text{ m/s}, \ f = 5 \text{ m/s}^2, \ s = 50 \text{ m}$

(c) $u = 75 \text{ m/s}, \ f = -10 \text{ m/s}^2, \ s = 25 \text{ m}$

2.6 Changing the Subject

1. Make s the subject of each of the following:

(a) $2s - 8p = 14$ (b) $28 = 4s + r - s$ (c) $10 - 2s = 12r + 2s$

2. In each of the following, make y the subject:

(a) $y + x = 6$ (b) $m + y = 2 - n$ (c) $\dfrac{k}{5} = \dfrac{y}{3}$

(d) $3 + m = d + y$ (e) $5 = y - 3m$ (f) $2y + 6 = 48 + 2x$

3. (a) Given that $4a + b = c - a$, express a in terms of b and c.

(b) Given that $x - y = 3z$, express y in terms of x and z.

(c) Given that $pq = r$, express q in terms of p and r.

(d) Given that $a + b = 8c + 7$, express c in terms of a and b.

4. Make a the subject of the following formulae:

(a) $a + x = b$ (b) $a + h = k$ (c) $a - m = n$

(d) $a - k = h$ (e) $a - b = c + d$ (f) $a + c = d + e$

(g) $y + a = x$ (h) $z - a = 2k$ (i) $p = a - q$

(j) $5k = p - a$ (k) $7k = p + a$ (l) $a - b - c = k^2$

(m) $b - a + k = h^3$ (n) $m + n + a = k$ (o) $m - n - a = h$

(p) $7k - h - a = 2a$ (q) $5pq - a = p^2 - q$ (r) $3xy + a = x^2y$

(s) $5a = 15$ (t) $ax = 3y$ (u) $xay = 3k$

(v) $2xy = 3ak$ (w) $ak = p - q + k$ (x) $ax^2 = 5y - 4$

5. Make a the subject of the given formula.

(a) $ax = y$ (b) $a(p - 4) = q$ (c) $ax + by = c$

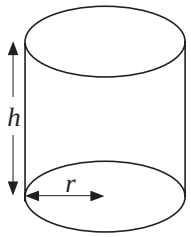
(d) $p(a + b) = c$ (e) $2a - 3m = 4a - 7$ (f) $5b - 2a = 3c$

(g) $\dfrac{a}{m} + b = c$ (h) $x = \dfrac{2a}{3} + 5z$ (i) $\dfrac{p + a}{5} = 3p$

(j) $R = m(a + g)$ (k) $2b = ax + a$ (l) $2m = 65 - 4a$

6. (a) The volume of a cylinder is given by

$$V = \pi r^2 h$$



(i) Make h the subject of this equation.

(ii) Find h when $r = 3$ cm and $V = 350$ cm^3.

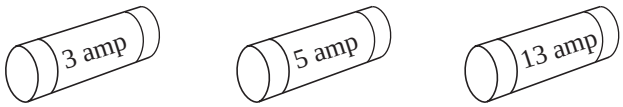
(b) The total surface area is given by

$$s = 2\pi r^2 + 2\pi rh$$

(i) Make h the subject of this equation.

(ii) Find h when $r = 3$ cm and $s = 300$ cm^2.

7. Electrical fuses are available as shown.



The correct fuse to use for an electrical appliance can be calculated using this formula,

$$F = \frac{P}{240}$$

where F = Fuse rating in amps,
P = Power rating in watts.

(a) Which fuse should be fitted for a toaster with power rating 1100 watts?

(b) An electric heater needs a 13 amp fuse. What is the largest power rating the heater could have?

(SEG)

8. The length of a man's forearm (f cm) and his height (h cm) are approximately related by the formula

$$h = 3f + 90$$

(a) Part of the skeleton of a man is found and the forearm is 20 cm long. Use the formula to estimate the man's height.

(b) A man's height is 162 cm. Use the formula to estimate the length of his forearm.

(c) George is 1 year old and he is 70 cm tall. Find the value the formula gives for the length of his forearm and state why this value is impossible.

(d) Use the formula to find an expression for f in terms of h.

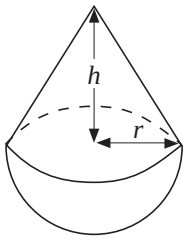
(MEG)

2.7 Further Change of Subject

1. The volume of a cylinder is given by

$$V = \pi r^2 h$$

where r is the base radius and h the height.



(a) Make r the subject of the formula.

(b) Find r when $V = 300 \text{ cm}^3$ and $h = 5 \text{ cm}$

2. The volume of a toy, consisting of a base hemisphere and cone top, is given by

$$V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

Make h the subject of this equation and find h when $V = 300 \text{ cm}^3$ and $r = 3 \text{ cm}$.

3. The surface area of a sphere is given by

$$S = 4\pi r^2$$

(a) Make r the subject of this equation.

(b) Find r when (i) $S = 100 \text{ cm}^2$ (ii) $S = 200 \text{ cm}^2$

By what factor does the radius change when the surface area is doubled?

4. Make x the subject of

(a) $y = 4x + 2$ (b) $y = 1 - 3x$ (c) $y = mx + c$

(d) $y = \dfrac{1}{x+1}$ (e) $y = 1 + \sqrt{x}$ (f) $y = \dfrac{1}{1+\sqrt{x}}$

(g) $y = \sqrt{\dfrac{5x}{a}}$ (h) $y = \sqrt{x+1}$ (i) $\dfrac{1}{y} = \dfrac{1}{x} + 1$

(j) $\dfrac{1}{y} = \dfrac{2}{3} - \dfrac{1}{x}$ (k) $y = \dfrac{1}{4} + \dfrac{1}{x}$ (l) $y = \dfrac{4}{\sqrt{2+x}}$

5. If $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$, make u the subject of this formula. Find u when

(a) $f = 5$ and $v = 1$ (b) $f = 3$ and $v = -2$

6. The percentage profit, p, on the sale of an item is given by the formula

$$p = \frac{100(s-c)}{c}$$

where s is the selling price and c is the cost price.
Express c in terms of s and p.

(MEG)

7. Students conduct an experiment to find g, the acceleration due to gravity.

They measure the time, T seconds, for one complete swing of a pendulum of length L centimetres.

The formula for g is

$$g = \frac{4\pi^2 L}{T^2}$$

(a) Find g when $L = 39.24$ and $T = 1.26$.
Take $\pi = 3.142$ or use the π button on your calculator.

(b) Rearrange the formula to express T in terms of L, π and g.

(SEG)

2.8 Expansion of Brackets

1. Copy and complete the following multiplication tables. Some have been done for you.

(a)

x	$y-2$	$3y$	$\frac{6}{y}$	$4-3y$
-1		$-3y$		
$\frac{1}{4}$				$1-\frac{3}{4}y$
2				
$-\frac{1}{2}$			$-\frac{3}{y}$	

(b)

x	$-k$	$\frac{2k}{3}$	$\frac{2}{3k}$	$2-2k$
6	$-6k$			
-4			$-\frac{8}{3k}$	
3				
$-\frac{1}{2}$				

2. Remove the brackets in each of the following algebraic expressions.

(a) $2(u-3)$ (b) $8(v+7)$

(c) $4(2x+3y)$ (d) $6(5a-b)$

(e) $-2(p-q)$ (f) $-5(a+b)$

(g) $-3(-2u-3v)$ (h) $8(-2u-3v)$

(i) $\frac{1}{2}(10p-6q)$ (j) $\frac{1}{5}(20x-15)$

(k) $-(b+c)$ (l) $-(p-q)$

(m) $-x(p+q)$ (n) $-y(-x+y)$

(o) $-(-p-q)$ (p) $-(-t+r)$

(q) $\frac{1}{2}\left(\frac{2}{3}a-\frac{4}{5}b\right)$ (r) $6a\left(\frac{1}{3}b-\frac{5}{6}c\right)$

3. Simplify each of the following algebraic expressions.

(a) $(3x-2y)+(4x-y)$ (b) $(p-m)+(m-2p)$

(c) $5(x-2)+3(4-x)$ (d) $(3a+2b)-(a-b)$

(e) $2(3m+n)-3(m-3n)$ (f) $(x-y)-(y-z)-(z-x)$

(g) $3a(b-c)+(3b-2)a$ (h) $m(m-n)-n(n-m)$

(i) $x(y-z)+y(z-x)+z(x-y)$ (j) $3(2y+5z)-4(2y-x)$

4. Multiply out and simplify each of the following expressions.

(a) $6(3x+y)$ (b) $5z(z-2y)$

(c) $\frac{1}{2}(2xy-4yz)$ (d) $q(p+2r-3s)$

(e) $(p+q)(r+s)$ (f) $(x+y)(z+2w)$

(g) $(3a+b)(a+c)$ (h) $(m+2n)(2p+3q)$

(i) $(a-b)(c+d)$ (j) $(2e-f)(2g-h)$

(k) $(3p-4q)(s+2t)$ (l) $(a+7)(2b+5)$

(m) $(x+3)(x+4)$ (n) $(a+5)(a-3)$

(o) $(x-7)(x-6)$ (p) $(3+c)(6-c)$

(q) $(1-3x)(4+3x)$ (r) $(2p+3)(p+5)$

(s) $(4x + 5y)(2x + 3y)$ (t) $(d - 7)(d - 5)$

(u) $(a + 5)^2$ (v) $(x - 3)^2$

(w) $(b + 2)^2$ (x) $(e - 4)^2$

(y) $(2x + 1)^2$ (z) $(3x - 2)^2$

2.9 Factorisation

1. Factorise the following:

(a) $2x + 4$ (b) $9 - 3x$

(c) $2 + 10x$ (d) $-5 - 15x$

(e) $x^2 + 2x$ (f) $x - 3x^2$

(g) $4x + 2x^2$ (h) $3x^2 - 9x$

(i) $10x - 5x^2$ (j) $7x^2 + 21$

(k) $3x^2 - x^3$ (l) $2x + 8x^3$

(m) $2x^3 + 10x^2$ (n) $4x^2 - 4$

2. The following expressions have been partly factorised.
Complete the factorisation.

(a) $2x^2 - 4x = 2(x^2 - 2x) = \ ?$ (b) $10x - 5x^2 = x(10 - 5x) = \ ?$

(c) $4x^3 + 8x = 4(x^3 + 2x) = \ ?$ (d) $8xy + 16x^2 = x(8y + 16x) = \ ?$

(e) $5xy + 10x^2y^2 = 5(xy + 2x^2y^2) = \ ?$

3. Factorise the following:

(a) $10a - 15b$ (b) $50py - 120p$

(c) $24abc - 8ab$ (d) $6abc + 12bcd$

(e) $16m^2 + 12n^2$ (f) $p^2y + p^2y^2$

(g) $18s^2t - 12st^2$ (h) $10a + 15a^2$

(i) $c - c^2$ (j) $2a^2b^2 - 8a^2b$

(k) $m^2n - mnl$ (l) $6xy - 3y + 9x$

(m) $pqr + p^2 + pr$ (n) $abc + a^2b + bc$

(o) $8abc + 6ab^2c + 4abc^2$ (p) $5s^2t - 3st - 4st^2$