## 14 <br> Loci and Transformations

### 14.1 Drawing and Symmetry

1. Draw accurately rectangles with the following sizes:
(a) 4 cm by 5 cm
(b) 9 cm by 2.5 cm
2. Make accurate drawings of each of the shapes below and answer the question below each shape.
(a)

What is the length of the sloping side?
(b)


> What is the length of one of the sloping sides?
(c)


What is the length of the longest straight line which can be drawn inside the shape?
3. Each shape below includes a semi-circle. Make an accurate drawing of each shape and state the radius of the semi-circle.
(a)

(b)

4. For each shape below:
(i) state the order of rotational symmetry,
(ii) copy the shape and draw any line of symmetry.
(a)

(b)

(c)

(d)

(e)

5.


Make 4 copies of this shape. Shade part of the shape to produce shapes with:
(a) exactly 2 lines of symmetry
(b) exactly 1 line of symmetry
(c) rotational symmetry of order 1
(d) rotational symmetry of order 2 .
6. Which of the shapes below have:
(a) rotational symmetry of order 1
(b) no lines of symmetry
(c) more than 2 lines of symmetry
(d) rotational symmetry of order 2
(e) rotational symmetry of order greater than 2 ?
A

B





### 14.2 Scale Drawings

1. The scale drawing of a shop, shown below, has been drawn on a scale of $1: 200$.


Find :
(a) the actual sizes of the stock room and each changing room.
(b) the area of the counter
(c) the width of the shop entrance.
2. A rough sketch is shown below of the ground floor of a house.


Use the information given to produce a scale drawing with a scale of $1: 200$.
3. The drawing opposite represents a scale drawing of a garden.
It is drawn with a scale of $1: 120$.
Find:
(a) the dimensions of the shed
(b) the area of the vegetable garden
(c) the dimensions of the flower bed
(d) the radius of the pond.
(e) the area of the land grassed.

4. A room is rectangular, with width 5 m and length 6 m . What would be the size of the rectangle on a scale drawing with a scale of:
(a) $1: 50$
(b) $1: 100$
(c) $1: 200$ ?
5. The sketch drawing shows the plan of a field.


Using a scale of 1 centimetre to 10 metres, make an accurate scale drawing.
Write down the length of the side CD to the nearest metre.
(MEG)
6. The scale drawing shows the position of an airport tower, T , and a radio mast, M. 1 cm on the diagram represents 20 km .

(a) (i) Measure, in centimetres, the distance TM.
(ii) Work out the distance in km of the airport tower from the radio mast.
(b) (i) Measure and write down the bearing of the airport tower from the radio mast.
(ii) Write down the bearing of the radio mast from the airport tower.

A plane is 80 km from the radio mast on a bearing of $220^{\circ}$.
(c) On a copy of the digram, plot the position of the plane, using a scale of 1 cm to 20 km .

## 14.3 <br> Constructing Triangles and Other Shapes

1. Draw triangles with sides of the following lengths:
(a) $12 \mathrm{~cm}, 8 \mathrm{~cm}, 7 \mathrm{~cm}$
(b) $8 \mathrm{~cm}, 5 \mathrm{~cm}, 6 \mathrm{~cm}$.
2. Draw accurately the triangles shown in the rough sketches below and then answer the question about each sketch.
(a)

What is the length of BC ?
(b)

How long are the sides AC and BC ?

(d) What is the size of the angle ACB?

3. Draw accurately an equilateral triangle with sides of length 6 cm .
4. An isosceles triangle has a base of length 8 cm and base angle of $37^{\circ}$.

Make an accurate drawing of the triangle and use it to estimate the lengths of the other sides of the triangle.
5. An isosceles triangle has 2 sides of length 6 cm and one side of length 10 cm . Find the sizes of all the angles in the triangle.
6. For each rough sketch below, draw accurately two possible triangles.
(a)

(b)

7. Draw a rhombus, ABCD , with $\mathrm{AB}=6 \mathrm{~cm}$ and $\mathrm{DA} B=40^{\circ}$. What are the lengths of the diagonals?
8.


A small window is made of a semicircle of radius 30 cm and a straight section of height 50 cm .

Construct an accurate drawing of this window.

### 14.4 Enlargements

1. Copy the diagrams below on to squared paper. Enlarge each shape with scale factor 2, using the point marked as the centre of enlargement.

(b)

(c)

(d)

2. In each diagram below, the smaller shape has been enlarged to obtain the larger shape. For each example, copy the diagram on to squared paper, state the scale factor and find the centre of enlargement.
(a)

(b)

(c)

(d)

3. Copy the diagram opposite and enlarge it with a scale factor of
(a) 2
(b) 3 .

4. On a copy of the grid below, enlarge the shaded shape by a scale factor of 3 . Start your enlargement at point B.

5. Enlarge the shape with scale factor 2 and centre $(0,3)$.

6. Enlarge the shaded figure, using scale factor 3 and centre of enlargement E.

7. In the diagram, triangle $T$ is an enlargement of triangle $S$ from a centre $C$.

(a) In the diagram, mark and label the centre of enlargement C.
(b) Write down the scale factor of the enlargement.
(MEG)

### 14.5 Reflections

1. Copy each diagram below and draw the reflection of each object.
(a)
Mirror Line

(c)

(d)
Mirror Line

(e)


(b)

(f)

2. Copy the diagram below.


Draw the mirror line for each of the following reflections.
(a) $\mathrm{A} \rightarrow \mathrm{B}$
(b) $\mathrm{A} \rightarrow \mathrm{D}$
(c) $\mathrm{B} \rightarrow \mathrm{C}$
(d) $\mathrm{C} \rightarrow \mathrm{D}$
3. Copy the diagram opposite.

Draw the mirror line for each of the following reflections:
(a) $\mathrm{A} \rightarrow \mathrm{B}$
(b) $\mathrm{A} \rightarrow \mathrm{D}$
(c) $\mathrm{B} \rightarrow \mathrm{C}$
(d) $\mathrm{C} \rightarrow \mathrm{D}$


4

(a) Copy the axis and the shape shown opposite.
(b) (i) Reflect the shape in the mirror line shown.
(ii) Reflect the new shape in the $x$-axis.
(iii) Reflect the new shape in the $y$-axis.
(iv) Reflect the new shape in the $x$-axis.
(c) Describe two reflections needed to take the shape defined in (iv) above back to the original shape.
5. Reflect each of the shapes below in the mirror line shown.
(a)

(b)


### 14.6 Construction of Loci

1. The line AB is 5 cm long. Draw the locus of a point which is always 2 cm from the line $A B$.
2. (a) Copy the diagram opposite.
(b) Construct the locus of a point which is equidistant from both lines.

3. Two oil rigs, A and B , are in the sea near the port of C .

(a) A third oil rig is the same distance from A and from B .

Copy the diagram and construct the locus of possible positions of the third oil rig on the diagram.
(b) The third oil rig, D , is also the same distance from C as it is from A and B . Mark with a cross the position of D .
4. The diagram shows a field, ABCD , drawn to a scale of 1 cm to 10 m . Treasure is hidden in the field.

(a) The treasure is at equal distances from the sides, AB and AD .

Copy the diagram and construct the locus of points for which this is true.
(b) The treasure is also 60 m from the corner, C. Construct the locus of points for which this is true.
(c) Mark with an X the position of the treasure.
5. The diagram below is a scale drawing of the floor of a room.

In the diagram, 2 cm represents 1 m .
X marks the position of an electric socket.


A vacuum cleaner is attached by a cable to the socket and can clean the floor up to 3 metres from the socket.

Copy the diagram and shade the part of the floor which can be cleaned by the vacuum cleaner.
6. Jason has to sail his ship between two rocks so that his ship is always the same distance from Point A on the first rock and Point B on the second rock.

The diagram below shows the rocks.


On a copy of the diagram, construct accurately the path along which Jason must sail his ship.
7.

## Layton •

- Moorby
- Newdon

The map above, drawn to a scale of 4 cm to represent 1 km , shows the positions of three villages, Layton, Moorby and Newdon.

Simon's house is the same distance from Moorby as it is from Layton.
The house is also less than $\frac{3}{4} \mathrm{~km}$ from Newdon.
Draw a copy of the map and mark on your drawing the possible positions of Simon's house. Show your construction lines clearly.
8. Signals from a radio mast, M, can be received up to a distance of 100 km . Use a scale drawing of 1 cm to represent 20 km to answer the following questions.
(a) Shade the region in which signals from the radio mast can be received.

The distance of a helicopter from the radio mast is 70 km , correct to the nearest kilometre.
(b) Write down
(i) the maximum distance the helicopter could be from the radio mast
(ii) the minimum distance the helicopter could be from the radio mast.
(LON)
9.
 - Y

The diagram represents a box which is to be moved across a floor XY.
$\mathrm{AD}=30 \mathrm{~cm}$ and $\mathrm{AB}=20 \mathrm{~cm}$.
First the box is rotated about the point A so that BC becomes vertical. Then the box is rotated about the new position of the point B so that CD becomes vertical.
(a) Copy the diagram and draw the locus of the point C .
(b) Calculate the maximum height of C above the floor. Give your answer correct to one decimal plece.
(A measurement from the scale drawing is unacceptable.)
10. (a) (i) Draw a straight line, $\mathrm{AB}, 8 \mathrm{~cm}$ long.
(ii) Draw the locus of points, P , which lie above the line AB such that the area of triangle ABP is $12 \mathrm{~cm}^{2}$.
(b) On the same diagram, construct the locus of points, Q , which lie above the line $A B$ such that angle $A Q B$ is $90^{\circ}$.
(c) Hence draw all triangles ABC which have C above AB , an area of $12 \mathrm{~cm}^{2}$ and an angle of $90^{\circ}$.
(MEG)

### 14.7 Enlargements which Reduce

1. For each pair of objects, state the scale factor of the enlargement which produces the smaller image from the larger one.
(a)

(b)

(c)

(d)

2. For each pair of objects below, the smaller shape has been obtained from the larger shape by an enlargement. For each example, state the scale factor and give the coordinates of the centre of enlargement.

3. Copy each shape and enlarge using the given centre of enlargement and the specified scale factor.
(a)

Scale factor $\frac{1}{2}$
(b)

Scale factor $\frac{1}{3}$

4. The larger rectangle is reduced in size to the smaller rectangle.

(a) What is the scale factor of the enlargement?
(b) What is the length of $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$ ?
5. Each diagram below shows a shape and its image after enlargement. In each case, state the scale factor and find the unknown lengths in the image.

(c)

(b)

(d)


6. Shape $A$ is shown in the diagram. Shape $A$ is enlarged to obtain the shape $B$. One side of shape B has been drawn.

(a) Write down the scale factor of the enlargement.
(b) Copy the drawing and complete the shape B on your diagram.

### 14.8 Further Reflections

1. Copy the diagrams below and draw the reflection of each shape in the mirror line.
(a)

(c)
Mirror Line

(b)

Mirror Line
(d)

(e)

(f)

2. The diagram shows the positions of the shapes A, B, C, D and E.


What is the equation of the line of reflection required to transform from:
(a) A to B
(b) B to D
(c) D to C
(d) C to A
(e) A to E ?
3. Copy the diagrams and reflect each of the shapes in the mirror lines given.
(a)

(b)

(LON)
4. Copy the set of axes opposite and the shape labelled A.
(a) Reflect A in the line $x=5$ to obtain B.
(b) Reflect B in the line $y=-2$ to obtain C .
(c) Reflect C in the line $x=-3$ to obtain D .
(d) Reflect D in the line $y=-2$ to obtain E .

(e) Name two reflections of E which would bring the shape back to A.
5. The diagram shows a number of shapes, some of which have been reflected in various lines.


State whether each mapping is a reflection and if so, give the equation of the mirror line.
(a) A to B
(b) A to D
(c) A to C
(d) D to E
(e) E to F
(f) $\quad \mathrm{B}$ to G
(g) C to G
(h) F to D
(i) B to E .

## Rotations

1. Copy the axes and shape A shown.
(a) Rotate the shape A through $90^{\circ}$ anticlockwise around ( 0,0 ) to obtain B.
(b) Rotate the shape A through $180^{\circ}$ clockwise around $(0,0)$ to obtain C .

(c) What rotation is needed to obtain the shape C from B ?
2. Copy the axes and shape shown below.

(a) Rotate the original shape through $90^{\circ}$ clockwise about the point $(1,0)$.
(b) Rotate the original shape through $180^{\circ}$ about the point $(5,2)$.
(c) Describe the rotation that takes the shape in (b) to the shape in (a).
(d) Rotate the original shape through $90^{\circ}$ anti-clockwise about the point $(-2,-1)$.
3. The diagram shows the position of a shape labelled $A$ and other shapes which were obtained by rotating A .

(a) Describe how each shape can be obtained from A by a rotation.
(b) Which shapes can be obtained by rotating the shape B ?
4. 


(a) Describe fully the single transformation which will transform the shape labelled A to the shaded shape.
(b) On a copy of the grid, draw the shaded shape after it has been reflected in the line $y=x$.
(NEAB)
5.

(a) The square ABCD is reflected in the line $x=1$.

What are the new coordinates of $C$ ?
(b) The square ABCD is rotated through $180^{\circ}$ about A .

What are the new coordinates of C ?
(SEG)
6. (a) Copy the diagram below.


Mirror Line
Reflect the shape A in the mirror line. Label the reflection B.
(b)


Describe fully the transformation which maps the triangle $C$ onto the triangle D.
7. The sketch shows the position of a rectangle ABCD .

(a) The rectangle ABCD is reflected in the line $x=4$ to give rectangle $\mathrm{A}_{1} \mathrm{~B}_{1} \mathrm{C}_{1} \mathrm{D}_{1}$. What are the coordinates of $\mathrm{C}_{1}$ ?
(b) The rectangle ABCD is rotated about A anticlockwise through $90^{\circ}$ to give $\mathrm{A}_{2} \mathrm{~B}_{2} \mathrm{C}_{2} \mathrm{D}_{2}$. What are the coordinates of $\mathrm{B}_{2}$ ?
(c) The rectangle ABCD is enlarged by scale factor 2, centre A .

What are the coordinates of the new position of $B$ ?

### 14.10 Translations

1. The shaded shape has been moved to each of the other positions by a translation. Give the vector used for each translation.

2. For the shapes shown, describe the translation which maps:
(a) $\mathrm{A} \rightarrow \mathrm{B}$
(b) $\mathrm{A} \rightarrow \mathrm{C}$
(c) $\mathrm{B} \rightarrow \mathrm{C}$
(d) $\mathrm{E} \rightarrow \mathrm{A}$
(e) $\mathrm{E} \rightarrow \mathrm{B}$
(f) $\quad \mathrm{B} \rightarrow \mathrm{A}$.

3. 



Copy this diagram. Draw the image of this shape when translated using:
(a) $\binom{5}{0}$
(b) $\binom{-3}{3}$
(c) $\binom{7}{2}$.
4. Copy the shape opposite.
(a) Translate the shape using the vector $\binom{7}{0}$.

(b) Translate the new shape using the vector $\binom{0}{-3}$.
(c) What translation is needed to translate the original shape to the final shape?
5. The points A, B and C have coordinates $(3,2),(-2,5)$ and $(-3,-1)$ respectively. Find the vector needed to translate:
(a) A to B
(b) B to C
(c) A to C.
14.11 Combined Translations

1. Copy the set of axes and shape shown below.

(a) Reflect this shape in the line $x=5$.
(b) Rotate the original shape $90^{\circ}$ clockwise about $(5,0)$.
(c) Reflect the shape in (b) about the line $x=2$.
(d) Describe a single transformation of the shape in part (a) to the shape formed in part (c).
2. 


(a) Describe a single transfomation from
(i) A to B
(ii) F to B
(iii) A to D
(iv) E to A .
(b) Describe two combined transformations for
(i) B to C
(ii) C to E
(iii) C to F .
3. (a) The point A is reflected in the $y$-axis. The image is the point B . Write down the coordinates of B.
(b) The point A is rotated through $90^{\circ}$ anticlockwise about O.
The image is the point C .
Write down the coordinates of C .
(c) The point B can be mapped onto point C by a translation.
Write down the column vector of this translation.

(MEG)
4.

(a) Copy the diagram and reflect the triangle A in the $x$-axis. Label the reflection B.
(b) Reflect the triangle B in the line $y=x$. Label the reflection C.
(c) Describe fully the single transformation which maps triangle A onto triangle C.
(d) Write down the equation of the line which is parallel to $y=x$ and which passes through the point $(0,8)$.

This line crosses the $x$-axis at the point P .
(e) Calculate the coordinates of P .
5. The parallelogram ABCD has vertices at $(6,3),(9,3),(12,9)$ and $(9,9)$ respectively.

(a) An enlargement, scale factor $\frac{1}{3}$ and centre ( 0,0 ), transforms parallelogram $A B C D$ onto $A_{1} B_{1} C_{1} D_{1}$. Copy the diagram and draw the parallelogram $A_{1} B_{1} C_{1} D_{1}$.
(b) The parallelogram $A_{1} B_{1} C_{1} D_{1}$ is translated by the vector $\binom{0}{2}$ onto $\mathrm{A}_{2} \mathrm{~B}_{2} \mathrm{C}_{2} \mathrm{D}_{2}$. What are the coordinates of $\mathrm{C}_{2}$ ?
(c) The parallelogram $\mathrm{A}_{2} \mathrm{~B}_{2} \mathrm{C}_{2} \mathrm{D}_{2}$ can be transformed onto ABCD by an enlargement. Give the scale factor and centre of this enlargement.
(SEG)
6. The diagram shows triangles T , S and U .

S is the image of T under a reflection in the line $x=3$.

U is the image of S under reflection in the line $x=6$.

A reflection in the line $x=3 n$, where $n$ is an integer, is denoted
 by $R_{n}$,

So S is the image of T under $R_{1}$ and U is the image of T under $R_{1}$ followed by $R_{2}$. V is the image of T under the successive transformations $R_{1}$ followed by $R_{2}$ followed by $R_{3}$.
(a) Draw V on a copy of the diagram above.
(b) Describe fully the single transformation that will map T to V .

W is the image of T under the successive transformation $R_{1}$, followed by $R_{2}$, followed by $R_{3}$ and so on to $R_{n}$.
(c) Describe fully the single transformation that will map T to W :
(i) when $n$ is even
(ii) when $n$ is odd.

### 14.12 Congruence

1. 



From the shapes in the diagram, write down the letters of three pairs of congruent shapes.
2. State which pairs of triangles are congruent.






3. For each question below, determine whether the triangles are congruent.

If the triangles are congruent, justify your answer.
(a)


(b)

(c)

(d)

(e)

(f)

4. Pythagoras wrote about a triangle where

$$
\begin{aligned}
& \mathrm{AB}=1 \text { unit, } \\
& \mathrm{BC}=1 \text { unit, and } \\
& \mathrm{ABC}=90^{\circ} .
\end{aligned}
$$

(a) Pythagoras claimed that the length of AC is a rational number.

Was he correct? Explain your answer.
(b) The triangle PQR has $\mathrm{PQ}=1$ unit, $\mathrm{QR}=\sqrt{3}$ units and $\mathrm{QPR}=90^{\circ}$, as shown.


Is the triangle PQR congruent to triangle ABC ? Explain your answer.

### 14.13 <br> Similarity

1. The diagram shows two similar triangles.

(a) the size of angle DFE
(b) the length of DE
(c) the ratio $\mathrm{AC}: \mathrm{DF}$ ?
2. A cylinder has a height of 10 cm and has volume $300 \mathrm{~cm}^{3}$. Find the volumes of similar cylinders of heights:
(a) 5 cm
(b) 20 cm .
3. Two cubes have volumes $729 \mathrm{~cm}^{3}$ and $1331 \mathrm{~cm}^{3}$. What is the ratio of:
(a) the side length of the cubes
(b) the surface areas of the cubes?
4. In the diagram, $\mathrm{CD}=4$ metres, $\mathrm{CE}=3$ metres and $\mathrm{BC}=5$ metres.
$A B$ is parallel to $D E$.
$A C E$ and BCD are straight lines.
(a) Explain why triangle ABC is similar to triangle EDC.
(b) Calculate the length of AC.

5. (a) Show clearly that these two triangles are similar.
(b) Calculate the value of $x$.

(NEAB)
6. All these triangles are similar.
(a) Calculate the length $x$.
(b) Calculate the length $y$.

(SEG)
7. 


(LON)
8.


The diagram shows a simplified ironing board. The feet, A and D , are 75 cm apart. The supports at $B$ and $C$ are 50 cm apart. The legs, AC and BD , are equal in length and are pivoted at $\mathrm{O} . \mathrm{BC}$ is parallel to AD .

The height of the ironing board above the floor is 85 cm .
(a) Use similar triangles to calculate the height, $h \mathrm{~cm}$, of O above the floor.
(b) Calculate the value of $x$, the angle between AC and the floor.
(c) Calculate the length of AC.
9. The diagram shows a symmetrical framework for a bridge.

$$
\begin{aligned}
& \mathrm{AC}=100 \mathrm{~m} \\
& \mathrm{AB}=\mathrm{BC}=70 \mathrm{~m}
\end{aligned}
$$

(a) (i) Calculate the angle BAD.
(ii) Calculate the length ED.


A similar framework is made with the length corresponding to $A C=180 \mathrm{~m}$.
(b) (i) Calculate the length corresponding to AB .
(ii) What is the size of the angle corresponding to angle BAD ?
10. (a) The model of the cross-section of a roof is illustrated below.

(i) Calculate the length of CE.
(ii) Triangles ABE and DCE are similar triangles with angle BAE equal to angle CDE. Calculate the length of AB .
(b) When the roof is constructed, the actual length of BC is 4.5 m .

Calculate the area of the cross-section of the actual roof space.
(SEG)
11. (a) Paul's ladder is 4 m long.
(i) Paul leans his ladder against a vertical wall, with the end, A on horizontal ground.

The angle between the ladder and the ground is $70^{\circ}$.
Calculate the distance of A from the wall.

(ii) Pamela moves the ladder and uses it to reach a windowsill which is 3.8 m above the gournd.

For safety, the angle between the ladder and the ground should be within $2^{\circ}$ of $70^{\circ}$.
Is the ladder safely placed?
(You must show some calculation to explain your answer.)

(b) Ranjit has a stepladder.

P is the midpoint of $\mathrm{RS} . \mathrm{Q}$ is the midpoint of TS. The length of PQ is 60 cm .
(i) Explain why triangles SPQ and SRT are similar.
(ii) Calculate the length of RT.

12.


A child builds a tower from three similar cylindricular blocks. The smallest block, A, has radius 2.5 cm and height 6 cm .
(a) Find the volume of the smallest block.
(b) Block B is an enlargement of A and block C is an enlargement of $B$, each with a scale factor of $1 \frac{3}{4}$. Find the total height of the tower.
(MEG)
13. Two similar solid shapes are made. The height of the smaller shape is 7 cm .

The width of the smaller shape is 6 cm . The width of the larger shape is 9.6 cm .

(a) Calculate the height of the larger shape.
(b) The volume of the larger shape is $695 \mathrm{~cm}^{3}$. Find the volume of the smaller shape.
(SEG)
14.


This is a scale model of the proposed Millennium Tower. The model is a cone of height 2 m and base radius 0.1 m . The angle between the slanting side and the vertical is $\theta$.
(a) What is the value of $\theta$, to the nearest degree?
(b) The proposed Millennium Tower is 1000 m high. What is its base radius?
(c) Calculate, in cubic metres, the volume of the model.
(d) How many times larger than the scale model is the volume of the proposed Millennium Tower?

### 14.14 Enlargements with Negative Scale Factors

1. The diagram below shows the original shape (shaded) and the images obtained by enlargements with different scale factors.

State the scale factor for each enlargement.

2. The shaded shape has been enlarged to give the other images. For each image, find the scale factor and the coordinates of the centre of enlargement.

3. Copy each diagram below. Enlarge each shape using the scale factor and centre of enlargement given.
(a)

Scale factor -2
Scale factor $-\frac{1}{2}$
(b)

(c)

(d)

Scale factor -3
4. Copy the axes and shape shown below.


Draw the image obtained when the original shape is enlarged with:
(a) scale factor -2 , centre of enlargement $(0,0)$
(b) scale factor -1 , centre of enlargement $(0,2)$
(c) scale factor -2 , centre of enlargement $(4,0)$.
5. Triangle $P$ is mapped to triangle $Q$ by an enlargement of scale factor -0.4 .

If BC is of length 5 cm , what is the length of DF?


