15 Variation

15.1 Simple Ratios

1. Simplify each of the following

(a)	6 : 2	(b)	8 : 4	(c)	3 : 9
(d)	10 : 5	(e)	15 : 30	(f)	7 : 14
(g)	10 : 100	(h)	4 : 16	(i)	5 : 25
(j)	1.3 : 2.6	(k)	24 : 15	(l)	8 : 24
(m)	9 : 36	(n)	1.5 : 6	(o)	4.5 : 18

2. A train contains 50 1st class passengers and 150 standard class passengers.
Find
   (a) the ratio of 1st class to standard class passengers,
   (b) the ratio of standard to 1st class passengers.

3. The sides of a room are of lengths 6 m and 4.5 m. Find the ratio of the larger to the shorter side.

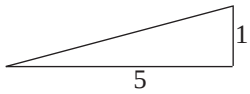
4. A ballet corps has 55 female members and 10 male members.
What is the ratio of
   (a) female to male members,
   (b) male to female members?

5. Lemonade and orange juice are mixed in the ratio of 2 : 1.
How much lemonade is mixed with
   (a) 10 cm^3 of orange juice,
   (b) 50 cm^3 of orange juice?

6. In a bean salad the ratio of red beans to haricot beans is 3 : 1.
If there are 20 haricot beans in a serving, how many
   (a) red beans are there,
   (b) beans in total are there?

7. The ratio of RED Smarties in a box to other colours is 1 : 8.
   (a) If there are 4 RED Smarties, how many Smarties are there altogether?
   (b) If there are 27 Smarties in the box, how many RED ones would you expect?

8. The ratio of vertical to horizontal distance travelled up a hill is 1 in 5. If you have travelled 20 m horizontally, how far have you travelled vertically?



9. For a cake, the ratio of egg to flour is 1 egg to 4 oz of flour.
   (a) If you are making a cake with 4 eggs, how much flour do you need?
   (b) If the recipe says 12 ozs of flour, how many eggs does it need?

15.2 Proportion and Ratio

1. For some jam making the ratio of weight of sugar to weight of fruit is 2 : 5.
   (a) If you have 10 lbs of fruit, how much sugar do you need?
   (b) If you have 3 lbs of sugar, what is the greatest weight of fruit you could use?

2. A shop sells, on average, 2.5 television sets for every video sold.
   (a) If, during one week, 10 videos were sold, estimate how many television sets were also sold in that week.
   (b) If 15 television sets were sold one week, estimate how many videos were also sold in that week.

3. Kitchen flooring is sold at £8.00 per 1 metre length.
   Find the cost of
   (a) 5 m (b) 50 cm (c) 80 cm.

4. A school employs teachers in the ratio 1 : 20 to the number of children.
   (a) If the school has 600 children, how many teachers does it need?
   (b) If the school has 15 teachers, how many children can it take?

5. The weights of two tins of fruit are in the ratio of 3 : 5
   Find the weight of the smaller tin as a percentage of the weight of the larger tin.
   (SEG)

6. The chart below gives the distances, in kilometres, between some towns and cities.

Aberdeen			
684	Birmingham		
779	184	Cardiff	
888	290	284	Dover

8 kilometres is approximately 5 miles.

Estimate the distance between Birmingham and Cardiff **in miles**.

(SEG)

7. This is a list of the ingredients needed to make 24 scones.

600 g flour	100 g dried fruit
250 g butter	water to mix

(a) How much dried fruit is needed for 6 scones?

(b) How much flour would you need for 40 scones?

(SEG)

8. 1 kilogram = 2.2 pounds.

1 pound = 16 ounces.

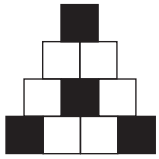
A recipe needs 10 ounces of flour. Convert this to grams.

(SEG)

9. (a) What fraction of the diagram is shaded?

(b) What percentage of the diagram is shaded?

(c) The ratio of shaded squares to unshaded squares is to be 4 : 1.
How many more squares must be shaded?



(NEAB)

10. These are the ingredients needed for making 18 rock cakes.

9 ounces of flour	6 ounces of sugar
6 ounces of margarine	8 ounces of mixed dried fruit
2 large eggs	

Larry wants to make 12 rock cakes.

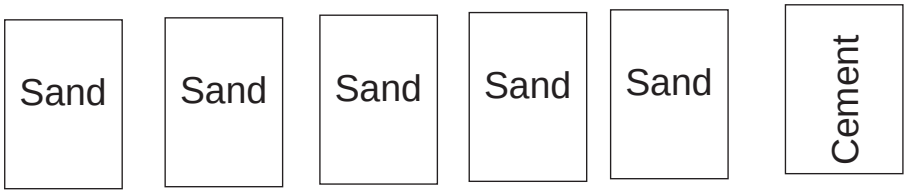
(a) How much of each ingredient will he need?

Kelly has 9 ounces of margarine, but plenty of all the other ingredients.

(b) What is the greatest number of rock cakes she can make?

(LON)

11.



Mortar is made by mixing 5 parts by weight of sand with 1 part by weight of cement.

How much sand is needed to make 8400 kg of mortar?

(LON)

12. A tourist drives a car which travels 35 miles on one gallon of petrol.

(a) How many kilometres will this car travel on one gallon of petrol?
(Take 5 miles to be equal to 8 kilometres.)

(b) This tourist drives a distance of 840 kilometres.
How many litres of petrol are used on this journey?
(Take 1 gallon to be equal to 4.5 litres)

(NEAB)

13. (a) Milk contains 87.5% water, 3.7% fat and 4.8% lactose.
The remaining contents are other substances.

What percentage of the contents are other substances?

(b) The average yield of a Friesian cow is 2700 litres of milk per year, but this can be increased by 400 litres with good feeding.

What percentage increase is this?

(c) In Britain, the ratio Friesian cows : All other breeds = 4 : 1
There are about 3 million cows in Britain altogether.

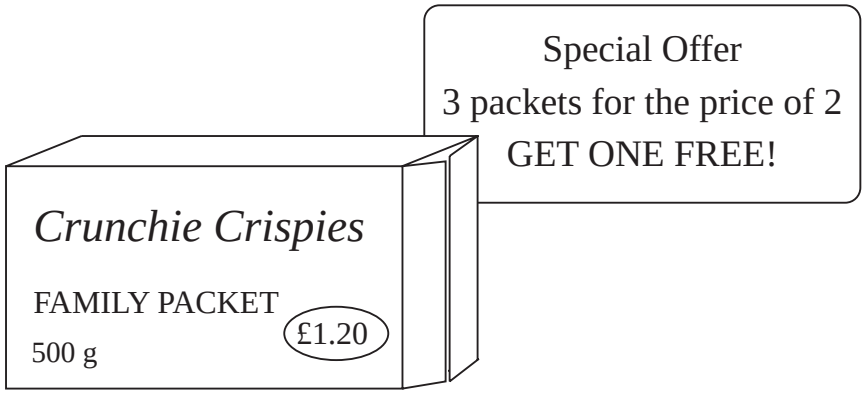
How many of these are Friesian?

(d) When Lisa was on holiday in Spain she paid 138 pesetas for a glass of milk.
She knew that £1 = 193 pesetas and estimated that the glass of milk cost 70p.

Show clearly, **without using a calculator**, how Lisa could have done this.

(MEG)

14. A family packet of Crunchy Crispies weighs 500 g and costs £1.20.



(a) Crunchy Crispies contain nuts and cornflakes.
The nuts and cornflakes are mixed by weight in the ratio 1: 3.

What is the weight of the nuts in each family packet?

(b) Sally gets three family packets of Crunchy Crispies at the special offer price.

What is the cost per kilogram of these Crunchy Crispies?

(c) The contents of each family packet is given to the nearest 10 g.

What is the minimum possible weight of one of these packets?

(SEG)

15.3 Map Scales and Ratio

1. Exeter and Plymouth are 80 km apart.

   Find the scale of a map that represents this distance by

   (a) 10 cm (b) 5 cm (c) 25 cm.

2. The distance between London and Birmingham is 144 km.

   What would be the map distance between these two cities using a scale of

   (a) 1 : 1000 000 (b) 1 : 500 000 (c) 1 : 80 000 ?

3. An ordinance survey map has a scale of 1 : 5000.

   What is the actual distance, in km, between two places measured on the map as

   (a) 5 cm (b) 9.5 cm (c) 21 cm ?

4. A plan of a school is drawn using a ratio of 1 : 100.

   (a) What will be the dimensions on the plan of the school playing-field which actually measures 100 metres by 50 metres?

   (b) On the plan, the dimensions of the school hall are 50 cm by 25 cm. What are the actual dimensions of the school hall?

5. The distance between Budapest and Prague is 500 km.

   (a) If a map has a scale of 1 : 500 000, what is the distance, on the map, between these two cities?

   (b) On another map the distance from Budapest to Prague measures 10 cm. What is the scale used on this map?

6. A classroom is drawn on a plan using a scale of 1 : 50.

   (a) On the plan, how many centimetres represent one metre?

   (b) The width of the classroom is 6.7 m. What would this width measure on the plan?

   (SEG)

7. The plan of a house is drawn to a scale of 1 : 50.

   (a) On the plan the length of the hall is 15 cm. What is the actual length of the hall in metres?

   (b) The actual width of the kitchen is 3.5 metres. What is the approximate width of the kitchen in feet?

   (SEG)

15.4 Proportional Division

1. The proceeds of a sale, £162, are divided between the two organisers in the ratio of 5 : 4.
How much does each organiser get?

2. A drink contains lime, orange and apple juices in the ratio 2 : 7 : 6.
Find the volume of each type of juice contained in 300 cm^3 of the drink.

3. An inheritance of £50 000 is shared by 4 relations in the ratio 1 : 2 : 3 : 4.
How much does each one receive?

4. On a motorway there are three lanes: an inside lane, a middle and an outside lane. One day, at midday, the speed of the traffic on these three lanes was in the ratio 3 : 4 : 5.
The speed in the outside lane was 70 miles per hour.
Calculate the speed on the inside lane.

(NEAB)

5. Ann and Bill share £400 in the ratio 5 : 3.

   (a) How much does each receive?

   (b) Bill gives £45 of his share to a charity.
   What percentage of his share is this?

(MEG)

6. A map is enlarged in the ratio 2 : 3.
On the first map a church measures 5 cm.
What will the church measure on the second (enlarged) map?

(SEG)

7. Alan scored 24 marks in a test which was marked out of 75.

   (a) Calculate Alan's percentage mark.

   (b) The marks were increased in the ratio 2 : 3.

      (i) Calculate Alan's new mark.

      (ii) Clare's new mark was 46.5.
      Calculate her original mark in the test.

(SEG)

8. (a)

Leaded petrol	**52.4p per litre**
Unleaded petrol	**49.6p per litre**

   I filled the petrol tank of my car with unleaded petrol. It cost me £18.60.

   (i) How many litres did I buy?

   (ii) How much more would it have cost me if I had bought leaded petrol instead?

(b) Last year the amounts I spent on road tax, car insurance and petrol were in the ratio 1 : 3 : 7.

I spent a total of £1430 on these three items.

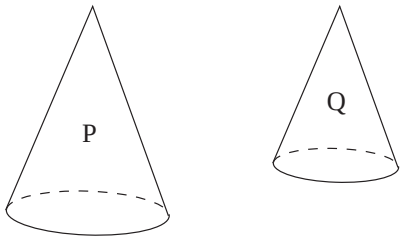
Calculate how much I spent on petrol.

(NEAB)

9. (a) The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.

Rearrange the formula to give r in terms of V and h.

(b) These cones are similar.



Not to scale

The ratio of the height of P to the height of Q is 3 : 2.

The volume of P is 5.4 cm^3.

Calculate the volume of Q.

(SEG)

15.5 Direct Proportion

1. Use the data given to check whether or not it agrees with the statement given.

(a)

x	1	3	7	11
y	4	12	28	44

$y \propto x$

(b)

q	0	1	2	3
p	0	$\frac{1}{2}$	1	$\frac{3}{2}$

$p \propto q$

(c)

x	1	5	10
y	0.1	0.5	1

$y \propto x$

(d)

t	0.1	1	2
s	0.5	5	10

$s \propto t$

2. Copy and complete each table using the statement given.

(a) $y \propto x$

x	1	4	9	
y	5	?	?	

(b) $s \propto t$

t	$\frac{1}{2}$	1	3	5
s	?	2	?	?

(c) $q \propto p$

p	1	3	5	9
q	?	9	?	?

(d) $y \propto x$

x	$\frac{1}{2}$	1	5	
y	1	?	?	

3. The yield Y of a tomato crop is directly proportional to the quantity of fertiliser F used. 5 kg of fertiliser produces 30 kg of tomatoes.

   (a) Find the relationship between Y and F.

   (b) What is the yield when 12 kg of fertiliser are used?

   (c) How much fertiliser was used to produce a yield of 42 kg?

4. A spring stretches when a mass is attached to one end.
   The extension x is directly proportional to the magnitude of the mass, m.
   When a mass of 50 g is attached, the extension is 1 cm.

   (a) Find the relationship between x and m.

   (b) When a mass of 120 g is attached, what is the extension?

   (c) What mass will produce an extension of 3.2 cm?

5. A launched rocket travels at constant acceleration.
   Its speed v ms^{-1}, is proportional to the time t, in seconds, since launch.
   After 5 seconds its speed is 120 ms^{-1}.

   (a) Find a relationship between v and t.

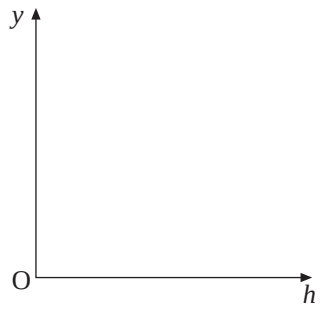
   (b) What is its speed when t equals

   (i) 1 second (ii) 10 seconds (iii) 60 seconds?

   (c) How long will it be before its speed is 20 000 ms^{-1}?

6. A ball is dropped to the floor from a height of h centimetres.
   It bounces to a height of y centimetres.
   y is directly proportional to h.

   (a) Sketch a graph to show the relationship between y and h.



   When $h = 120$, $y = 80$

   (b) Find y when $h = 150$.

(LON)

15.6 Inverse Proportion

1. For each table of values below, determine whether they agree with the relationship stated.

(a)

x	1	2	4
y	12	6	3

$y \propto \frac{1}{x}$

(b)

q	1	2	5
p	2	1	0.5

$p \propto \frac{1}{q}$

(c)

r	$\frac{1}{2}$	1	2
s	2	1	$\frac{1}{2}$

$s \propto \frac{1}{r}$

(d)

x	$\frac{1}{2}$	1	5
y	10	5	1

$y \propto \frac{1}{x}$

2. Copy and complete each of these tables to match the stated relationship.

(a) $y \propto \frac{1}{x}$

x	10	20	40
y	2	?	?

(b) $p \propto \frac{1}{q}$

q	1	2	8
p	?	2	?

(c) $s \propto \frac{1}{r}$

r	1	2	5
s	5	?	?

(d) $v \propto \frac{1}{u}$

u	10	20	100
v	1	?	?

3. Two quantities, x and y, are such that y is inversely proportional to x. Also note that $y = 4$ when $x = 2$.

   (a) Find the relationship between x and y.

   (b) What is the value of y when $x = 4$?

4. The value of a TV set is assumed to be inversely proportional to its age. When it is a year old it is sold for £400.

   (a) What will its value be when it is 2 years old?

   (b) How many years old will it be when its value is first less than £100?

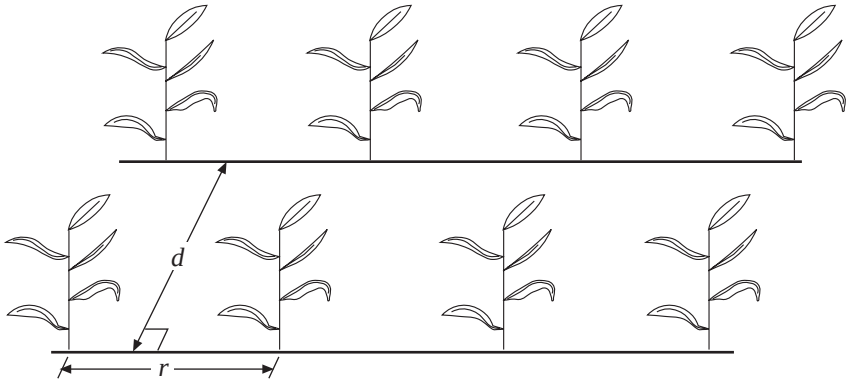
   (c) Is the assumption made here a reasonable one?

5. The value, v, of a train is assumed to be inversely proportional to its age, x. It was sold for £500 000 when it was 4 years old.

   (a) Find the relationship between v and x.

   (b) What is its value when it is 10 years old?

   (c) How many years old is it when its value is first less than £100 000?

6. 

The diagram is taken from a book about growing maize.
The distance between the rows of plants is d metres.
The spacing between the plants in the rows is r metres.

The number, P, of plants per hectare is given by the formula $P = \dfrac{10\,000}{dr}$.

$d = 0.8$ and $r = 0.45$.

(a) Calculate the value of P.
Give your answer to 2 significant figures.

The value of d is inversely proportional to the value of r and $d = 0.9$ when $r = 0.4$.

(b) Calculate the value of r when $d = 1.2$.

(LON)