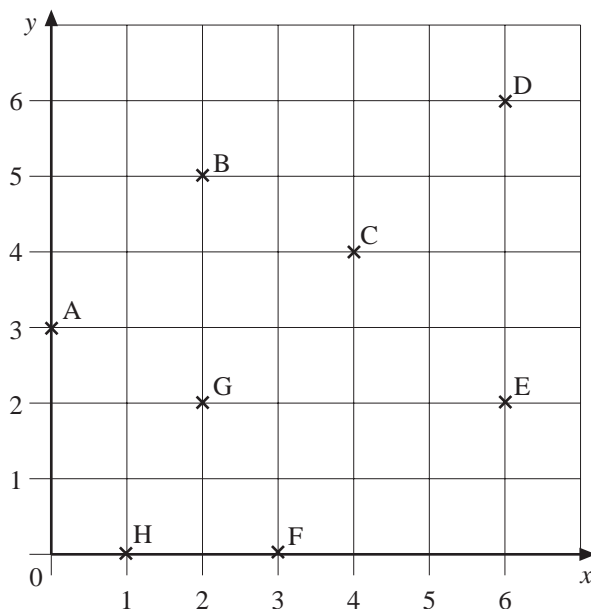


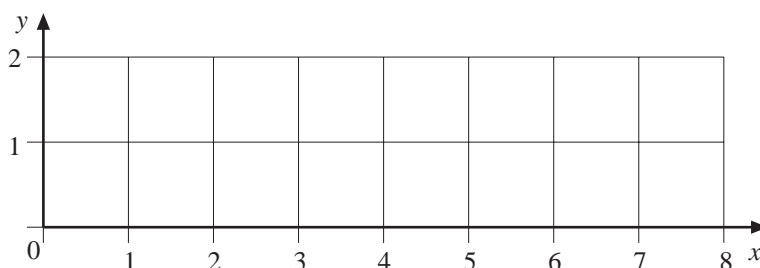
# 13 Graphs

## 13.1 Positive Coordinates

1. Write down the coordinates of each point marked on the grid below.

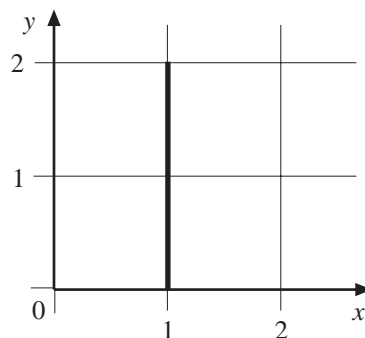


2. On a grid like the one below, draw lines
- (a) from  $(0, 0)$  to  $(0, 2)$  to  $(1, 1)$  to  $(2, 2)$  to  $(2, 0)$ .
  - (b) from  $(5, 0)$  to  $(3, 0)$  to  $(3, 1)$  to  $(4, 1)$  to  $(3, 1)$  to  $(3, 2)$  to  $(5, 2)$ .
  - (c) from  $(6, 0)$  to  $(6, 2)$  to  $(8, 2)$  to  $(8, 1)$  to  $(6, 1)$ .

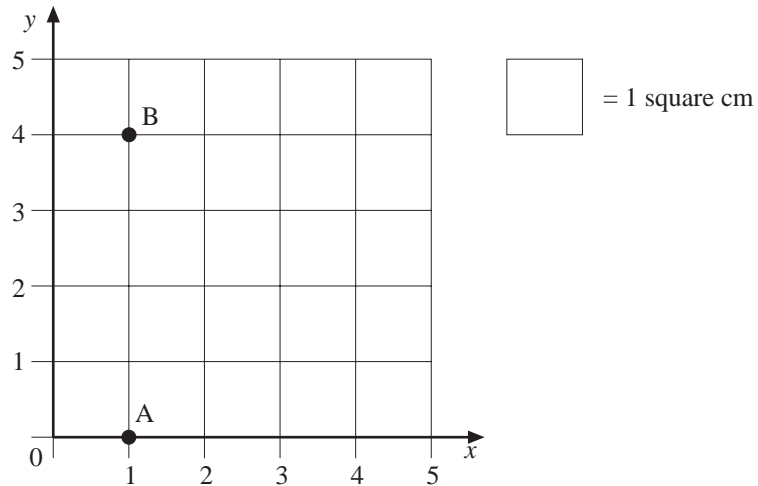


3. Using a grid like the one opposite you can write the number 1 by joining  $(1, 0)$  to  $(1, 2)$ .

Write out instructions for the digits 2, 3 and 4, using just the points shown on the grid for the vertices.



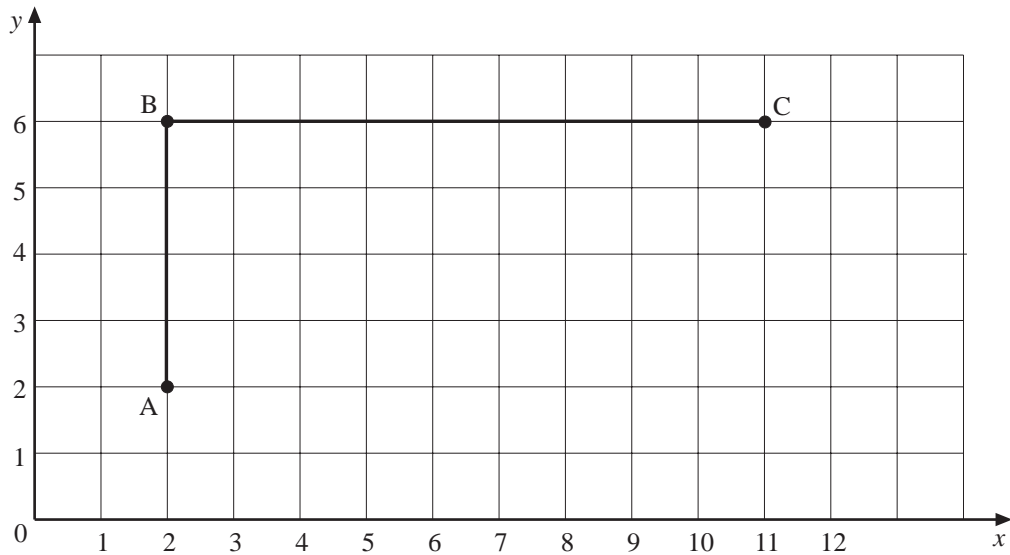
4.



- (a) Write down the co-ordinates of the point
  - (i) A
  - (ii) B
- (b) On a similar grid, plot and label the points  
C (3, 4) and D (3, 2)
- (c) Work out the area of the shape ABCD.

(LON)

5.

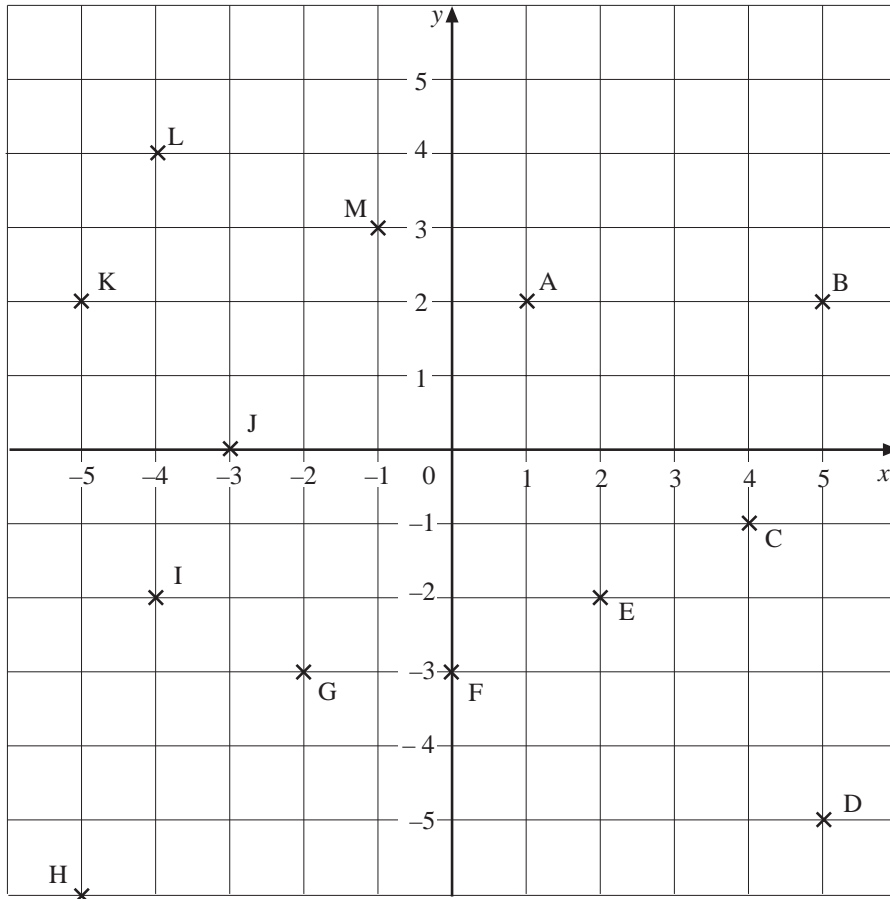


- (a) Write down the coordinates of the mid-point of AC.
- (b) Mark and label a point D on the diagram so that ABCD is a rectangle.
- (c)
  - (i) Find the perimeter of the rectangle ABCD.
  - (ii) Find the area of the rectangle ABCD.
- (d) The rectangle has reflective (line) symmetry.  
Describe another type of symmetry that it has.

(MEG)

# 13.2 Coordinates

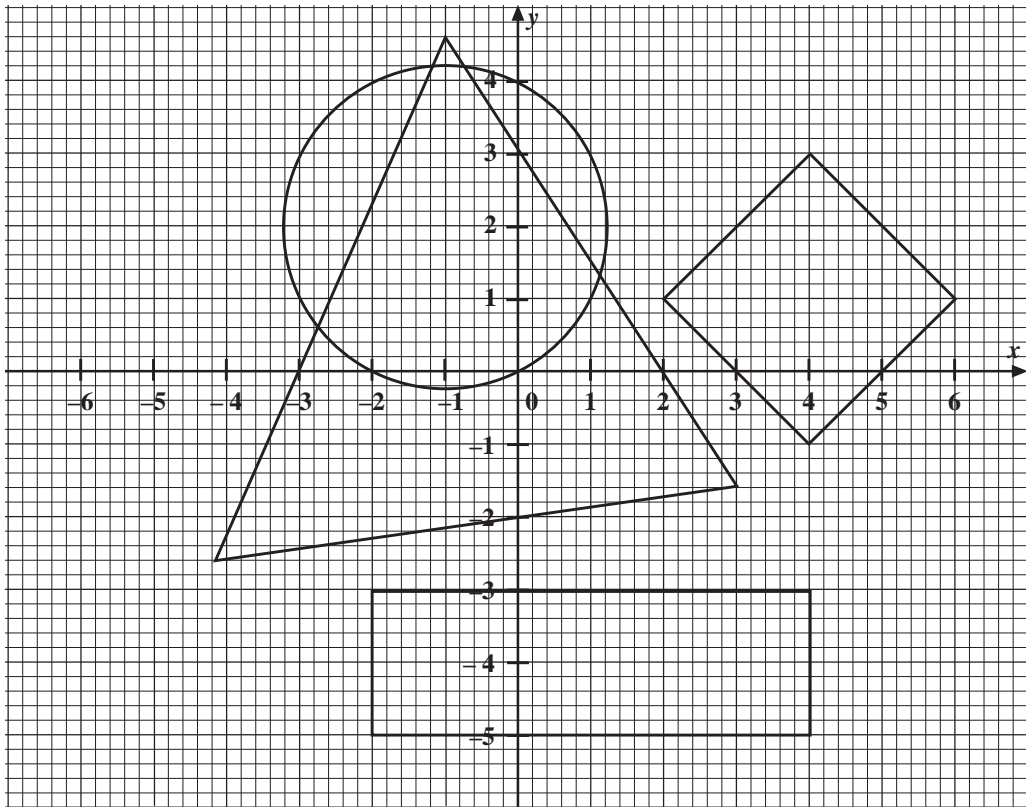
1. Write down the coordinates of each point marked on the grid below.



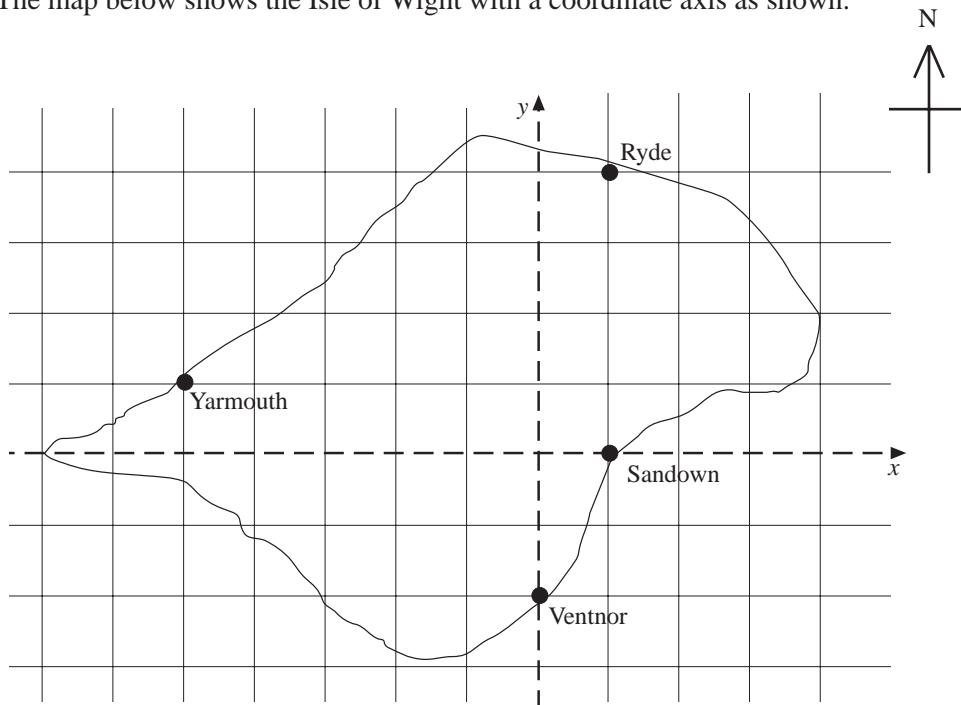
2. The following diagram shows several shapes.

Write down the coordinates of

- the centre of the circle,
- the points of intersection of the circle and the axes,
- the vertices of the square,
- the points of intersection of the triangle and the  $y$ -axis,
- the point of intersection of the diagonals of the rectangle.



3. The map below shows the Isle of Wight with a coordinate axis as shown.



- (a) Write down the coordinates of the towns
  - (i) Ryde      (ii) Sandown      (iii) Ventnor      (iv) Yarmouth
- (b) What are the coordinates of the points furthest
  - (i) west      (ii) east?

4. On a suitable set of coordinate axes, draw the triangle with vertices at
- $(0, 0)$ ,  $(5, 0)$ ,  $(5, 4)$
  - $(8, 4)$ ,  $(0, -4)$ ,  $(2, 4)$
  - $(1, -1)$ ,  $(-4, 3)$ ,  $(-6, -2)$
  - $(-1, 4)$ ,  $(-4, -2)$ ,  $(3, -4)$
5. On a set of coordinate axes join the points
- E  $(-3, 4)$  to F  $(6, -2)$
  - G  $(2, 4)$  to H  $(1, -2)$
- Write down the coordinates of the point of intersection of the two lines.
6. For each of the following, plot the given points on a suitable coordinate grid. For each question, join the points in order with straight lines and identify the geometrical shape.
- $(6, 4)$ ,  $(-6, 4)$ ,  $(-6, -4)$ ,  $(6, -4)$
  - $(0, 5)$ ,  $(-6, 0)$ ,  $(0, -5)$ ,  $(6, 0)$
  - $(0, 0)$ ,  $(0, 8)$ ,  $(5, 4)$
  - $(1, 0)$ ,  $(0, 3)$ ,  $(-1, 4)$ ,  $(-6, 1)$
  - $(4, 0)$ ,  $(0, 5)$ ,  $(-4, 0)$
  - $(5, 2)$ ,  $(-1, 3)$ ,  $(-1, -3)$ ,  $(5, -2)$
  - $(0, -3)$ ,  $(6, -3)$ ,  $(4, 2)$ ,  $(-2, 2)$
7. The points A  $(-3, 4)$ , B  $(2, 1)$  and C  $(3, -1)$  are three vertices of a parallelogram ABCD. Draw the parallelogram and write down the coordinates of D.
8. The points A  $(1, -1)$ , B  $(-3, -1)$  and C  $(-1, -4)$  are three vertices of a rhombus ACBD. Draw the rhombus ACBD and write down the coordinates of D.

## 13.3 Plotting Straight Lines

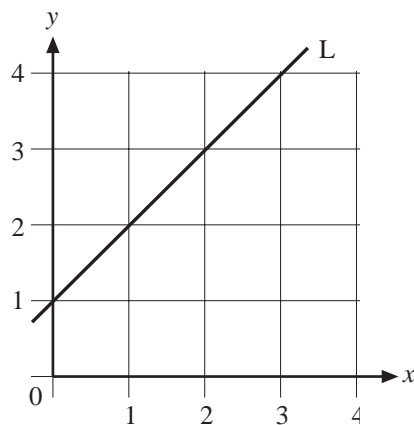
1. (a) Given the equation  $y = 2x$ , copy and complete the table below.

$x$	-4	-2	0	2	4
$y$	-8	?	?	4	?

- Using a set of axes with  $x$  values from  $-4$  to  $4$  and  $y$  values from  $-8$  to  $8$ , plot the points in (a) and draw a straight line through them.
- Use your graph to find
  - the value of  $y$  when  $x = 3$ ,
  - the value of  $x$  when  $y = -6$ .

2. (a) Use the equation  $y = 2x - 1$  to complete the coordinates below.  
 $(-2, ?)$ ,  $(0, ?)$ ,  $(2, ?)$
- (b) Plot these points on a set of axes and draw a straight line through the points.
- (c) Write down the coordinates of two other points on the line.  
 Check that they satisfy the equation  $y = 2x - 1$ .
3. On the set of axes with  $x$  values from  $-5$  to  $5$  and  $y$  values from  $-5$  to  $5$ , draw the graph of each of the following equations.
- (a)  $y = x + 2$
- (b)  $y = x - 1$
- (c)  $y = 2x$
- (d)  $y = 2x - 1$
- (e)  $y = 3x + 1$

4.



The point  $(2, b)$  is on the line marked L.

What is the value of  $b$ ?

(SEG)

5. (a) Copy and complete this table of values for  $y = 2x - 3$ .

$x$	$-4$	$-2$	$0$	$2$	$4$
$y$	$-11$	$?$	$?$	$1$	$?$

- (b) On a suitable grid, plot the points in the table and join them with a straight line.
- (b) From your graph, what is the value of
- (i)  $y$  when  $x = -1$
- (ii)  $x$  when  $y = 3$ ?

6. On the same coordinate grid, draw the graphs of

(a)  $y = x$                       (b)  $y = 2x$                       (c)  $y = \frac{1}{2}x$

(d)  $y = -x$                       (e)  $y = -2x$

What do you notice about all these lines?

7. On the same coordinate grid, draw graphs of

(a)  $y = 2x + 1$                       (b)  $y = 2x - 1$                       (c)  $y = 2x + 2$                       (d)  $y = 2x$

What do you notice about the four graphs?

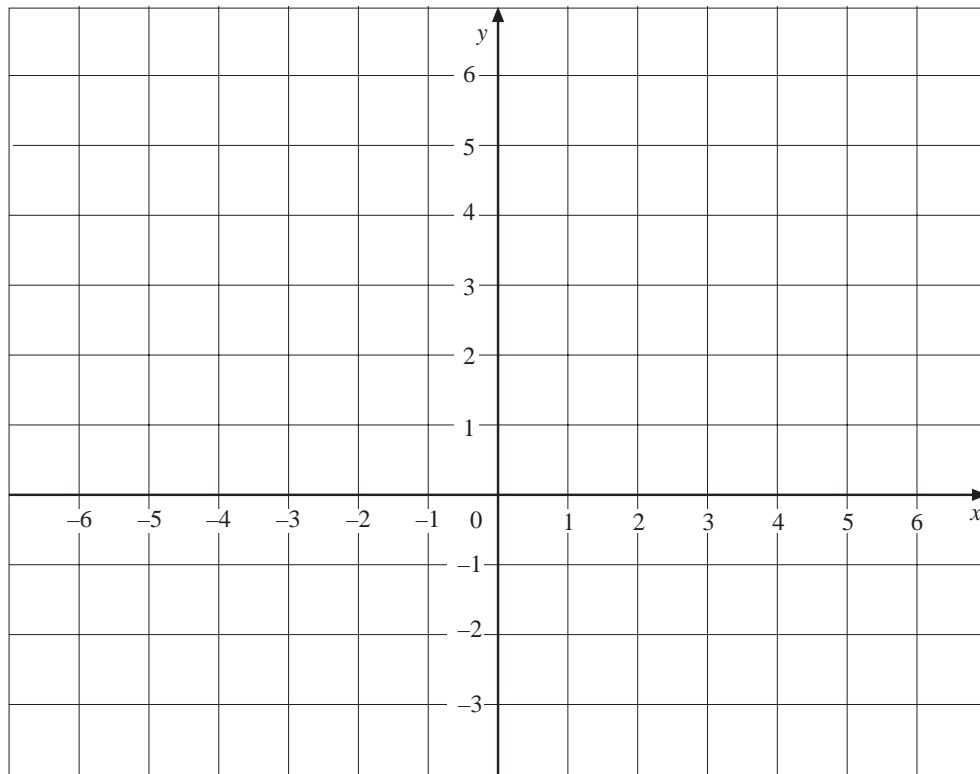
8. On the same coordinate grid, draw graphs of

(a)  $y = x + 1$                       (b)  $y = \frac{1}{2}x + 1$                       (c)  $y = 2x + 1$

What do you notice about the three graphs?

9. (a) Complete this table of values for  $y = x + 2$ .

$x$	-3	-2	-1	0	1	2	3
$y$			1		3	4	



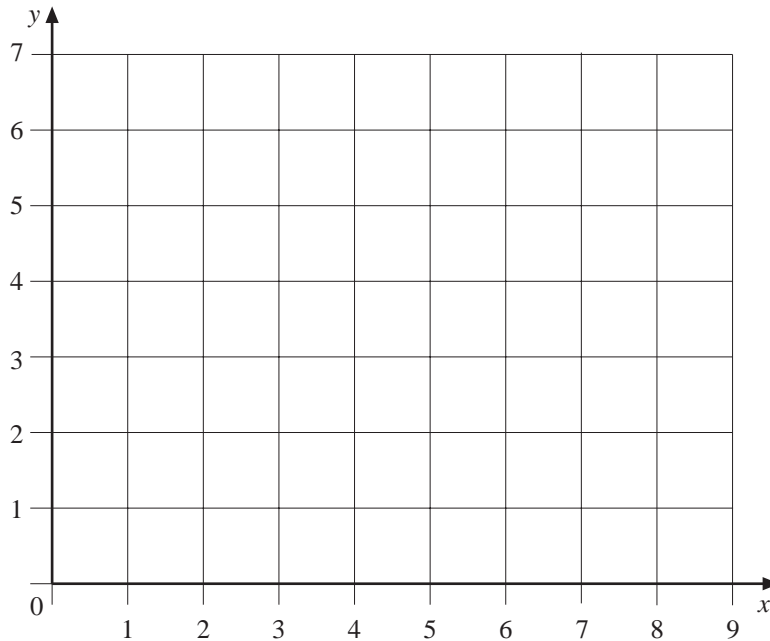
(b) On a grid like the one above, plot the points given by the values in your table.

Join the points.

(c) Work out the value of  $x$  when  $y = 2.5$ .

(LON)

10. (a) (i) Plot the points  $(1, 3)$  and  $(4, 6)$  on a grid like the one below.  
Join the points with a straight line.



- (ii) The point  $P(a, 5)$  lies on the line.  
What is the value of  $a$ ?
- (iii) This line is extended.  
Copy and complete the following mapping for points on this line.

1     →     3  
4     →     6  
10    →    .....  
 $x$     →    .....

- (b) On the same axes draw the graph of  $y = 6 - x$ .
- (c) Solve the equation  $x + 2 = 6 - x$ .

(SEG)

## 13.4 Plotting Curves

1. Copy and complete the table below for  $y = x^2 - 1$ .

$x$	-3	-2	-1	0	1	2	3
$y$		3					8

Plot these points on a suitable grid and draw a smooth curve through the points.



2. Copy and complete the table below for  $y = 2 - x^2$ .

$x$	-3	-2	-1	0	1	2	3
$y$	-7				1		

Plot these points on a suitable grid and draw a smooth curve through the points.

3. Copy and complete the table below for  $y = x^2 - 5x + 4$ .

$x$	-1	0	1	2	3	4	5	6
$y$	10						4	

Plot these points on a suitable grid and draw a smooth curve through the points.

4. Copy and complete the table below for  $y = x^3 + x - 1$ .

$x$	-3	-2	-1	0	1	2	3
$y$	-31				1		

Plot these points on a suitable grid and draw a smooth curve through the points.

5. On the same grid, sketch the graphs of the functions

(a)  $y = x^2$       (b)  $y = \frac{1}{2}x^2$       (c)  $y = 2x^2$       (d)  $y = \frac{1}{4}x^2$

What do you notice?

6. Copy and complete the table below for  $y = \frac{1}{x}$ .

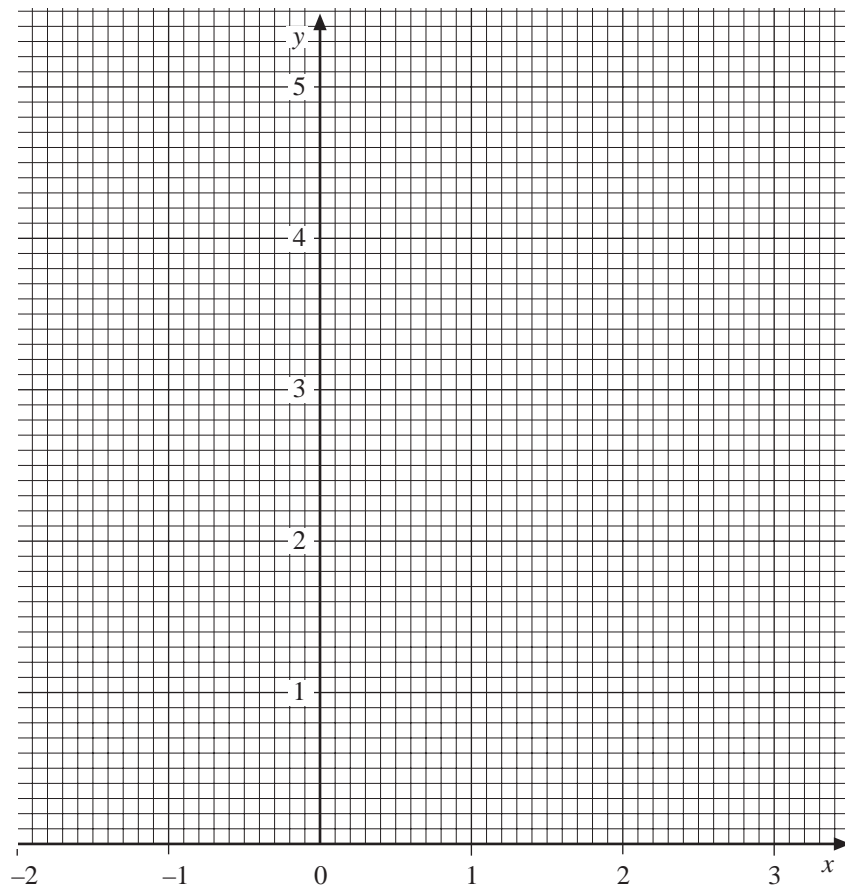
$x$	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
$y$				-2							$\frac{1}{3}$

Plot these points on a suitable grid and draw a smooth curve through the points.

What happens as  $x \rightarrow 0$ ? Justify your answer.

7. (a) Complete the table of values and draw the graph of the function  $y = x^2 + 1$  on a copy of the following grid.

$x$	-2	-1	0	1	2
$y = x^2 + 1$	5			2	



- (b) Use your graph to find the value of  $y$  when  $x = 0.5$ .

(SEG)

8. (a) Complete these tables of values:

- (i) for the equation  $y = 2x$

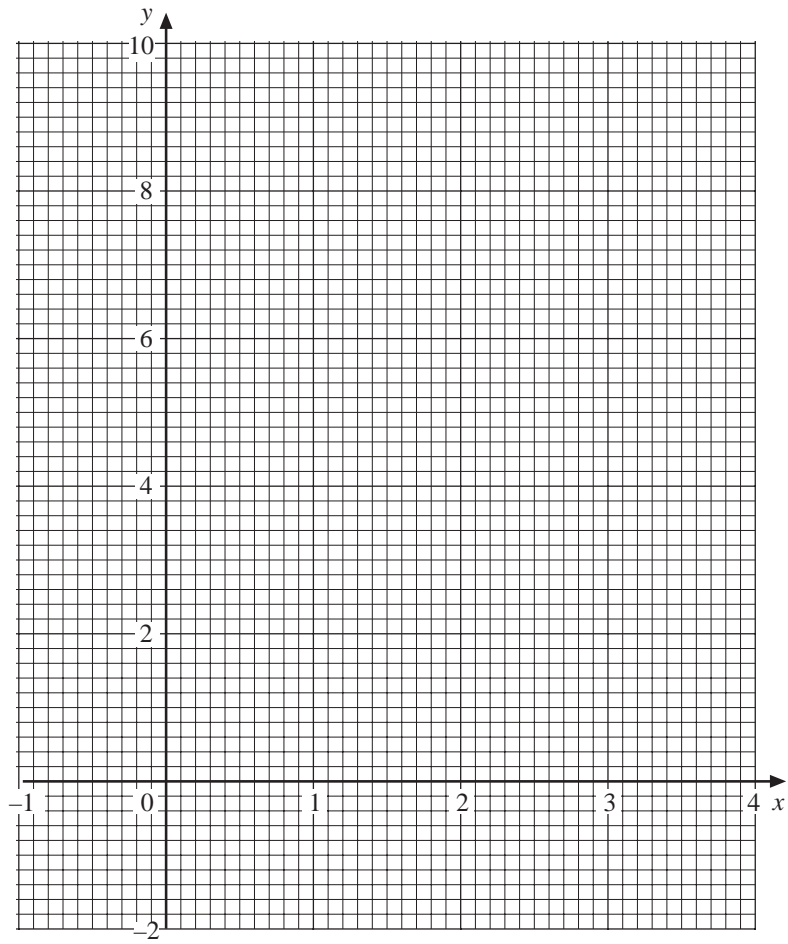
$x$	-1	1	3
$y$			

- (ii) for the equation  $y = x^2$

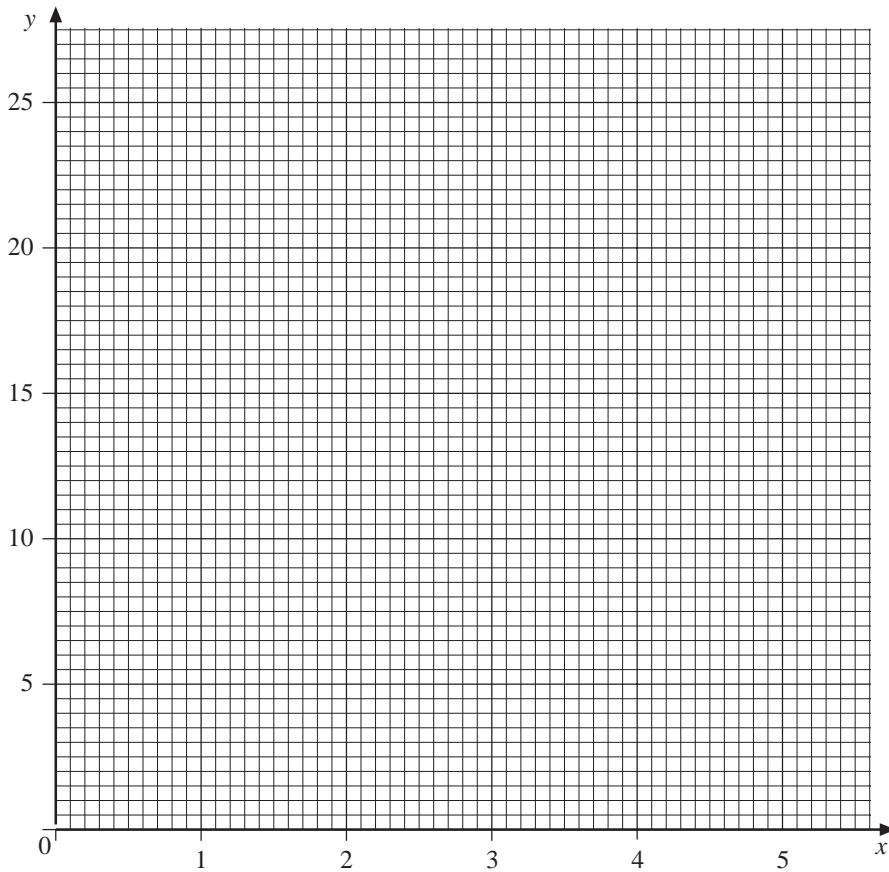
$x$	-1	0	1	2	3
$y$					

- (b) On a copy of the following grid draw the graphs of

$$y = 2x \text{ and } y = x^2.$$



9.



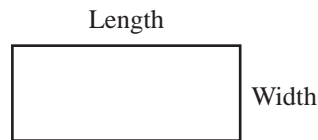
(NEAB)

$x$	0	1	2	3	4	5
$y$	0	1	4	9	16	25

- (a) Using the given table of values, draw the graph of  $y = x^2$  on a grid like the one above.
- (b) Use your graph to find the approximate value of  $x$  when  $y = 14$ .

(MEG)

10.



The width of the rectangle is  $x$  cm.

The length of the rectangle is 4 cm more than the width,

- (a) Write down an expression in terms of  $x$  for the length of the rectangle.

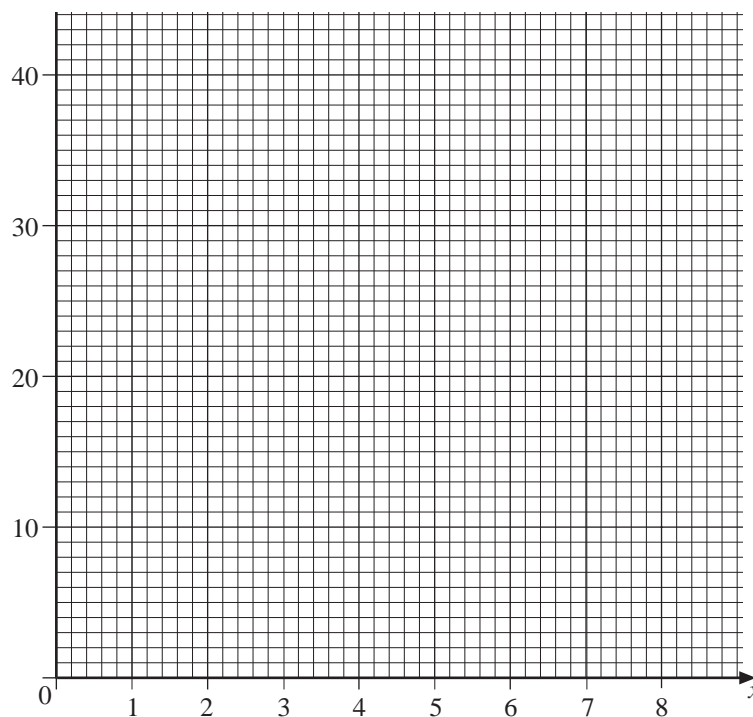
The perimeter of the rectangle is  $P$  cm.

- (b) Write down a formula for  $P$  in terms of  $x$ .

The table gives the values of  $P$  when  $x = 6$ .

$x$	2	4	6	8
$P$			32	

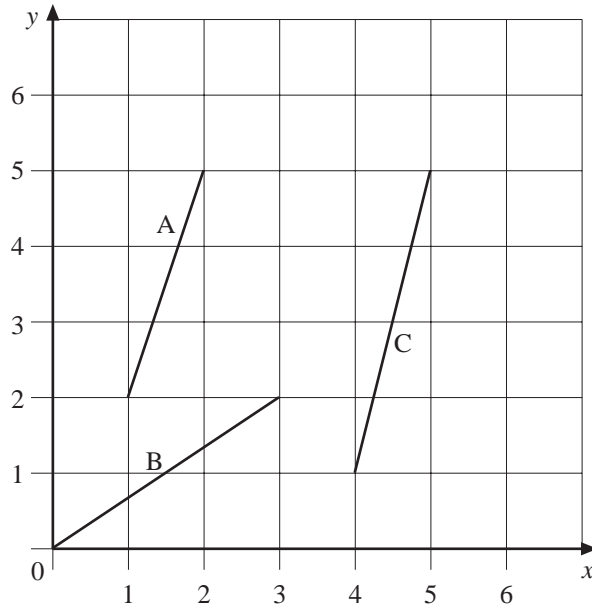
- (c) Complete a copy of the table for  $x = 2, 4$  and  $8$ .
- (d) On a grid like the one below, draw the graph of  $P$  against  $x$  for values of  $x$  from 2 to 8.



(LON)

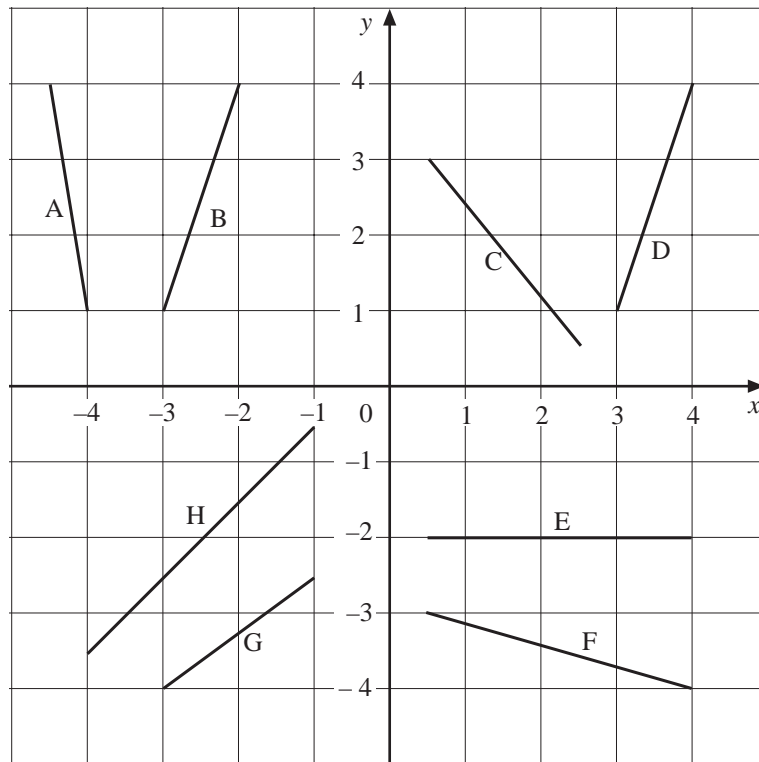
# 13.5 Gradient

1. Find the gradient of each line in the diagram below.

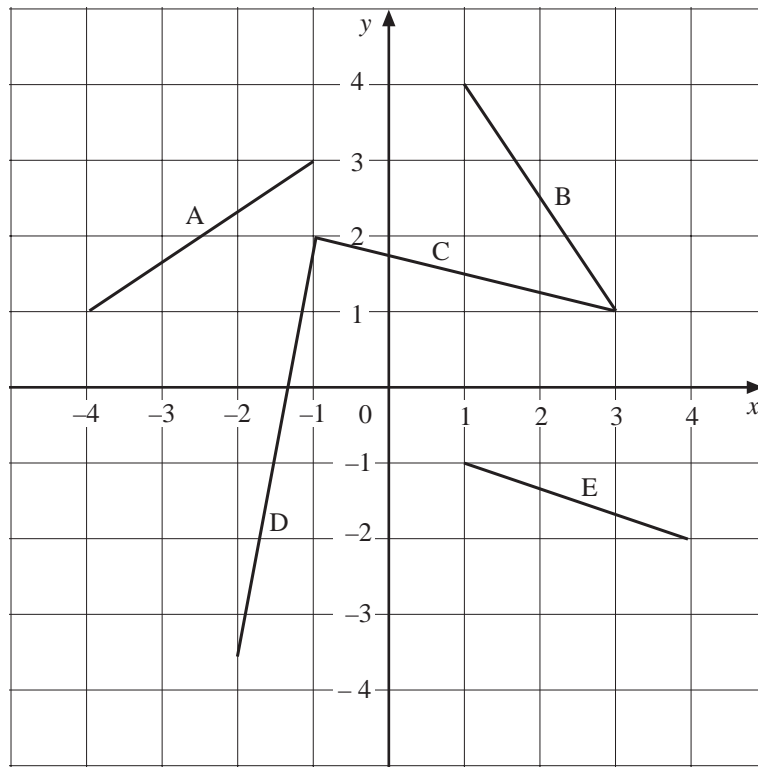


2. Which of the following lines have

- (a) positive                      (b) negative                      (c) zero  
gradient in the grid below?



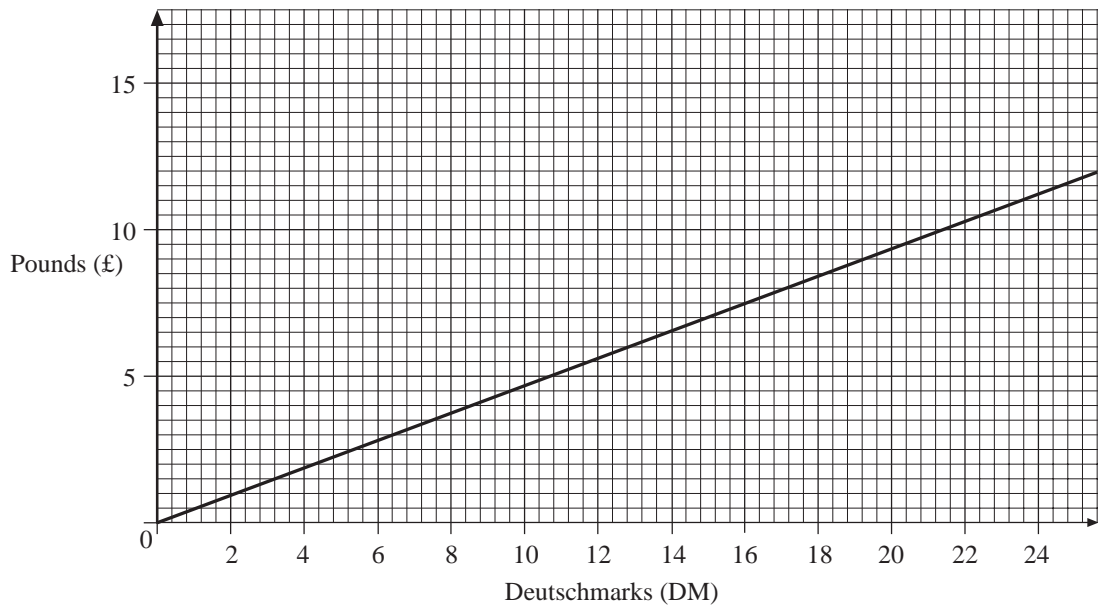
3. Find the gradient of the four lines shown on the grid below.



4. Find the gradient of the line that joins the points with coordinates
- |                          |                         |
|--------------------------|-------------------------|
| (a) (0, 0) to (4, 8)     | (b) (1, 3) to (7, 2)    |
| (c) (-1, 4) to (2, 10)   | (d) (-3, -2) to (1, -8) |
| (e) (4, -2) to (-1, -12) | (f) (5, -3) to (3, -5)  |
5. (a) For the line  $y = 3x - 1$ , complete the coordinates  
 A (1, ?), B (2, ?), C (3, ?), D (10, ?)
- (b) What is the gradient of the line  
 AB, BC, CD?
- What do you notice?
6. A quadrilateral is formed by joining the points A to B, B to C, C to D, D to A, where A, B, C, D have coordinates  
 A (-2, -2), B (-1, 3), C (3, 1), D (1, -3)
- (a) Which of the four lines found has the *largest* gradient?
- (b) Which of the four lines has the *smallest* gradient?

## 13.6 Application of Graphs

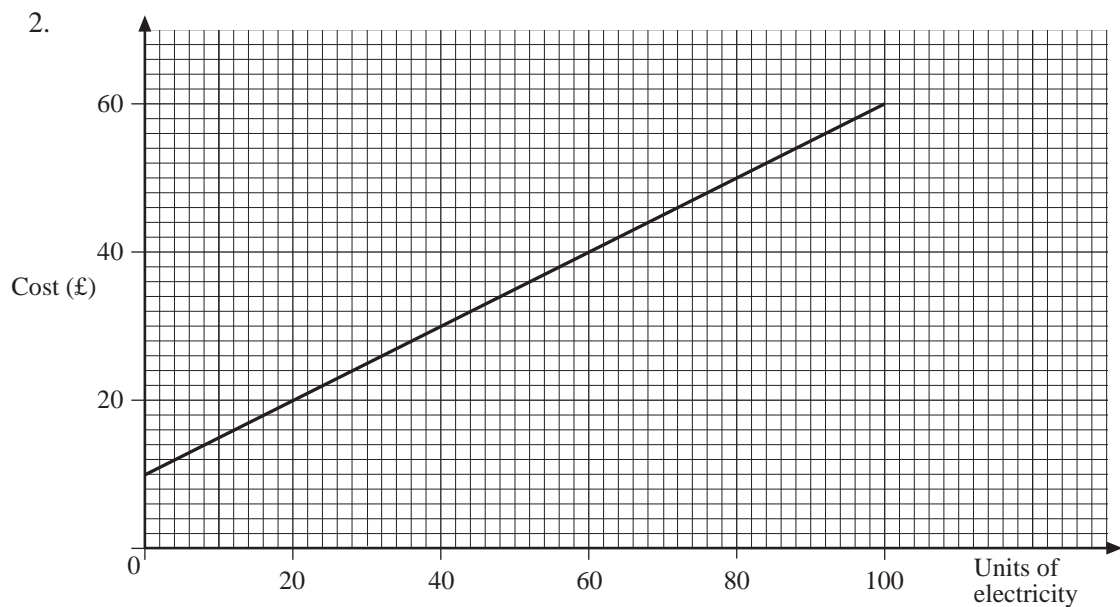
1. The diagram shows a conversion graph between Pounds (£) and German Deutschmarks (DM).



Use the graph to write down how many

- Deutschmarks can be exchanged for £10,
- Pounds can be exchanged for 14 Deutschmarks.

(LON)



The graph shows the cost, in pounds, of electricity used by one person.

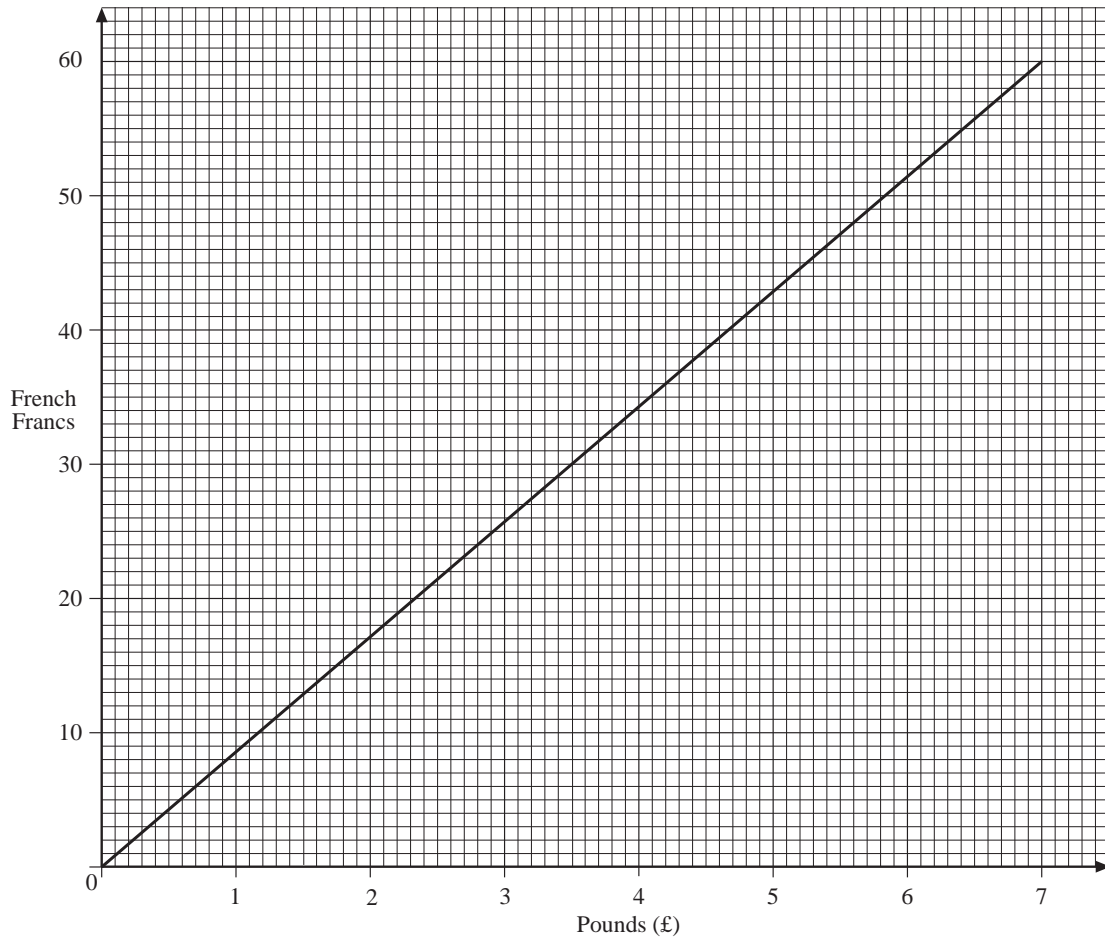
The cost is made up of a fixed standing charge, plus the cost of the number of units of electricity used.

Use the graph to find

- (a) the standing charge in pounds,  
 (b) the cost, in pence, of one unit of electricity.

(LON)

3. This graph can be used to convert between pounds (£) and French Francs.



Use the graph to convert

- (a) (i) £6 to French Francs,  
 (ii) 55 French Francs to pounds.

Jim is going to the USA. The bank will exchange 1.58 dollars for £1.

It also charges a commission of 1% for the exchange.

Jim wants to exchange £250 for dollars at the bank.

- (b) Work out how many dollars he would receive.

(LON)



4. The table below shows volumes expressed in imperial and metric measure.

Imperial measure in fluid ounces	Metric measure in litres
1	0.028
4	0.114
6	0.180
7	0.199
8	0.227
10	0.284

- (a) Plot these entries on a suitable grid.
- (b) One of the entries in the metric column is incorrect.
- (i) Draw the conversion graph showing the connection between imperial and metric measure and write down the incorrect metric entry.
- (ii) Use your graph to estimate the correct metric entry.

(NEAB)

5. Hannah goes to the shop to buy a loaf of bread.

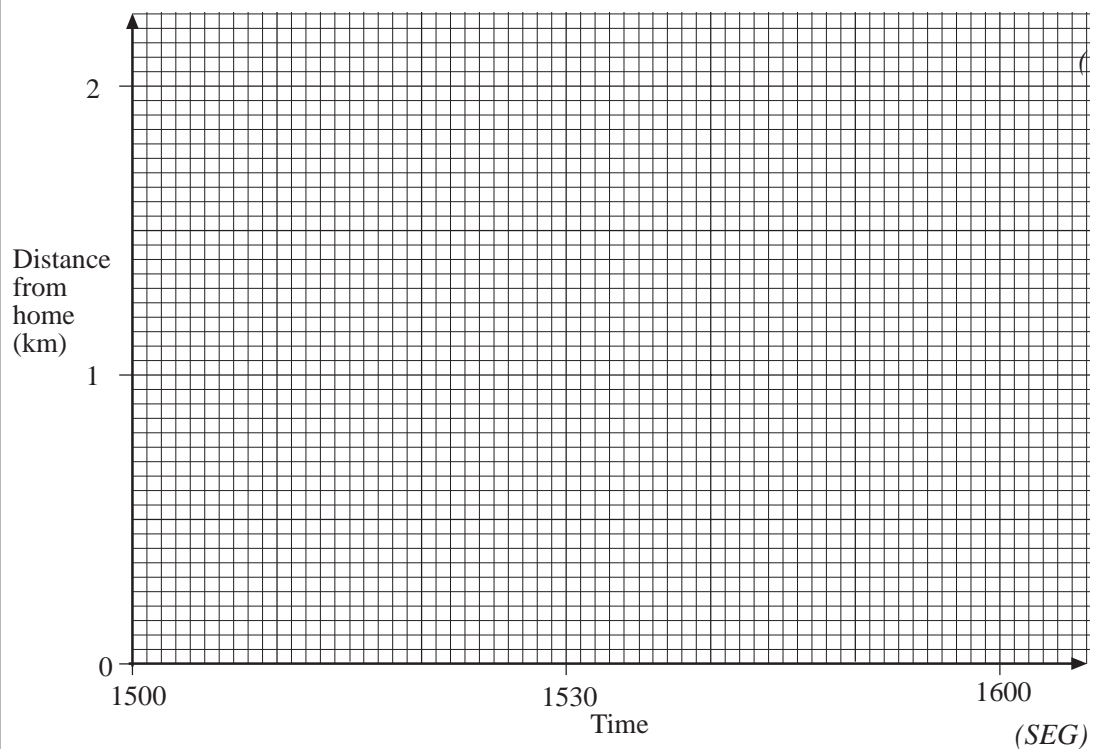
The shop is 800 m from the house.

She leaves home at 1512 and walks to the shop at a steady speed.

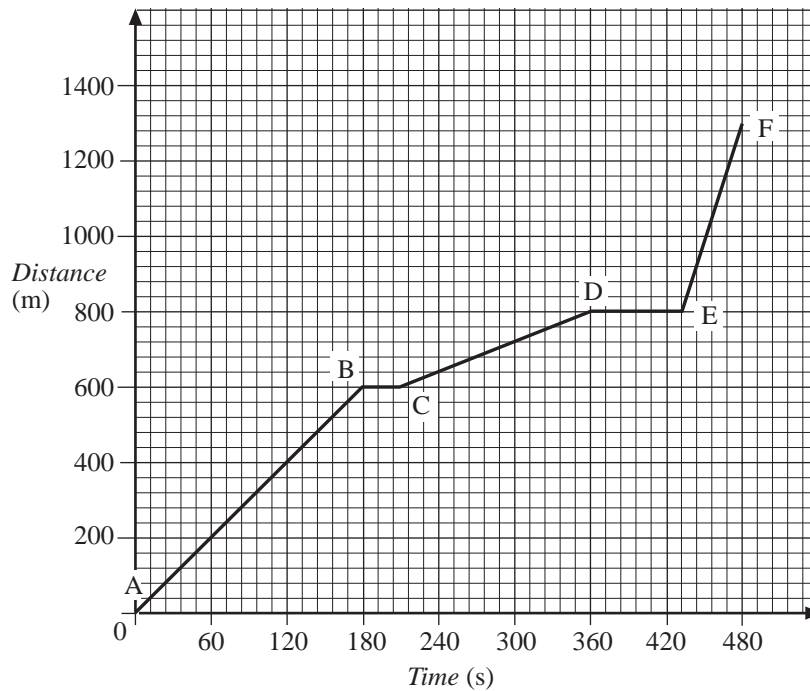
She takes 16 minutes to reach the shop and then 5 minutes to buy a loaf of bread.

She then walks home at a steady speed arriving at 1548.

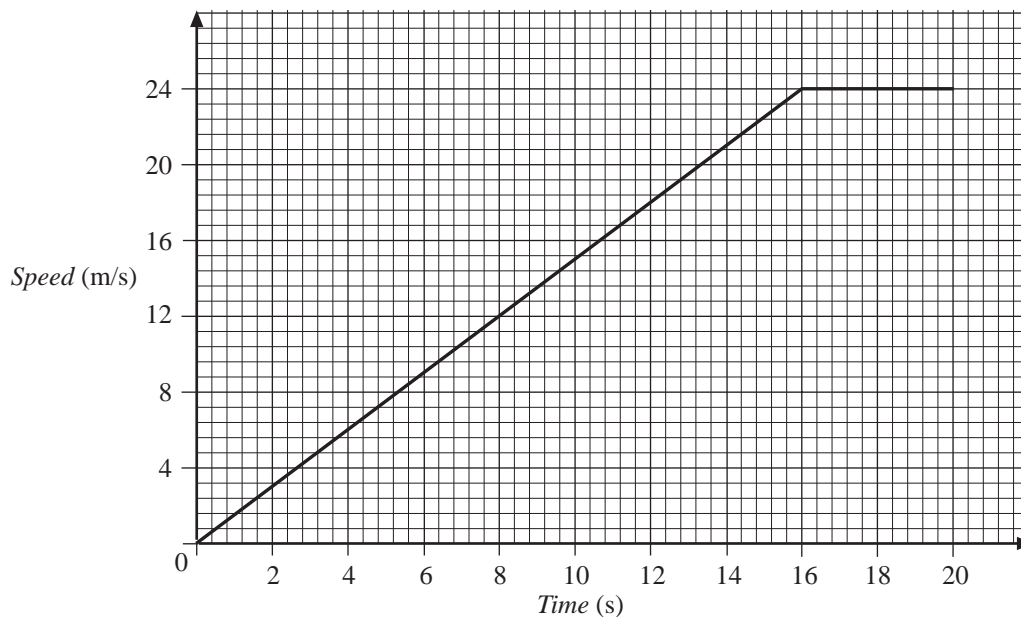
On a copy of the diagram draw a distance-time graph to represent her journey.



6. The graph below shows the distance travelled by a car on a journey to work.



- (a) The car stopped at two sets of traffic lights. How long did the car spend waiting at each set of lights?
- (b) On which part of the journey did the car travel fastest?
- (c) (i) How far did the car travel?  
(ii) How long did it take for the whole journey?  
(iii) What was the car's average speed for the whole journey?
7. The graph below shows the speed of a train as it sets off from a station.

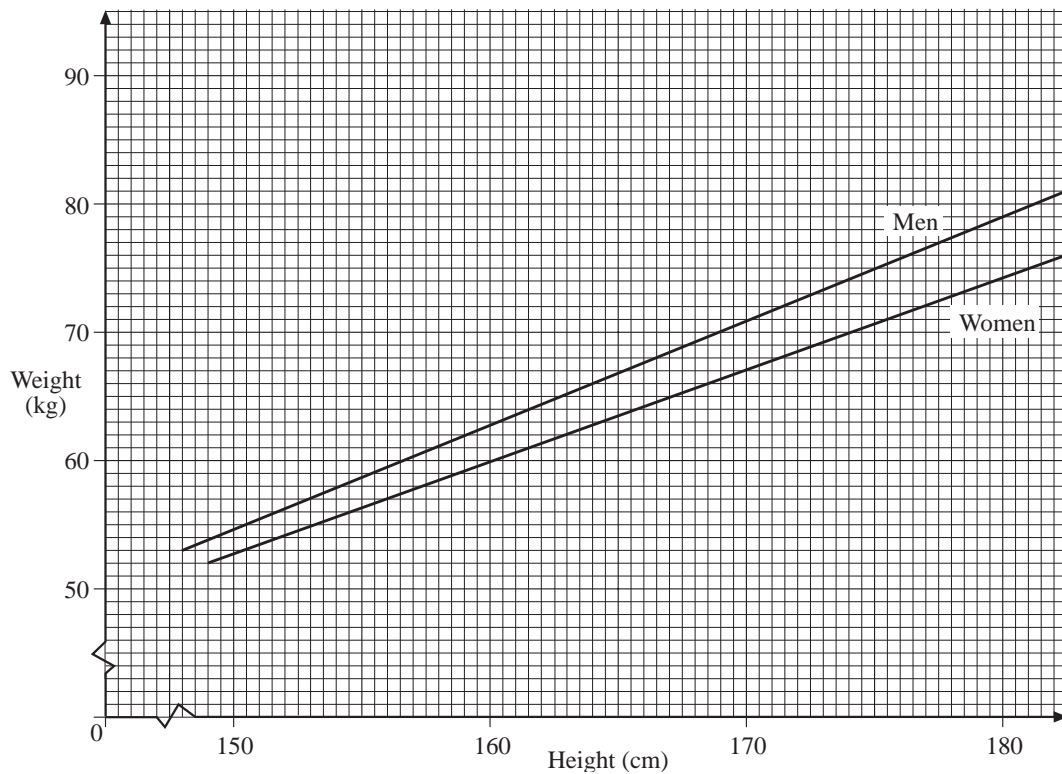


Find the distance travelled by the train after

- (a) 4 seconds      (b) 8 seconds      (c) 16 seconds

What is the formula that connects the time of travel and distance travelled for  $0 \leq t \leq 16$ ?

8. The graphs of the average weight for different heights for women and men are shown.



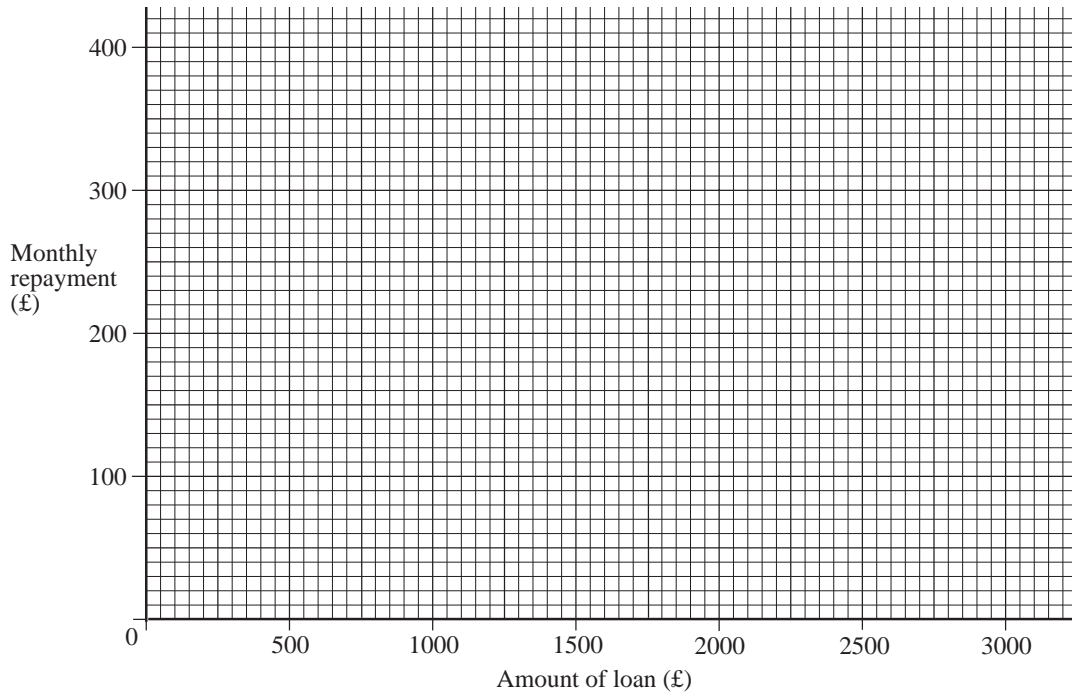
- (a) Jim and his wife Linda are both 160 cm in height.  
Use the graphs to estimate the difference in their weights.
- (b) Arthur and his wife Pam both weigh 75 kg.  
Use the graphs to estimate the difference in their heights. Show all your working.
- (c) The actual difference in the heights of Arthur and Pam is 12 cm.  
Give a possible reason why the graphs give a different answer.

(SEG)

9. The table shows the repayments required on loans of different amounts, for 1 year.

Amount of loan (£)	500	750	1500	2250	3000
Monthly Repayment (£)	60	85	160	235	310

- (a) Plot these pairs of values on a copy of the grid below. Join them with a straight line.



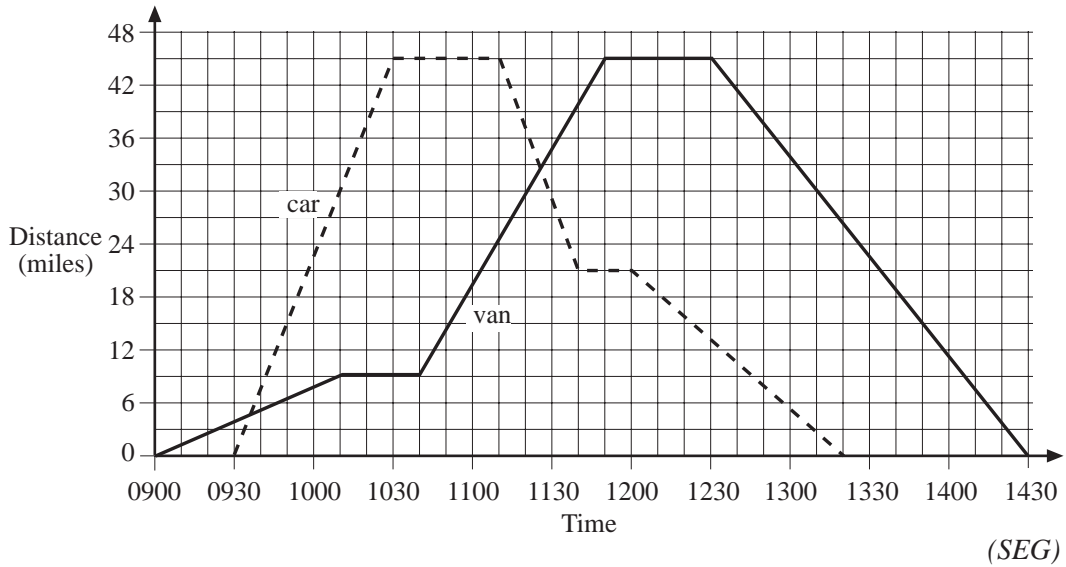
- (b) I can afford to repay £180 a month. Use the graph to find out the largest amount I could borrow.
- (c) Use your graph to find the monthly repayment on a loan of £1000.
- (d) Phil borrows £1500.

Altogether his 12 monthly repayments amount to more than £1500. How much more?

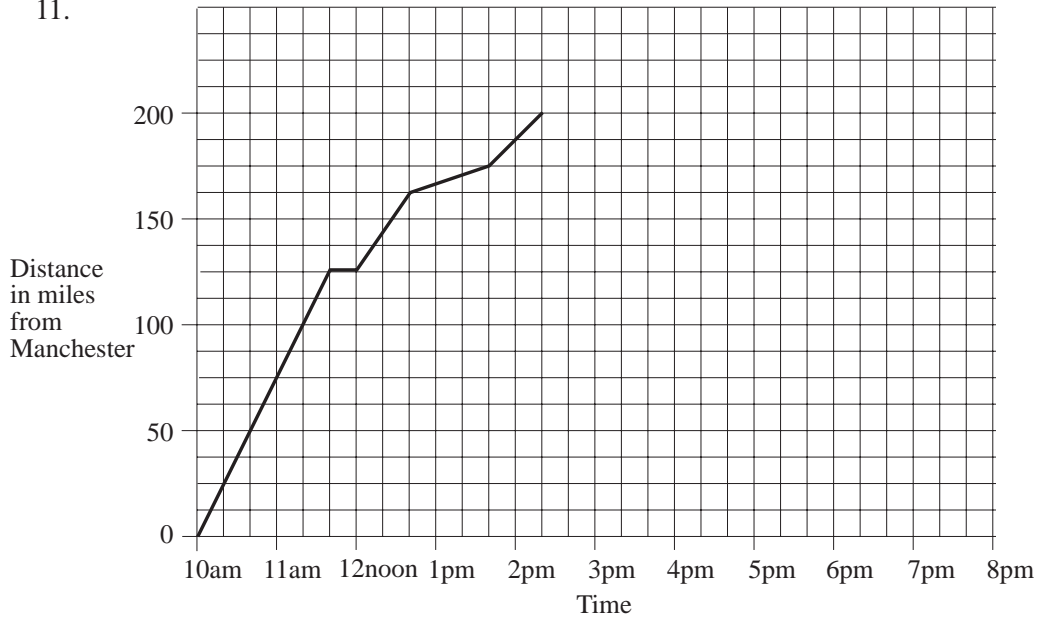
(MEG)

10. The following distance-time graph shows the journeys made by a van and a car starting from Oxford, travelling to Luton, and returning to Oxford.

- (a) How far had the car travelled when it met the van for the second time?
- (b) Calculate, in miles per hour, the average speed of the car between 0950 and 1000.
- (c) During which period of time was the van travelling at its greatest average speed?



11.



The graph represents part of Mrs Hinton's journey from Manchester to London. Mrs Hinton stopped for a rest at a service station.

- (a) (i) Write down the time at which she stopped.  
 (ii) For how long did she stop?

For part of her journey Mrs Hinton had to slow down because of a traffic queue.

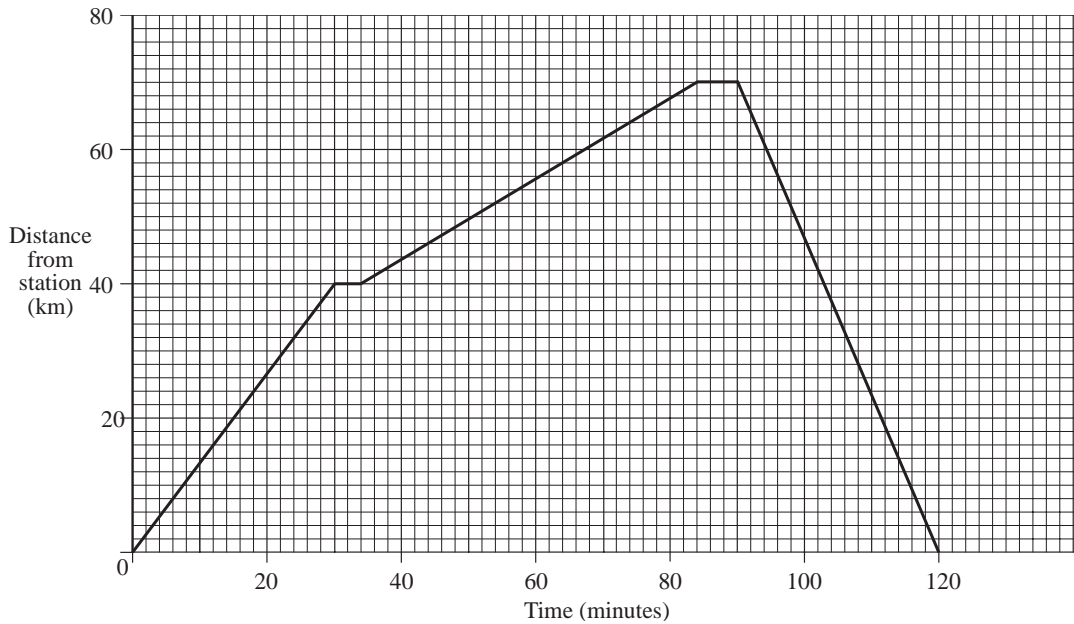
- (b) For how many miles did she travel at this lower speed?

Mrs Hinton spent an hour at a meeting in London. She then returned home to Manchester, travelling at a steady speed of 50 miles an hour.

- (c) Use this information to complete the graph of her journey.

(LON)

12. The diagram shows the travel graph of a train.



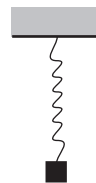
- (a) Find the greatest speed at which the train travelled.  
Give your answer in km/h.
- (b) Calculate the average speed for the whole journey in km/h.

(NEAB)

## 13.7 Scatter Plots and Lines of Best Fit

1. The extension of a spring for a variety of attached weights is given in the table below.

<i>Mass</i> (grams)	100	200	300	400	600
<i>Extension</i> (cms)	51	105	148	195	305

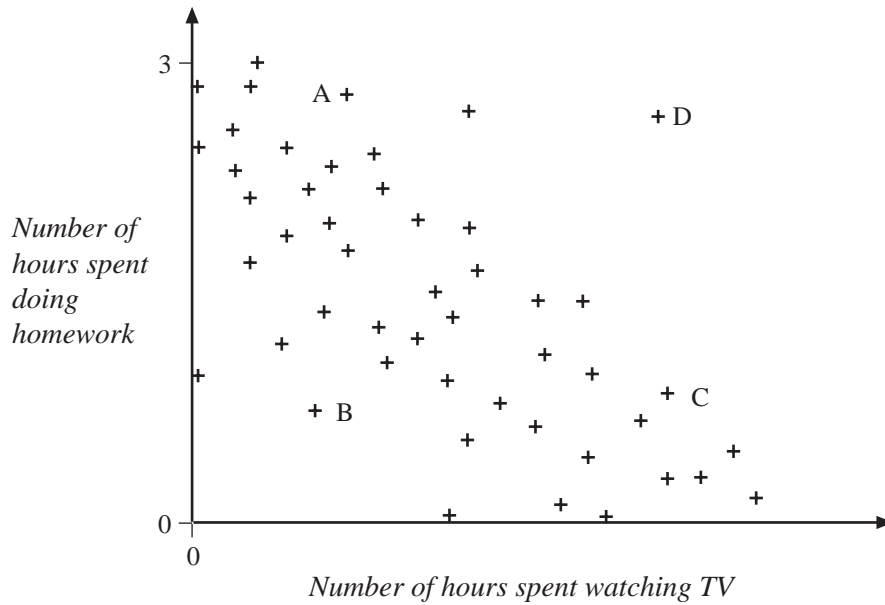


Plot the data on a scatter diagram. What type of correlation does this table show?

Draw a line of best fit. What do you think would be the extension for a mass of 500 grams?

2. Annie asked a four teenagers to say how much time they spent doing homework one evening, and how much time they spent watching TV.

Here is a scatter diagram to show the results.



- (a) Which of the four points, A, B, C or D, represents each of the statements shown below? Write down one letter for each of the names.

*LUCY* says

*I spent most of my evening doing homework. I only watched one programme on TV.*

*TOM* says

*I watched a lot of TV last night and I also did a lot of homework*

*CHRIS* says

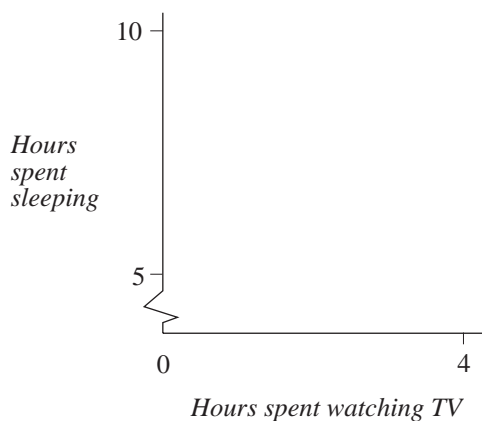
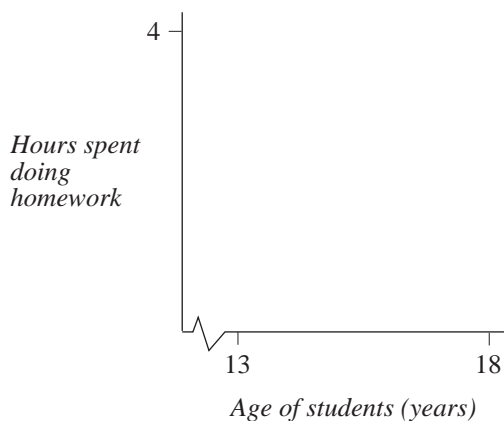
*I went out last night. I didn't do much homework or watch TV.*

- (b) Make up a statement which matches the fourth point.
- (c) What does the graph tell you about the relationship between time spent watching TV and time spent doing homework?
- (d) Annie also drew scatter diagrams which showed that:

Older students tend to spend more time doing homework than younger students.

There is no relationship between the time students spend watching TV and the time students spend sleeping.

On a copy of the axes below, show what Annie's scatter diagram may have looked like.

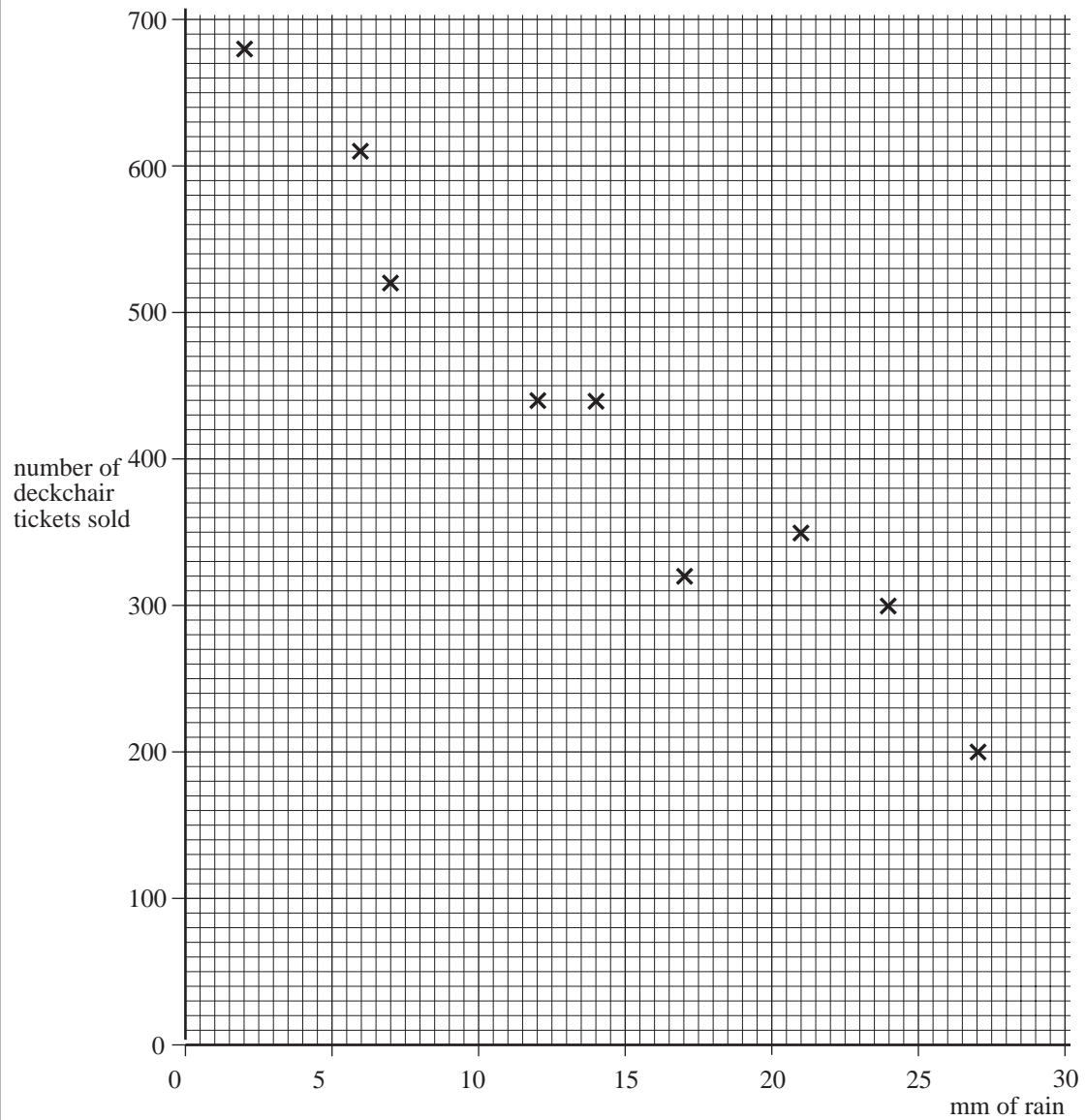


3. Each week during the summer season, a seaside resort recorded the rainfall and the number of deckchair tickets sold.

Some of the results are plotted on the scatter diagram on the next page.

- What does the scatter diagram tell you about the connection between the rainfall and the number of deckchair tickets sold?
- On a copy of the diagram draw in a line of best fit.
- In the first week of June only 250 deckchair tickets were sold.  
How much rain do you think the resort had that week?





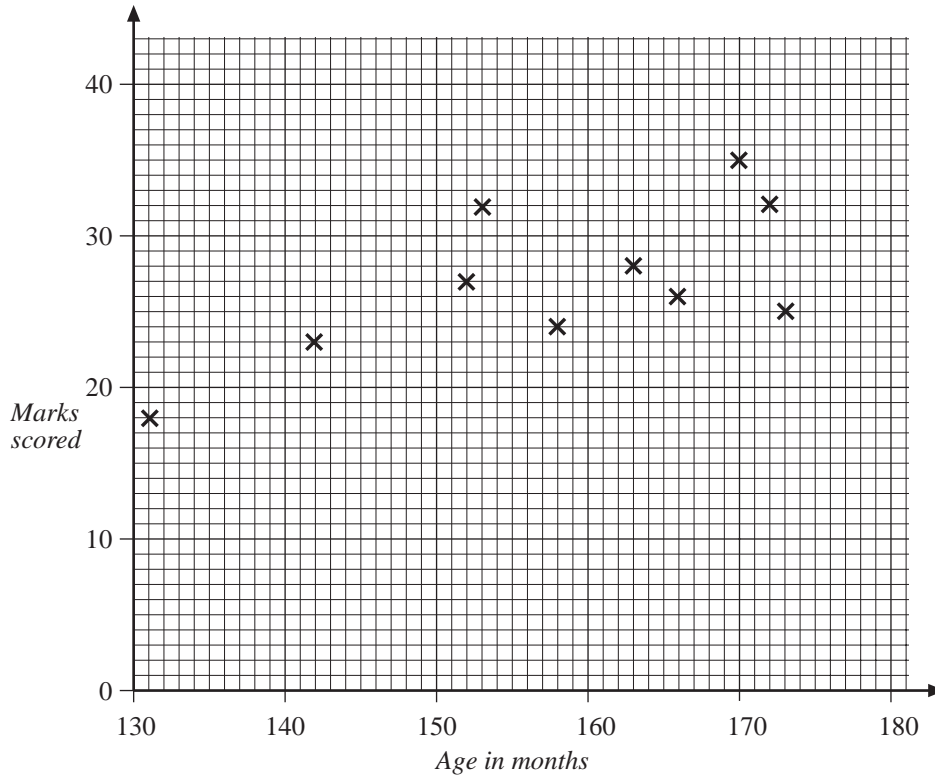
(NEAB)

4. Ten boys of different ages were set the same General Knowledge test. The results are shown in the table below.

Boy	A	B	C	D	E	F	G	H	I	J
Age (months)	131	142	152	153	158	163	166	170	172	173
Score	18	23	27	32	24	28	26	35	32	25

- (a) The mean of the ages of the boys is 158 months.  
Calculate the mean of their scores.

(b) The results in the table have been plotted on the following scatter graph.



- (i) Does the scatter graph show the sort of result you would expect? Explain your answer.
- (ii) On a copy of the scatter graph draw a line of best fit.
- (iii) Taking age into account, to which boy would you award a prize for the best performance?

5. The table below shows the number of Compact Discs (CDs) and the number of Long Playing Records (LPs) that were sold from 1984 to 1992.

	1984	1985	1986	1987	1988	1989	1990	1991	1992
Number of CDs (millions)	0	5	10	20	30	35	45	60	70
Number of LPs (millions)	55	55	50	50	45	40	20	15	5

- (a) 70 million CDs were sold in 1992. Write the number 70 million in figures.
- (b) In which year did the sale of CDs overtake the sale of LPs?
  - (i) Draw a scatter graph to show the sale of CDs against the sale of LPs.
  - (ii) What does your scatter diagram tell you about the connection between the sale of CDs and the sale of LPs?

6. Tom breeds hamsters. The number of hamsters is expected to treble each year. Tom had 20 hamsters on 1.1.96.

Date	1.1.96	1.1.97	1.1.98	1.1.99	1.1.00
Estimated number of hamsters	20	60	180	540	1620

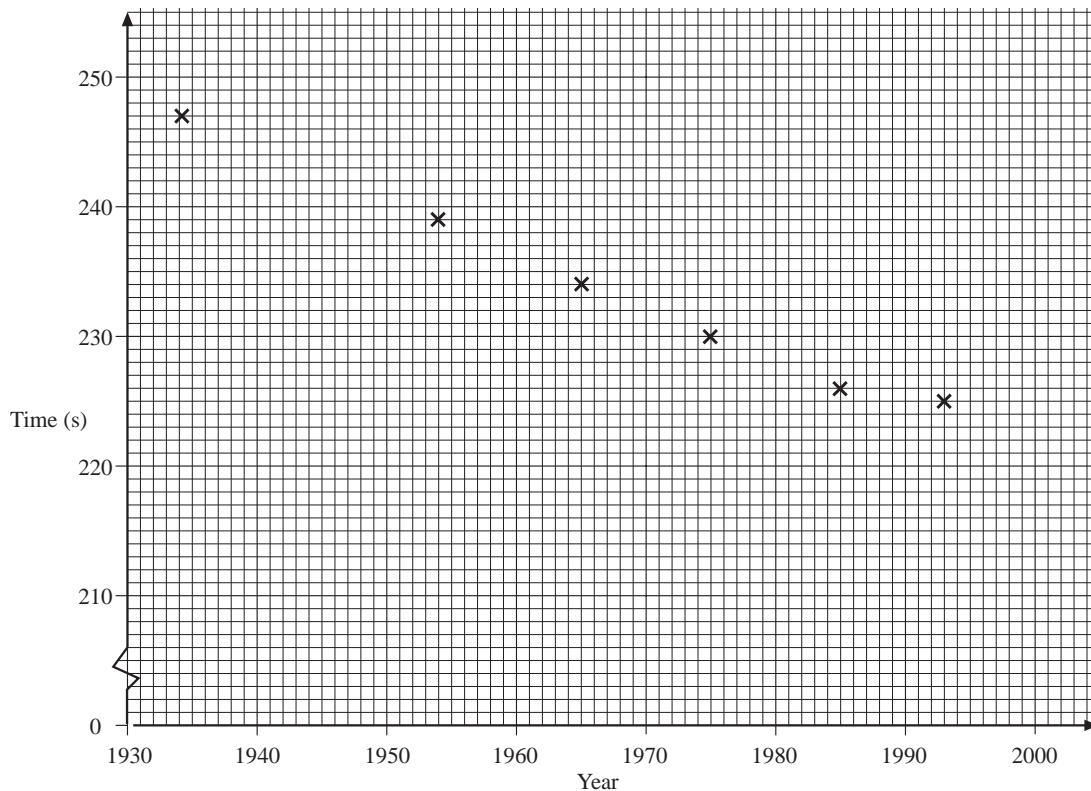
- (a) On a suitable grid, draw a smooth curve to represent this information.  
 (b) Use your grid to estimate the number of hamsters Tom would have on 1.7.98.  
 (c) Write down an expression to find the estimated number of hamsters  $n$  years after 1.1.96.

(LON)

7. Below are the years and times of some world records for running the mile.

Glen Cunningham	1934	247 sec
Roger Bannister	1954	239 sec
Michael Jazy	1965	234 sec
John Walker	1975	230 sec
Steve Cram	1985	226 sec
Noureddine Morcelli	1993	225 sec

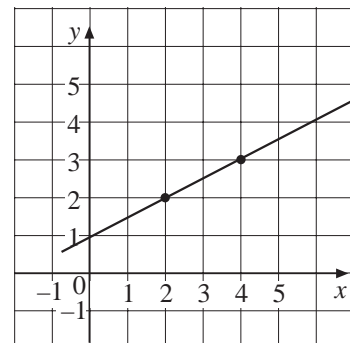
These data are used to plot a scatter diagram.



- (a) (i) On a copy of the scatter diagram, draw the line of best fit.  
 (ii) Sebastian Coe ran a new world record in 1979.  
 Use the line to estimate his time.  
 (iii) Explain why the line you have drawn can only be used to estimate times for a limited number of years.
- (b) Roger Bannister's actual time of 239.2 seconds is known to be correct to the nearest tenth of a second.  
 What is the shortest time that it could actually be?
- (c) In 1993 Noureddine Morcelli's time for running the mile was 225 seconds.  
 By taking 5 miles to be equal to 8 kilometres, calculate what his time for the 1500 metres would have been, assuming that his average speed was the same.  
 Give your answer to the nearest second.

## 13.8 The Equation of a Straight Line

1. Find the equation of the straight line with
- (a) gradient = 1 and y-intercept = 2  
 (b) gradient = 3 and y-intercept = -1  
 (c) gradient = -2 and y-intercept = 1  
 (d) gradient =  $-\frac{1}{2}$  and y-intercept = 2
2. Write down the gradient and y-intercept of each of the following lines.
- (a)  $y = 2x - 1$       (b)  $y = 3x + 4$       (c)  $y = 1 - \frac{1}{2}x$   
 (d)  $y = -x + \frac{1}{2}$       (e)  $y = \frac{1}{3}(x + 1)$       (f)  $y = -\frac{1}{3} + 2x$
3. The diagram shows the straight lines passing through the points (2, 2) and (4, 3).  
 Find
- (a) the gradient of the line,  
 (b) the y-intercept of the line,  
 (c) the equation of the line.

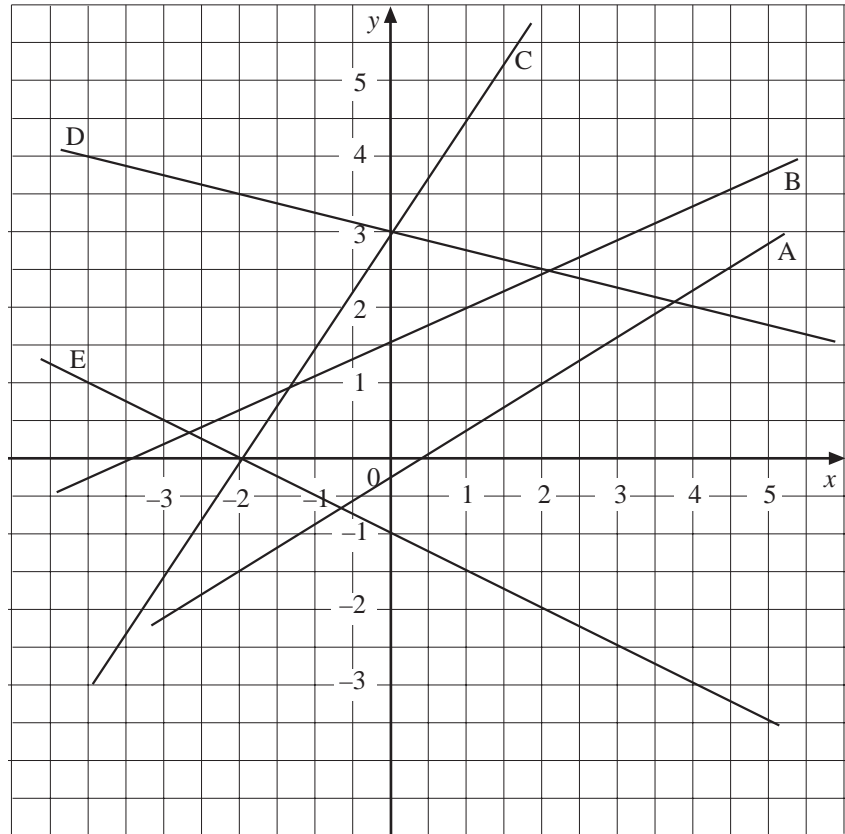


4. Write down the gradient and y-intercept of each of the following lines.

(a)  $y = x - 4$                       (b)  $y = 2x + 1$                       (c)  $y = \frac{1}{2}x - 4$

(d)  $y = 5 - 3x$                       (e)  $y = \frac{1}{2} - \frac{x}{4}$                       (f)  $y = -1 - \frac{x}{2}$

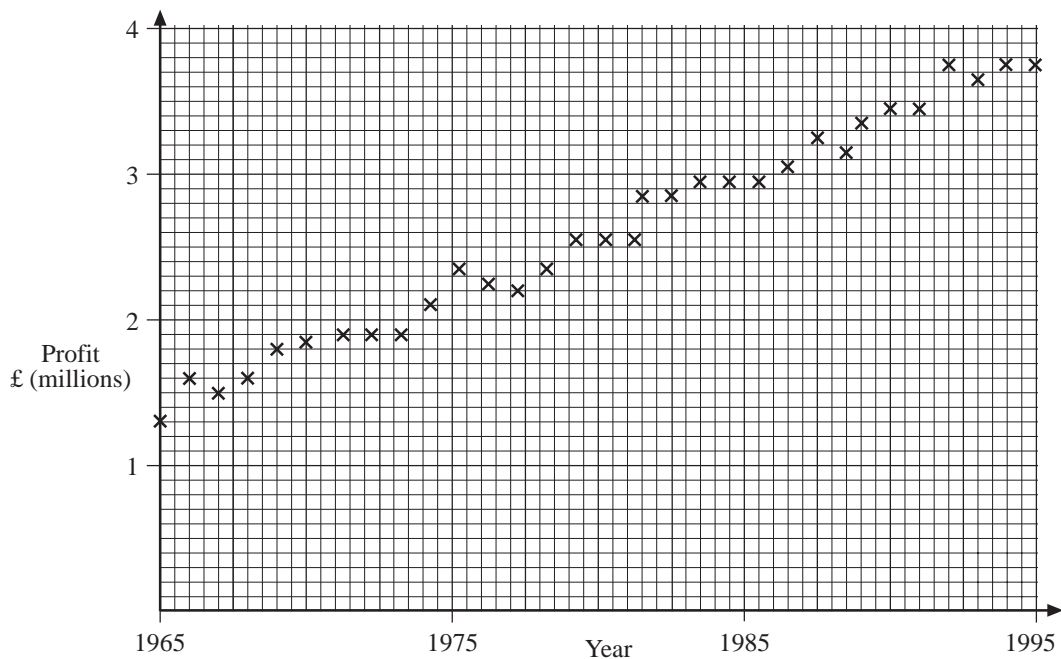
5. Find the equation of each line shown in the diagram below.



6. Find the equation of the line that passes through the points with coordinates:

- (a)  $(0, 1)$  and  $(2, 5)$   
 (b)  $(1, 5)$  and  $(3, 3)$   
 (c)  $(0, 2)$  and  $(4, 0)$   
 (d)  $(-1, -1)$  and  $(1, 3)$

7. The scatter diagram represents the profits made by a company over the years 1965 to 1995.



Use the diagram to calculate an estimate of the profit the company would expect to get in the year 2000 if this trend continues.

(NEAB)

8. The line  $y = 2x + c$  passes through the point  $(1, 8)$ .

Find the value of  $c$ .

9. The line  $y = mx + 3$  passes through the point  $(2, -7)$ .

Find the value of  $m$ .

10. The charges made by a removal firm consist of

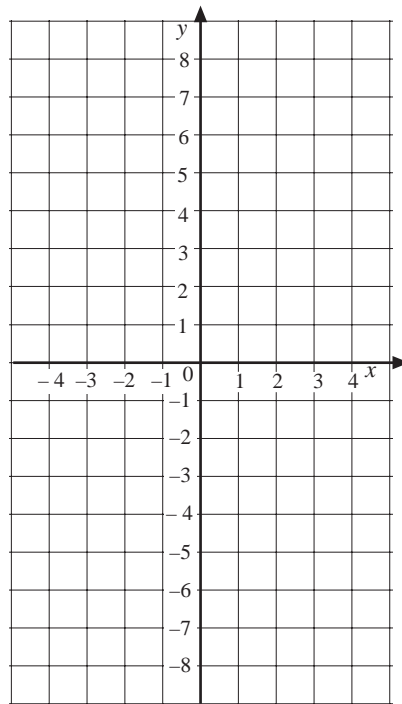
- (a) a fixed charge of £50, and
- (b) a variable charge of £5 per mile travelled.

Write down the formula for the total cost,  $y$ , in terms of the distance travelled,  $x$  miles. Draw a graph of this relationship for  $0 \leq x \leq 20$  and use it to estimate the distance travelled when the total cost is £120.

11. (a) Complete a copy of the table of values for  $y = 3x - 2$ .

$x$	-1	0	1	2	3
$y = 3x - 2$					

- (b) On a copy of the following grid plot your values for  $x$  and  $y$ .  
Join your points with a straight line.



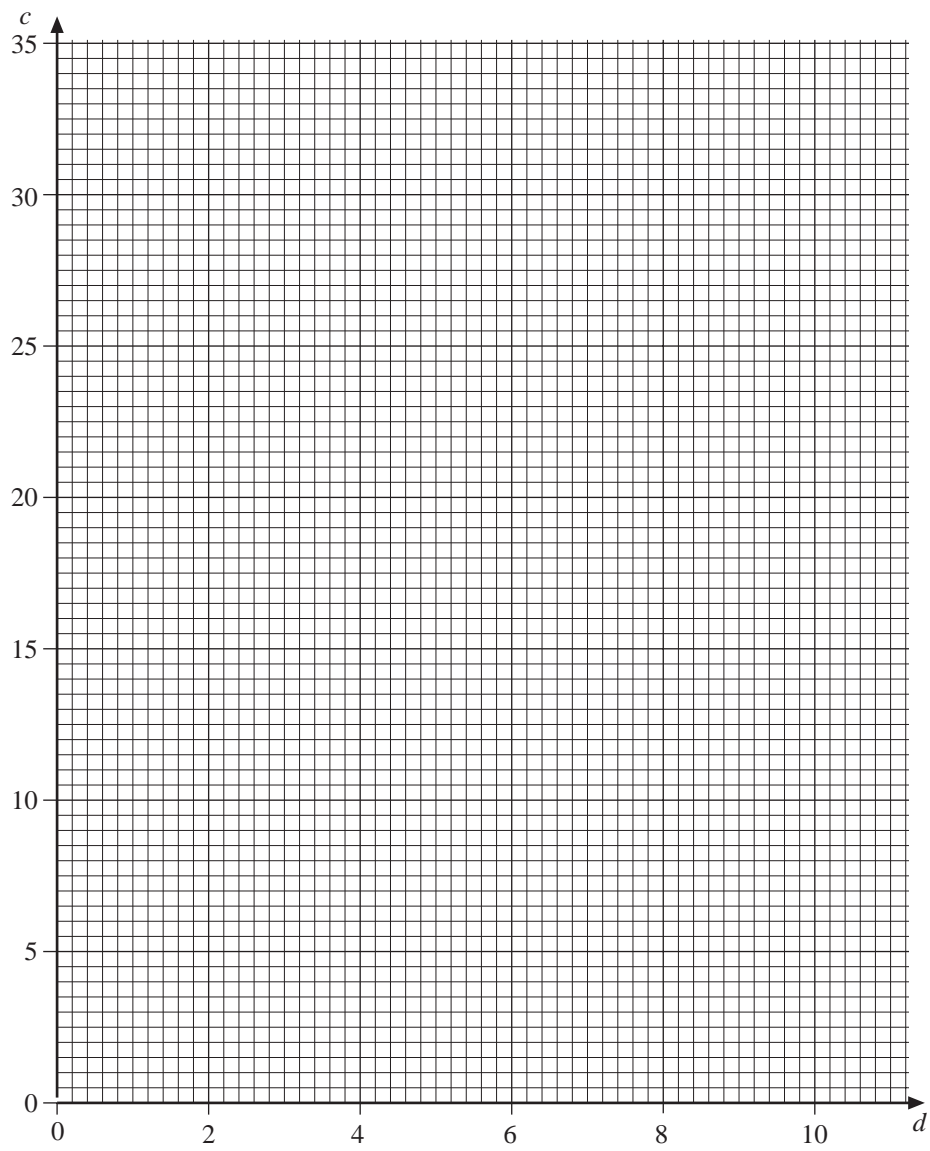
- (c) Write down the coordinates of the points where your graph crosses the  $y$ -axis.

(LON)

12. This table shows the diameter,  $d$ , and the circumference,  $c$ , of four circular objects. They have been measured to the nearest centimetre.

Object	$d$	$c$
2p coin	2 cm	6 cm
tin of beans	6 cm	18 cm
saucer	7 cm	21 cm
plate	10 cm	30 cm

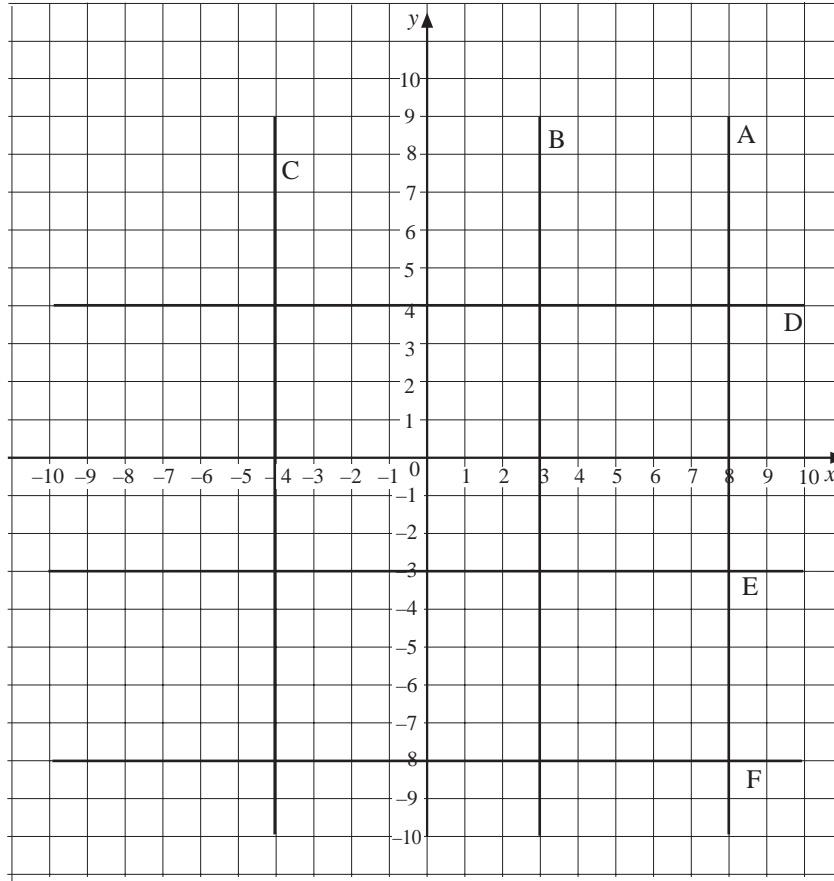
- (a) On a copy of the following grid, plot  $c$  against  $d$  for each object.  
Draw a straight line to show the relationship between  $c$  and  $d$ .
- (b) Write down the equation of this straight line.





# 13.9 Horizontal and Vertical Lines

1. Write down the equation of each line marked in the following diagram.



2. Draw the lines  $x = 3$ ,  $x = 7$ ,  $y = 1$ ,  $y = 10$ .

Write down the coordinates of the points where they cross and find the area of the rectangle enclosed by these lines.

3. Draw the lines  $x = -1$ ,  $x = 3$ ,  $y = -4$ ,  $y = 2$ .

What are the coordinates of the centre of the rectangle formed?

4. Draw the square which has corners at the points with coordinates

$$(-2, 1), (-2, 4), (1, 1) \text{ and } (1, 4)$$

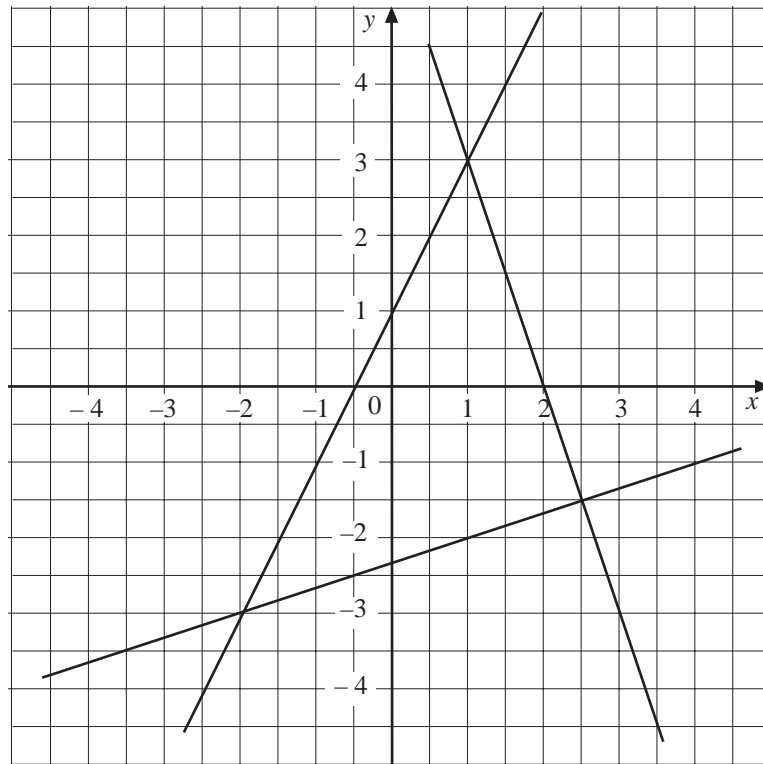
What are the equations of the lines that form the sides of the square?

5. Draw the lines  $y = \frac{1}{2}x$ ,  $x = 8$ ,  $y = 2$ .

What is the area of the triangle enclosed?

# 13.10 Solutions of Simultaneous Equations by Graphs

1.



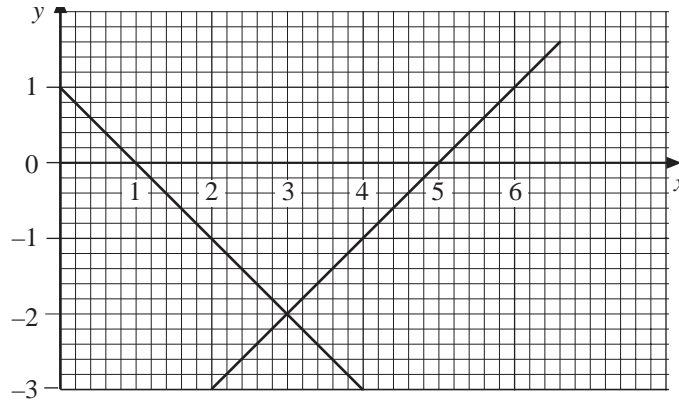
Use the graph above to solve

(a)	$y = 2x + 1$	(b)	$y = \frac{1}{3}x - \frac{7}{3}$	(c)	$y = -3x + 6$
	$y = \frac{1}{3}x - \frac{7}{3}$		$y = -3x + 6$		$y = 2x + 1$

2. Solve graphically the following sets of simultaneous equations.

(a)	$x + y = 4$	(b)	$x + 2y = 5$
	$2x + y = 10$		$2x - 3y = 3$
(c)	$2y - 5x = 3$	(d)	$4x + 3y = -2$
	$3y + 2x = -5$		$3x + 4y = 2$

3.



The diagram shows the graphs of the equations

$$x + y = 1 \quad \text{and} \quad y = x - 5$$

Use the diagram to solve the simultaneous equations

$$x + y = 1$$

$$y = x - 5$$

(LON)

4. Jane buys 3 litres of oil and 40 litres of petrol for £30.

Richard buys 2 litres of oil and 10 litres of petrol for £10.

The cost of 1 litre of oil is £ $x$ . The cost of 1 litre of petrol is £ $y$ .

Therefore  $3x + 40y = 30$

and  $2x + 10y = 10$

- Draw the graphs of these equations.
- What is the cost of one litre of petrol?

(SEG)

5. Fifty-one students went to a pop concert.

Let  $x$  represent the number of men.Let  $y$  represent the number of women.

The number of women is related to the number of men by the two equations

$$y + x = 51$$

$$y = 2x + 3$$

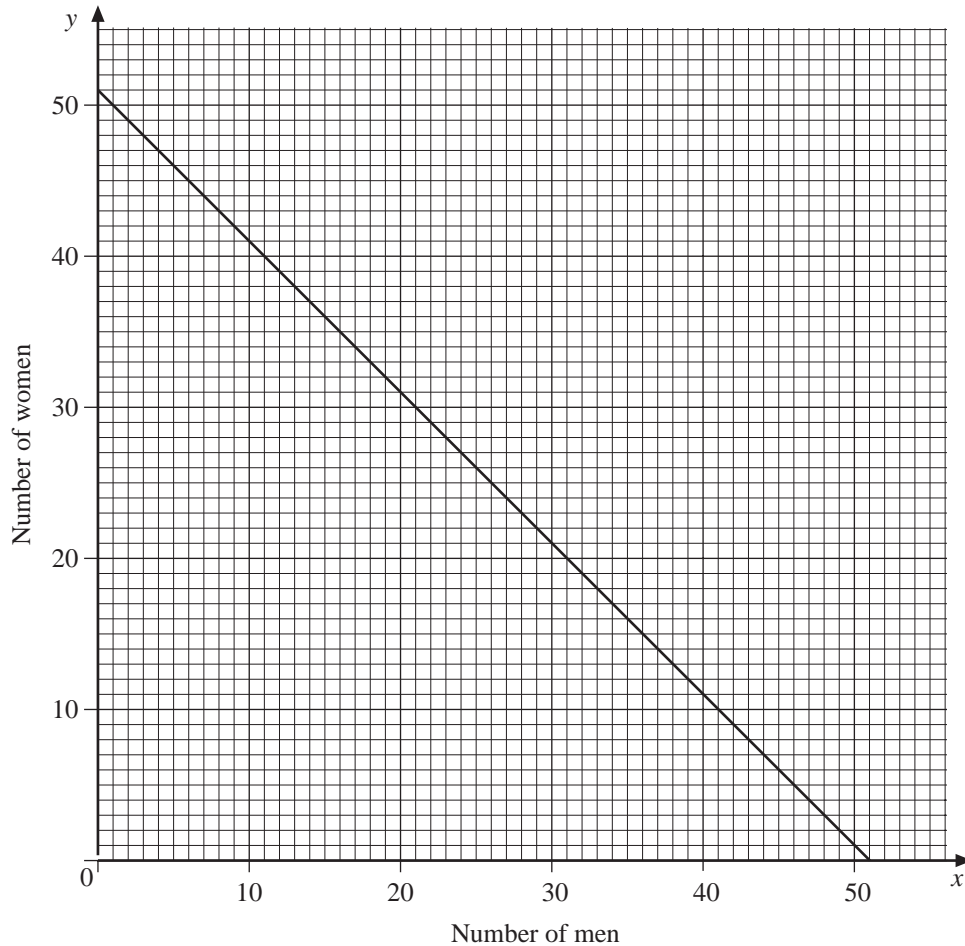
On the following graph, the line  $x + y = 51$  has been drawn

- Draw the line  $y = 2x + 3$  on a copy of the graph.
- Use the graph to write down the number of men and women who went to the pop concert.

- (c) At another pop concert there were fewer than 51 students but there were at least 20 men.

By drawing another line on your graph, find the region that represents the possible values of  $x$  and  $y$ .

Label the region with the letter  $R$ .

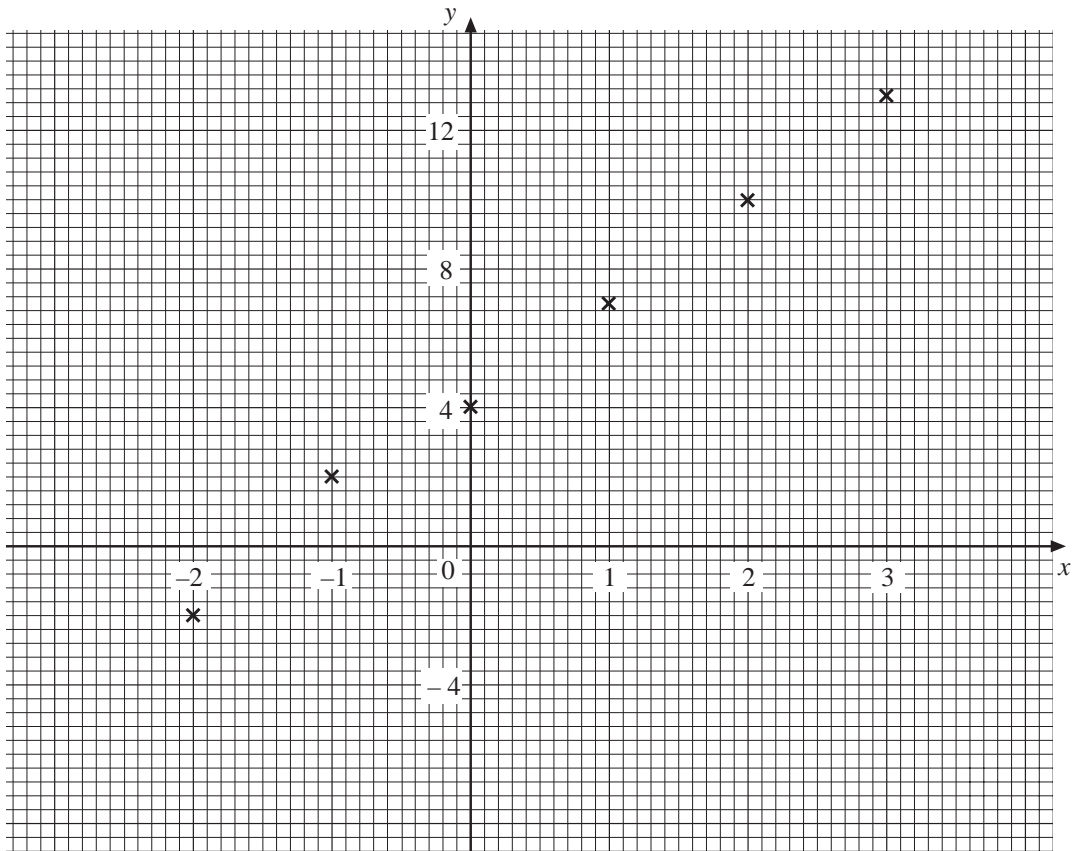


(SEG)

6. Beth was asked to draw the graph of  $y = 3x + 4$ .  
She plotted the six points shown in the following diagram.
- (a) (i) *From the shape of the graph, how can you tell that one of the points is in the wrong place?*
- (ii) On a copy of the diagram, draw the graph of  $y = 3x + 4$ .
- (b) By drawing another straight line on the diagram, solve the simultaneous equations

$$y = 3x + 4$$

$$y = 2 - x$$



(MEG)

7. Errol's house has a meter which measures the amount of water he uses.  
Errol can pay on Tariff A for the number of water units that he uses.

The graph on the next page can be used to find out how much he must pay for his water on Tariff A.

- (a) Use the graph to find how much he must pay when he used
- 60 units,
  - 104 units.

Errol uses  $x$  water units. This costs  $\pounds c$ .

- (b) Use the information from the graph to find a formula for  $c$  in terms of  $x$ .

Instead of Tariff A, Errol could pay for his water on Tariff B.

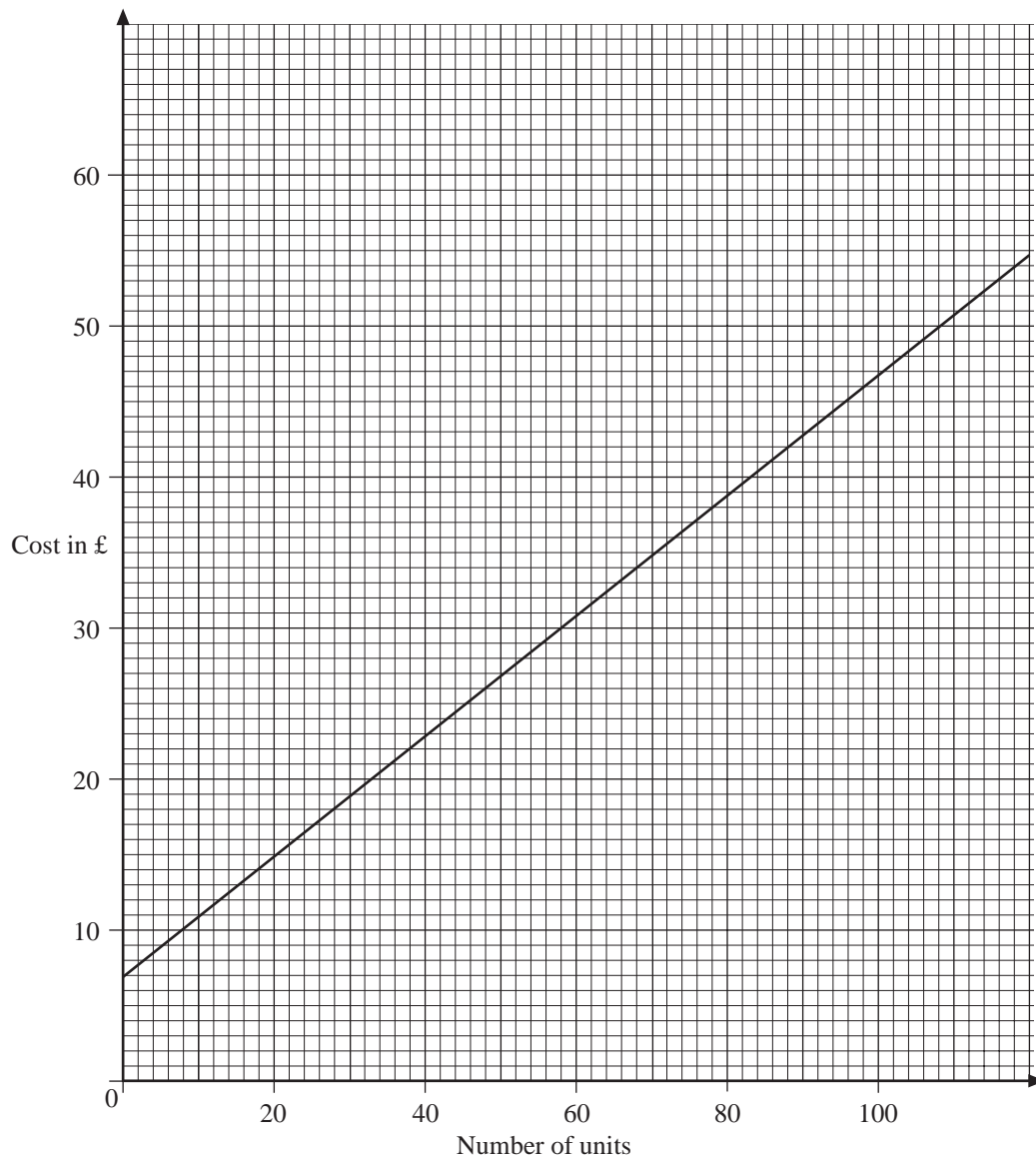
The table shows how much Errol would have to pay for his water on Tariff B.

Number of water units used ( $x$ )	0	20	40	60	80	100
Cost ( $\pounds c$ )	12	18	24	30	36	42

- (c) Plot a graph on a copy of the grid to show this information.

Errol wants to be charged the smaller amount for the water he uses.

- (d) Use the graph to find how many units Errol can use before Tariff A becomes dearer than Tariff B.



## 13.11 Graphs of Common Functions

1. State whether each equation below would produce the graph of a

*linear, quadratic, cubic or reciprocal*

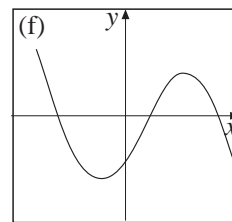
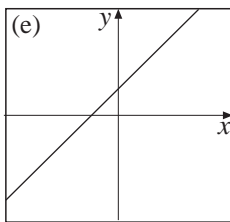
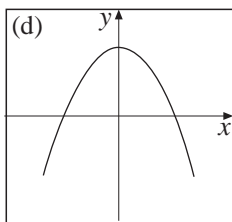
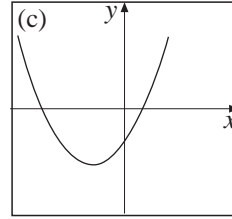
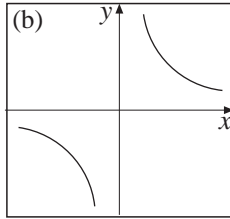
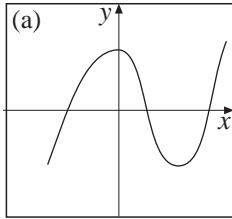
function.

- |                     |                        |                   |
|---------------------|------------------------|-------------------|
| (a) $y = 2 - x^2$   | (b) $y = \frac{3}{x}$  | (c) $y = 3 - 5x$  |
| (d) $y = x^3 - x^2$ | (e) $y = -\frac{1}{x}$ | (f) $y = x^2 + 4$ |

2. Each of the following graphs is the sketch of a

*linear, quadratic, cubic or reciprocal*

function. State which it is for each graph.



3. Each equation below has been sketched. Select the most suitable graph for each equation.

A:  $y = x^2 - 1$

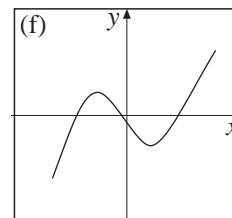
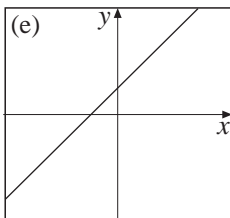
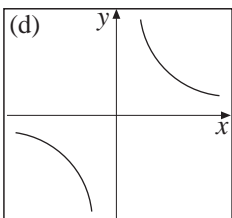
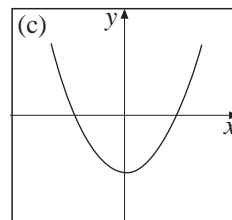
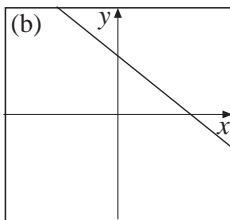
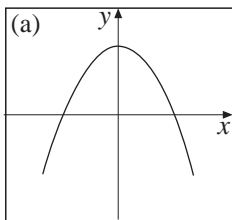
B:  $y = 2 - x$

C:  $y = \frac{2}{x}$

D:  $y = x^3 - x$

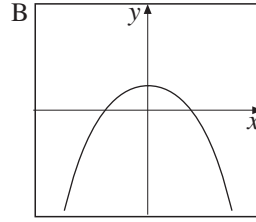
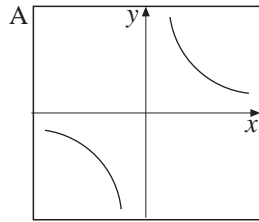
E:  $y = 2 - x^2$

F:  $y = x + 1$



4. Which of the following functions is illustrated by each of the graphs below?

$$y = x + 1, \quad y = x^2 + 1, \quad xy = 1, \quad y = x^3 + 1, \quad y = 1 - x^2$$

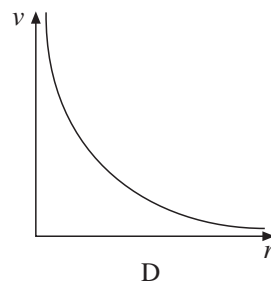
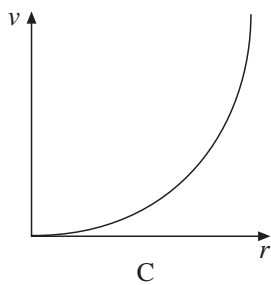
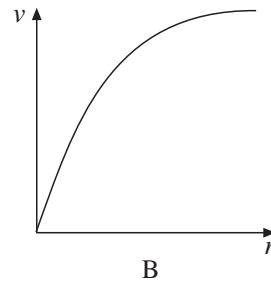
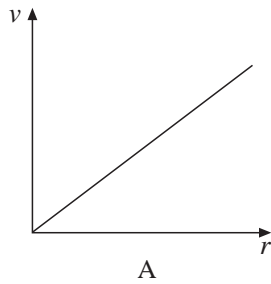


(SEG)

5. The radius,  $r$ , and value,  $v$ , of gold coins were measured and recorded.

$r$ (cm)	0.5	1	1.5	2	2.5
$v$ (£)	250	1000	2250	4000	6250

- (a) Which of these graphs represents the information shown in the table?



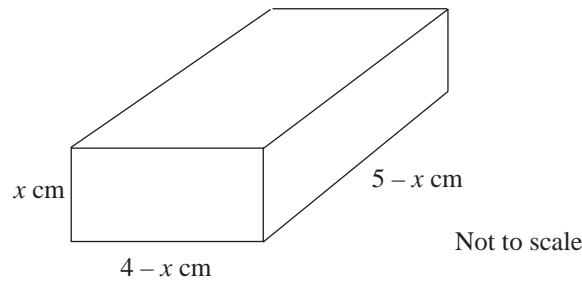
- (b) Which of these equations describes the information shown in the table?

$$v = k\sqrt{r}, \quad v = kr, \quad v = kr^2, \quad v = \frac{k}{r} \quad \text{where } k \text{ is a constant.}$$

(SEG)



6. A cuboid has the dimensions shown.



The volume,  $V$ , of the box is given by

$$V = x(4 - x)(5 - x)$$

- (a) Draw a graph of  $V$  against  $x$  for  $0 \leq x \leq 4$ .  
Use your graph to find the maximum volume of the cuboid.
- (b) Find the maximum total surface area of the cuboid when the volume is  $11 \text{ cm}^3$ .

(SEG)

7.  $x^2 + 4x + 9$  can be written in the form  $(x + p)^2 + q$ , where  $p$  and  $q$  are integers.

- (a) Find the value of
- $p$
  - $q$
- (b) Hence, or otherwise,
- sketch the graph of  $y = x^2 + 4x + 9$ .
  - write down the minimum value of  $x^2 + 4x + 9$ .

(LON)

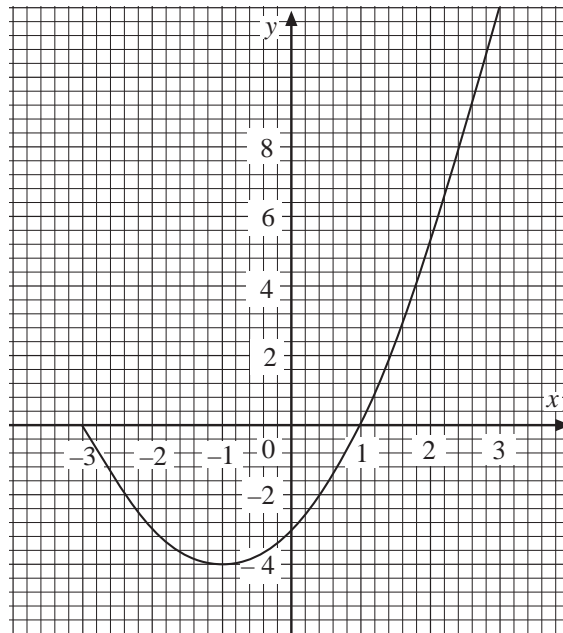
## 13.12 Graphical Solutions of Equations

1. Draw a graph of  $y = 2^x$  for  $0 \leq x \leq 3$ . Use your graph to solve the equations

(a)  $3 = 2^x$                       (b)  $6 = 2^x$

2. The following sketch shows the graph of

$$y = x^2 + 2x - 3$$



Use the graph to write down the value of the positive solution to

$$x^2 + 2x - 8 = 0$$

(LON)

3.  $y = x^3 - 4x - 1$

(a) Complete a copy of the table of values.

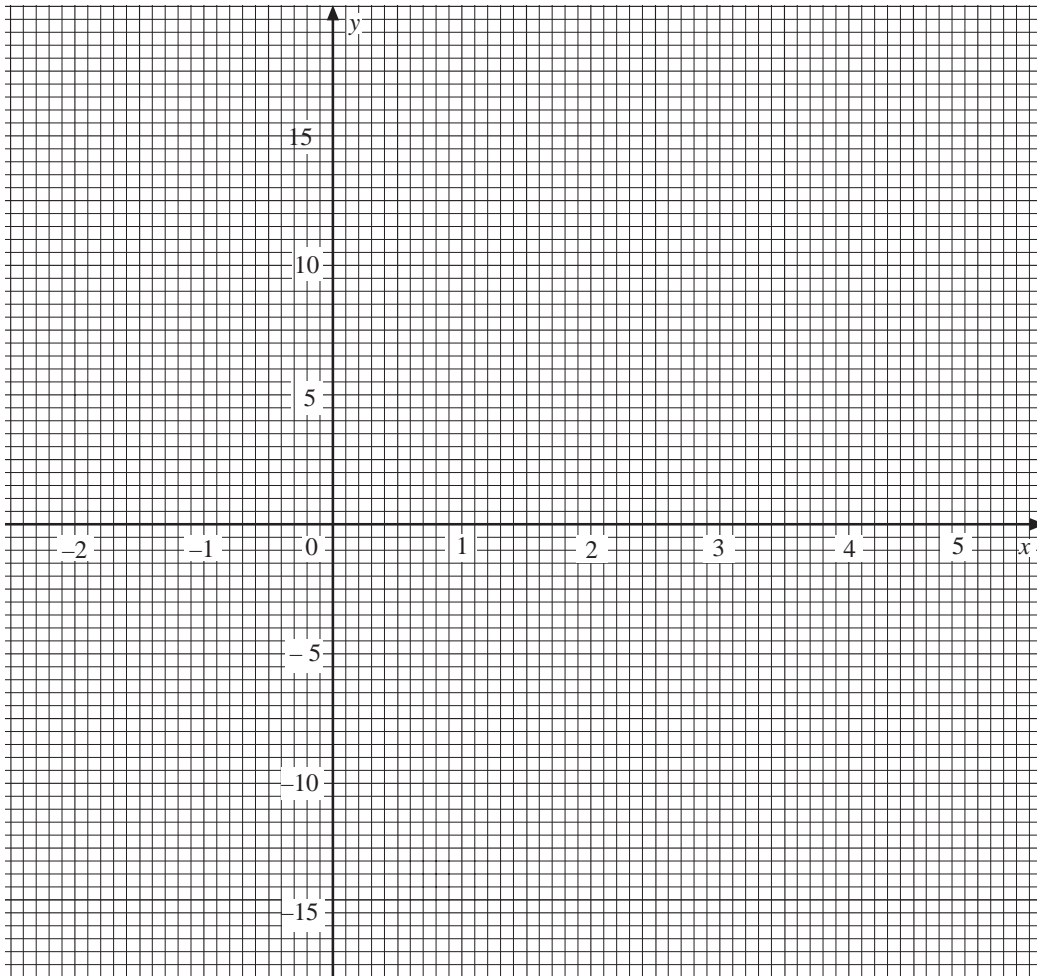
$x$	-2	-1	0	1	2	3
$y$		2				

(b) On a copy of the following grid draw the graph of  $y = x^3 - 4x - 1$ .

(c) By drawing a suitable straight line on the grid, solve the equation

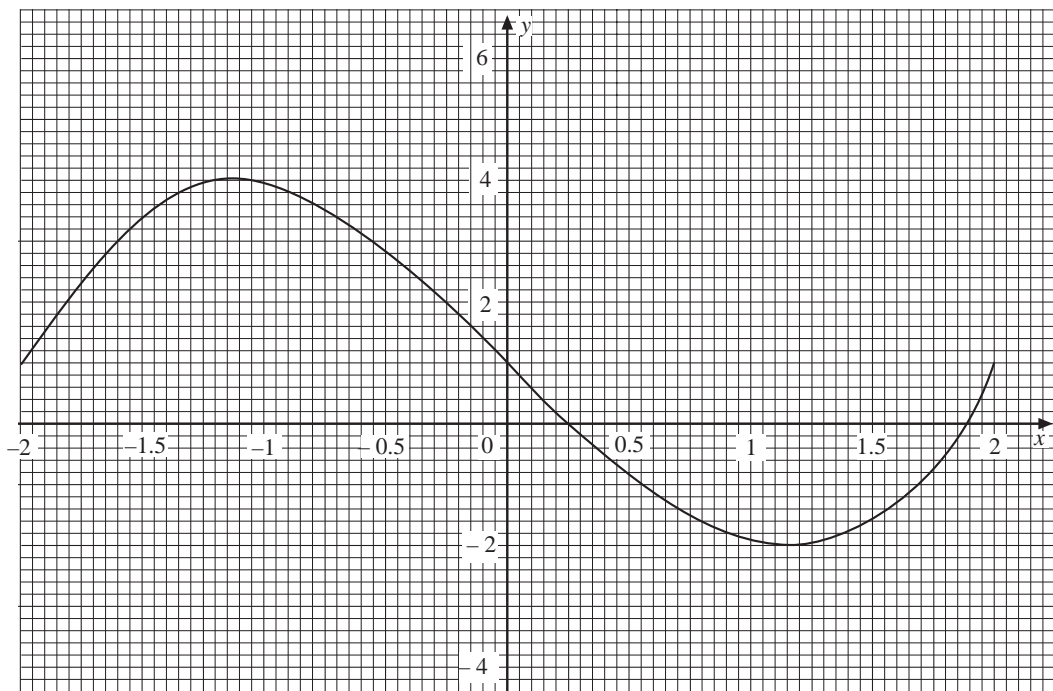
$$x^3 - 4x - 3 = -2$$

(d) Using the method of trial and improvement, or otherwise, solve the equation  $x^3 - 4x - 1 = 30$  correct to one decimal place.



(LON)

4.



The diagram shows part of the graph of  $y = x^3 - 4x + 1$ .

- (a) Use a copy of the graph to find approximate solutions in the range  $-2 < x < 2$  of  $x^3 - 4x + 1 = 0$
- (b) By drawing suitable straight lines on your grid, find approximate solutions in the range  $-2 < x < 2$  of the equations
- (i)  $x^3 - 4x - 1 = 0$
- (ii)  $x^3 - 5x + 3 = 0$

(LON)

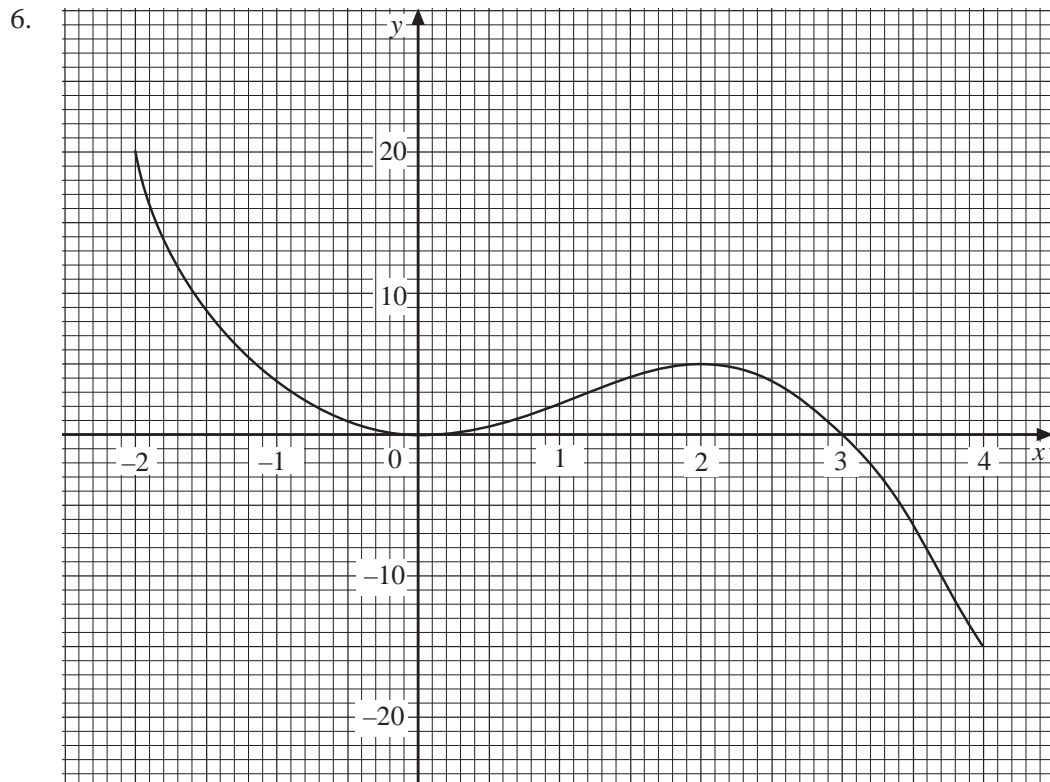
5. (a) Alex is using "trial and improvement" to solve the equation  $x^2 - 3x = 1$ .

First he tries  $x = 3$  and finds the value of  $x^2 - 3x$  is 0. By trying other values of  $x$  find a solution of the equation  $x^2 - 3x = 1$ , correct to one decimal place.

*You must show all your working.*

- (b) (i) Draw the graph of  $y = x^2 - 3x$ , for values of  $x$  from  $-1$  to  $4$ .
- (ii) By drawing suitable line on your graph show that the equation  $x^2 - 3x = 1$  has two solutions.

(SEG)



The diagram shows the graph of  $y = 3x^2 - x^3$  for values of  $x$  from  $-2$  to  $4$ .

- (a) Use a copy of the graph to find three values of  $x$  which satisfy the equation  $3x^2 - x^3 = 2$ .
- (b) The equation  $3x^2 - x^3 = k$  is satisfied by only one value of  $x$  between  $-2$  and  $4$ .

What can be said about the number  $k$ ?

- (c) (i) On a copy of the diagram, draw the reflection of the graph in the  $x$ -axis.
- (ii) Write down the equation of this reflection.

(MEG)

7. (a) Complete the table of values for the graphs of

$$y = x^3 - 2 \quad \text{and}$$

$$y = 3x^2 + 3x - 6$$

$x$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$
$y = x^3 - 2$		$-3$	$-2$	$-1$		$25$	$62$
$y = 3x^2 + 3x - 6$	$0$		$-6$	$0$	$12$	$30$	

- (b) (i) On graph paper draw the graphs of

$$y = x^3 - 2 \quad \text{and} \quad y = 3x^2 + 3x - 6$$

- (ii) Use your graphs to solve the equation  $x^3 - 3x^2 - 3x + 4 = 0$ .

(LON)

8. (a) Draw the graph of

$$y = 20 - 5x \quad \text{and} \quad y = x^3 \quad \text{for} \quad 0 \leq x \leq 3$$

- (b) Use your graph to estimate, correct to one decimal place, the solution of

$$x^3 + 5x - 20 = 0$$

(SEG)

9. A child's toy consists of a set of different sized blocks which are in the shape of cubes. The cost,  $C$  pence, of making each block is made up of two parts:

A fixed cost of 24 pence;

and a cost that is proportional to the cube of the length,  $x$  centimetres, of the block.

When  $x = 2$ ;  $C = 56$ .

- (a) Find the equation connecting  $x$  and  $C$ .
- (b) The cost,  $D$  pence, of decorating each block is given by the equation

$$D = 4 + 3x^2$$

Another child's toy is in the shape of a stick.

The cost,  $D$  pence, of decorating a stick of length,  $x$  centimetres, is given by the equation

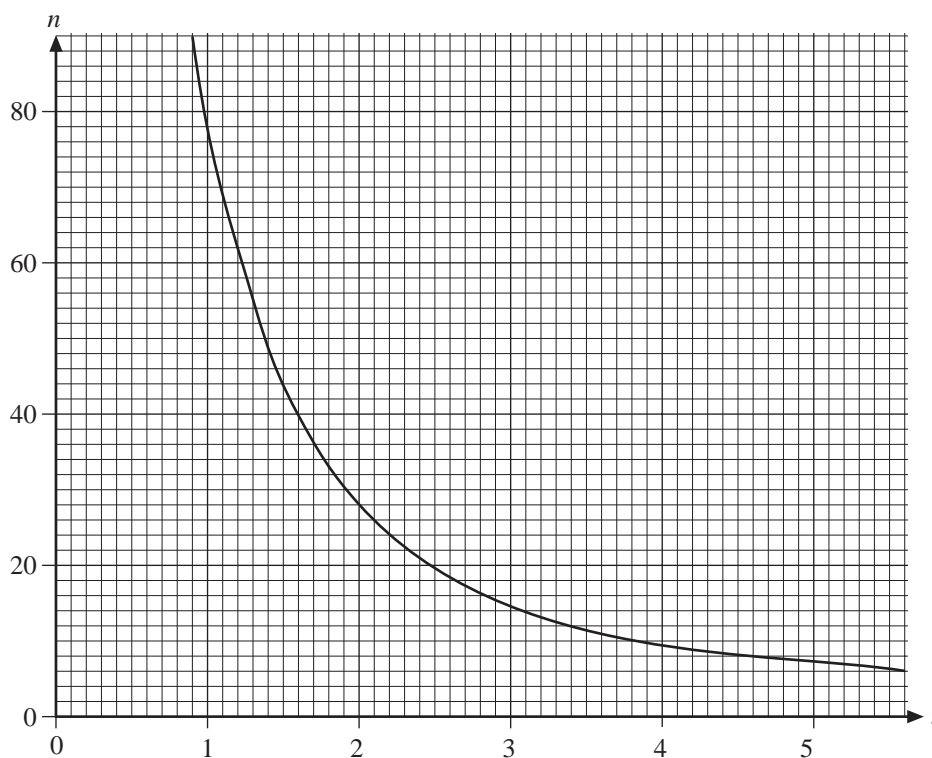
$$D = 15 + 8x$$

Use a graphical method to find the value of  $x$  when the cost of decorating the block and the stick is the same.

(SEG)

10. The graph of  $n = \frac{80}{t^{1.5}}$  is given.

- (a) On a copy of the grid below draw a graph so that the equation  $\frac{80}{t^{1.5}} = 2t^2$  can be solved.



- (b) (i) Use your graph to find the value of  $t$  at the point of intersection of the two curves.

(ii) Write the equation  $\frac{80}{t^{1.5}} = 2t^2$  in the form  $40 = \dots$

Simplify the right hand side as far as possible.

- (c) For the equation  $n = \frac{80}{t^{1.5}}$  estimate the rate of decrease of  $n$  when  $t = 2$ .

- (d) Estimate, by drawing, the gradient of the tangent at the point when  $t = 4$ , on the graph you have drawn.

(SEG)

11. John places a cake in his freezer. The temperature,  $T$  °C, of the cake after  $t$  minutes is given by the formula

$$T = 32(2^{-t}) - 18$$

- (a) Copy and complete the table below.

$t$ (minutes)	0	1	2	3	4
$T$ °C					

- (b) Draw the graph of  $T$  against  $t$ .
- (c) John knows that the cake's temperature is 14 °C when he places it in the freezer. He does not know the formula for its temperature after  $t$  minutes. He estimates that its temperature will fall by 10 °C every minute. On your grid, draw the graph showing how John thinks the temperature will vary during the first three minutes.
- (d) Use your graph to find the time when the estimated temperature is the same as the true temperature of the cake.