

# 10 Equations

## 10.1 Negative Numbers

1. Rewrite each of the following sets of numbers in increasing order.

(a)  $-5, 0, -10, 2, -1, 3$

(b)  $3, -3, 0, 10, -5, -12$

(c)  $-1, 4, -7, -3, 2, 10$

(d)  $-21, 41, 11, -31, -11, 21$

2. Write down all the integers that lie between:

(a)  $-5$  and  $-2$ ,      (b)  $-3$  and  $+3$ ,

(c)  $-7$  and  $0$ ,      (d)  $-25$  and  $-21$ .

3. Insert a "<" sign or a ">" sign between each pair of numbers so that it reads correctly.

(a)  $2 ? 5$       (b)  $-2 ? 4$       (c)  $-5 ? -8$       (d)  $10 ? -10$

## 10.2 Arithmetic with Negative Numbers

1. What is the value of:

(a)  $-7 + 4$       (b)  $-2 - 4$       (c)  $(-3) \times 2$

(d)  $-7 - (-2)$       (e)  $4 \times (-3)$       (f)  $(-1) \times (-4)$

(g)  $15 \div (-5)$       (h)  $(-15) \div 5$       (i)  $(-15) \div (-5)$

(j)  $(-12) \div 4$       (k)  $(-10) \div (-5)$       (l)  $4 \div (-2)$

(m)  $6 \times (-7)$       (n)  $(-8) \times (-4)$       (o)  $8 - (-2)$

(p)  $(-9) \div 3$       (q)  $-10 - (-12)$       (r)  $(-10) \times (-12)$

2. Calculate the value of each of the following expressions (first evaluate the expressions inside the brackets).

(a)  $(-6 - (-2)) \times (-5)$       (b)  $(7 - (-3)) \div (-2)$

(c)  $(-4 + 7) \times (-4 - 7)$       (d)  $(5 - (-2)) \times (-5)$

(e)  $(10 - (-5)) \div (3 - (-2))$       (f)  $(10 + (-10)) \times (4 - 2)$

(g)  $(7 \times (-3)) \times (5 - 2)$       (h)  $((-12) \div 4) \times (15 \div (-3))$

## 10.3 Simplifying Expressions

1. Simplify, as far as possible, each of these expressions:

(a)  $x + 4x$

(b)  $3x + 5x - 2x$

(c)  $x + 4 - 2x + 2$

(d)  $5a - 2 + a + 6$

(e)  $3a - 5b + 6a + 7b$

(f)  $-4a - 5y + 2a + 6y$

(g)  $x + 4 - y + 2x - y$

(h)  $2x + 7y - 4x + 3y + 1$

(i)  $4p + 8q - 4p + 8q$

(j)  $x + y - 10x - 11y$

2. Collect like terms together where possible in each of the following expressions.

(a)  $4x^2 + 2x + x^2 + 5x$

(b)  $3x^2 - 2x + 2x^2 - 2x$

(c)  $5y^2 + 3y - 4y^2 - 4y$

(d)  $x^2 + y^2 - 3x^2 - 3y^2$

(e)  $3x + x^2 - y^2 - 4x + y^2$

(f)  $x^2 - 2xy + y^2 + 4xy$

(g)  $3x^2 + x - 1 + x^2 - 3x + 2$

(h)  $4ab - 7bc + 5ab - 2ac$

3. Expand the following:

(a)  $4(x + 2)$

(b)  $5(3 + 2x)$

(c)  $6(2x + 1)$

(d)  $4(2x - 2)$

(e)  $3(x + y)$

(f)  $7(x - 2y)$

(g)  $x(1 + y)$

(h)  $x(x + 2y)$

(i)  $3(x + 2y - 2)$

(j)  $x(1 + x + y)$

(k)  $2x(1 + y)$

(l)  $3x(x + 2y)$

4. Simplify each of the following expressions by first removing the brackets.

(a)  $2(x + 1) + 4(2 + x)$

(b)  $3(2x - 1) + 2(x + 4)$

(c)  $3(x + y) + 2(2x - y)$

(d)  $5(x - y) - 2(x + y)$

(e)  $4(a + 2b) + 3(2a + b)$

(f)  $3(2x + 3y) - 4(x + 2y)$

## 10.4 Simple Equations

1. Solve each of the following equations:

(a)  $x + 5 = 8$

(b)  $x - 5 = 4$

(c)  $x + 5 = 10$

(d)  $x - 5 = 9$

(e)  $6 + x = 7$

(f)  $3x = 6$

(g)  $6x = 42$

(h)  $7x = 14$

(i)  $12x = 24$

(j)  $\frac{x}{2} = 6$

(k)  $\frac{x}{5} = 5$

(l)  $\frac{x}{2} - 1 = 4$

2. Solve each of these equations:
- |                   |                   |                    |
|-------------------|-------------------|--------------------|
| (a) $x + 4 = 2$   | (b) $5 + x = 3$   | (c) $x - 3 = -7$   |
| (d) $3x = -12$    | (e) $5x = -20$    | (f) $2x + 1 = -3$  |
| (g) $3x - 1 = 14$ | (h) $5x + 2 = -8$ | (i) $2x - 4 = 8$   |
| (j) $4x - 7 = -9$ | (k) $9 - 2x = 8$  | (l) $3x + 7 = -10$ |
3. One number is greater than another by 4, and their sum is 32. Find the two numbers.
4. When a number is doubled and 5 is taken from the result, the answer is 37. What is the number?
5. The sum of two numbers is 120. If the larger number is four times the smaller number, what are the two numbers?
6. Andrew is 5 years older than Tim. If Tim is aged 21, then write down an equation for  $x$ , the age of Andrew. Solve this equation for  $x$ .
7. Morag thought of a number. She doubled this number and added 10 to give the result 52. What number did Morag think of?
8. The sum of three *consecutive* numbers is 120. If  $x$  is the smallest of the three numbers, write down the equation that  $x$  satisfies. Hence, solve for  $x$ .
9. When 42 is added to twice a number, the result is 346. Find the number.
10. A man was 26 years old when his son was born. Now, he is three times as old as his son. How old is the son now?

## 10.5 Solving Equations

1. Solve the following equations.
- |                         |                       |                       |
|-------------------------|-----------------------|-----------------------|
| (a) $2x - 7 = 3$        | (b) $3x - 4 = 8$      | (c) $5x + 2 = 7$      |
| (d) $3x + 9 = 0$        | (e) $15 - 2x = 9$     | (f) $17 + 3x = -3$    |
| (g) $5x = -15 + x$      | (h) $-2x - 7 = -4$    | (i) $5x - 4 = 3x - 1$ |
| (j) $7x - 14 = 18 - 4x$ | (k) $8x - 7 = 5 + 4x$ | (l) $9x + 4 = 3x - 9$ |
2. Solve the following equations.
- |                            |                                       |                            |
|----------------------------|---------------------------------------|----------------------------|
| (a) $\frac{3}{4}x = 15$    | (b) $\frac{2}{5}x - 1 = 4$            | (c) $5 - \frac{x}{4} = 3$  |
| (d) $\frac{x}{3} + 5 = 15$ | (e) $2 + \frac{5}{7}x = 1\frac{1}{4}$ | (f) $\frac{2x + 4}{7} = 3$ |

(g)  $\frac{3x-4}{5} - 7 = 0$       (h)  $\frac{3x+4}{2} = x - 2$

(i)  $\frac{2x-1}{3} = 1 - x$       (j)  $7 + \frac{x-1}{2} = x$

3. Solve the following equations.

(a)  $3(x-4) = 7$

(b)  $9(x-4) = 3$

(c)  $5(2x+3) = 35$

(d)  $8(2+3x) = 4$

(e)  $7(x+4) = 2(x-4)$

(f)  $5(3x+5) = 2(7x-4)$

(g)  $2(5-2x) = 4(2-3x)$

(h)  $2(x+1) = 3(x-5) + 9$

(i)  $\frac{1}{4}(5x+4) = \frac{1}{3}(2x-1)$

(j)  $2[2(x-4) + 3] = 5$

(k)  $2x - [3 + (x-5)] = 6$

(l)  $17(x-3) = 3(7x-15)$

4. When a number  $x$  is multiplied by 5, it gives the same result as when 48 is added to twice the number. Write down an equation for  $x$ , and find its solution.

5. Ahmad is twice as old as Bobby. John is 7 years younger than Ahmad. If the sum of their ages is 38, how old are the three boys?

6. Janet is three times as old as her daughter, Mary. Five years ago Janet was four times as old as Mary. How old is Janet now? How old will Mary be in 7 years' time?

7. Two boys, A and B, are 600 m apart. They walk towards each other at speeds of 35 m per minute and 25 m per minute respectively. After how many minutes will they meet each other?

8. Two men, P and Q, start at the same point and travel in opposite directions by motorcycle. The speed at which P's motorcycle travels is 4 km/h faster than Q's. After 5 hours, they are 580 km apart. Find the speed at which P travels.

9. Solve the equations.

(a)  $4x + 2 = 26$

(b)  $19 + 4y = 9 - y$

10. Mrs. Root gave her maths class this problem:

"When 8 is added to a certain number, the result is 3 times as large as when 2 is subtracted from the number."

She asked the class to find the original number.

Paul solved the problem using the equation

$$x + 8 = 3(x - 2)$$

Solve this equation.

11. Brenda went out walking and running. She travelled 7 km. She walked part of the way at 6 km/hour, and ran the rest of the way at 12 km/hour.

The distance she ran was  $x$  km.

- (a) Write down an expression for the time taken running.

- (b) The time taken walking was  $\frac{(7-x)}{6}$  hours.

The total time spent walking and running was one hour.

- (i) Write down an equation in terms of  $x$ .  
(ii) Find the value of  $x$ .

## 10.6 Trial and Improvement Method

1. Solve, using a trial and improvement method, each of the following equations, giving your answer correct to 1 decimal place.

(a)  $x^3 - 4 = 5$

(b)  $x + \sqrt{x} = 10$

(c)  $x^3 - x = 6$

(d)  $x^3 + x = 4$

2. Use trial and improvement methods to find the solution of each of these equations, giving your answer to 2 decimal places.

(a)  $x^2 + 4x - 3 = 0$

(b)  $x^2 - 3x + 1 = 0$

(c)  $2x^2 + x - 4 = 0$

(d)  $x^2 + 5x + 2 = 0$

3. George has to find a solution to the equation  $x^2 + 2x = 10$ , correct to one decimal place.

First he tries  $x = 3.0$  and finds that the value of  $x^2 + 2x$  is 15.

By trying other values of  $x$  find a solution of the equation  $x^2 + 2x = 10$ , correct to one decimal place. You *must* show all your working.

(SEG)

4. (a) *Without using a calculator*, write down an estimate of the square root of 40. Give your estimate correct to one decimal place.  
(b) Explain how you obtained your estimate to the square root of 40.  
(c) Use a trial and improvement method to find the square root of 40 correct to two decimal places. Show your working clearly.

5.  $x$  is a number such that  $x(x + 1)(x - 1) = 20$ .
- Find the two consecutive whole numbers between which  $x$  must lie.
  - Use the method of trial and improvement to find the solution correct to 3 significant figures.
- (NEAB)
6. Dilip is using trial and improvement to solve equations.
- He finds the solution of a certain equation lies between 2.731 and 2.734.  
Write down an approximation to the solution, correct to as many significant figures as are justified so far.
  - The solution to another equation lies between 4.62 and 4.67.  
Write down an approximation to the solution, correct to as many significant figures as are justified so far.
- (SEG)

## 10.7 Expanding Brackets

1. Multiply out and simplify.
- |                        |                       |                      |
|------------------------|-----------------------|----------------------|
| (a) $x(1 + x)$         | (b) $2(2x + 1)$       | (c) $2x(x - 1)$      |
| (d) $4x(2 + x)$        | (e) $5x(3 - 2x)$      | (f) $x^2(1 + x)$     |
| (g) $(x + 1)(x + 2)$   | (h) $(x + 1)(x - 1)$  | (i) $(x + 2)(x - 1)$ |
| (j) $(x - 3)(x - 2)$   | (k) $(1 + a)(1 + 2a)$ | (l) $(x + y)(x - y)$ |
| (m) $(ax + b)(cx - d)$ | (n) $(x + 1)^2$       |                      |
2. Expand the following:
- |                          |                         |                              |
|--------------------------|-------------------------|------------------------------|
| (a) $6(3x + y)$          | (b) $5z(z - 2y)$        | (c) $\frac{1}{2}(2xy - 4yz)$ |
| (d) $q(p + 2r - 3s)$     | (e) $(p + q)(r + s)$    | (f) $(x + y)(z + 2w)$        |
| (g) $(3a + b)(a + c)$    | (h) $(m + 2n)(2p + 3q)$ | (i) $(a - b)(c + d)$         |
| (j) $(2e - f)(2g - h)$   | (k) $(3p - 4q)(s + t)$  | (l) $(a + 7)(2b + 5)$        |
| (m) $(x + 3)(x + 4)$     | (n) $(a + 5)(a - 3)$    | (o) $(x - 7)(x - 6)$         |
| (p) $(3 + c)(6 - c)$     | (q) $(1 - 3x)(4 + 3y)$  | (r) $(2p + 3)(p + 5)$        |
| (s) $(4x + 5y)(2x + 3y)$ | (t) $(d - 7)(d - 5)$    | (u) $(a + 5)^2$              |
| (v) $(x - 3)^2$          | (w) $(b + 2)^2$         | (x) $(e - 4)^2$              |
| (y) $(2x + 1)^2$         | (z) $(3x - 2)^2$        |                              |

3. Simplify these expressions as far as possible.

(a)  $(3p + 2q)^2$

(b)  $(4m - 3n)^2$

(c)  $(x + 5)(x - 5)$

(d)  $(y + 7)(y - 7)$

(e)  $(5a + 3)(5a - 3)$

(f)  $(6x + 5y)(6x - 5y)$

(g)  $(x - 2)(x + 2)$

(h)  $(x - a)(x + a)$

4. (a) Multiply out and simplify

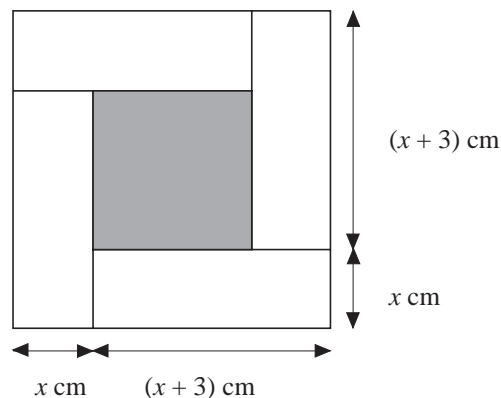
$$(3x - 1)(2x + 3)$$

(b) Show how you could use your answer to (a) to work out  $29 \times 23$ .

5. (a) (i) Multiply out  $4x(x + 3)$ .

(ii) Multiply out and simplify  $(2x + 3)(2x + 3)$

(b) Four identical rectangular tiles are placed around a square tile as shown in the diagram.



Using your answers to (a), or otherwise, find the area of the square tile.

## 10.8 Simultaneous Linear Equations

1. Solve each of the following pairs of simultaneous equations:

(a)  $x + y = 14$   
 $x - y = 4$

(b)  $x - y = -1$   
 $2x - y = 0$

(c)  $3x - y = 9$   
 $4x - y = -14$

(d)  $y - x = -1$   
 $3x - y = 5$

(e)  $5x + 4y = 4$   
 $3x + 4y = 8$

(f)  $3x + 5y = 5$   
 $3x + 9y = -3$

(g)  $3x + 2y = 0$   
 $-x + y = 5$

(h)  $3x - y = -2$   
 $x - 3y = 10$

(i)  $3x - 2y = 7$   
 $4x + y = 13$

(j)  $3a - b = 9$   
 $2a + 2b = 14$

(k)  $3x - 8y = 1$   
 $6x - 7y = 25$

(l)  $2m + 5n = 24$   
 $4m + 3n = 20$

$$\begin{array}{ll} \text{(m)} & 2x + 7y = 17 \\ & 5x + 3y = -1 \end{array} \quad \begin{array}{ll} \text{(n)} & 5u - 2v = 9 \\ & 7u - 5v = 28 \end{array}$$

2. Solve the simultaneous equations

$$\begin{array}{l} x + y = 4 \\ 15x + 25y = 76 \end{array}$$

3. Solve the following equations:

$$\begin{array}{lll} \text{(a)} & x + y = 7 & \text{(b)} \quad x - 3y = 7 & \text{(c)} \quad 3x + y = 13 \\ & x - y = 3 & & 5x - y = 35 \\ \text{(d)} & 3x + 3y = 15 & \text{(e)} \quad 3x + 2y + 7 = 0 & \text{(f)} \quad 3x + y = 17 \\ & 3x - 5y = -41 & 5x - 2y + 1 = 0 & 3x - y = 19 \\ \text{(g)} & 3x + 2y = 8 & \text{(h)} \quad 2x = 5 - y & \text{(i)} \quad 7x - y + 23 = 0 \\ & 2y - 5x = 8 & 3y = 1 - 2x & x + 2y - 1 = 0 \\ \text{(j)} & 3x - 5y = 31 & \text{(k)} \quad x + 3y = 7 & \text{(l)} \quad 3x + 7y = -15 \\ & x + 3y = 1 & y - 4x = 11 & x - 3y = 11 \end{array}$$

4. Suvinder spends £26 on 100 postage stamps. If  $x$  of them are 20p stamps and the remaining  $y$  are 35p stamps, write down two equations in  $x$  and  $y$  and solve them.
5. Harry pays £8.50 for 5 kg of flour and 3 kg of sugar. Sarah pays £13.20 for 8 kg of flour and 4 kg of sugar. If the cost of flour is £ $x$  per kg and the cost of sugar is £ $y$  per kg, write down two equations in  $x$  and  $y$  and solve them.
6. John and David have £14.00 altogether. If John's money is doubled and David's tripled, they will have £34.00 altogether. How much does each boy have?
7. A retailer can buy either two television sets and three video-recorders for £3750, or four television sets and one video-recorder for £4250. What is the cost of a television set? What is the cost of a video-recorder?
8. A toothbrush and a tube of toothpaste cost £4.15; the toothbrush costs 25p less than the tube of toothpaste. Find the cost of each item.
9. A grocer wants to mix a type of spice which costs £22 per kilogram with another type which costs £12 per kilogram, to obtain 20 kilograms of mixture which will cost £15 per kilogram. What quantity of each spice must the grocer take?
10. Mrs Rogers bought 3 blouses and 2 scarves. She paid £26.  
Mrs Summers bought 4 blouses and 1 scarf. She paid £28.  
The cost of a blouse was  $x$  pounds.  
The cost of a scarf was  $y$  pounds.
- (a) Use the information to write down two equations in  $x$  and  $y$ .
- (b) Solve these equations to find the cost of one blouse.