

6 Number System

6.5 Estimating Answers

1. Express each of the following correct to 3 significant figures:

- (a) 96.63 (b) 316.5 (c) 1.940 5
 (d) 0.004 681 (e) 50.92 (f) 0.000 604 8
 (g) 0.040 713 (h) 5.984 (i) 26.98

2. Write each of the following correct to the number of significant figures (s.f.) indicated.

- (a) 308.637 (4 s.f.) (b) 0.099 8 (1 s.f.)
 (c) 420.65 (3 s.f.) (d) 0.004 307 (2 s.f.)

3. Write 13.004 72 correct to

- (a) 5 s.f. (b) 4 s.f. (c) 2 s.f.

4. Nigel, Ali and Sue were given ten calculations to do.

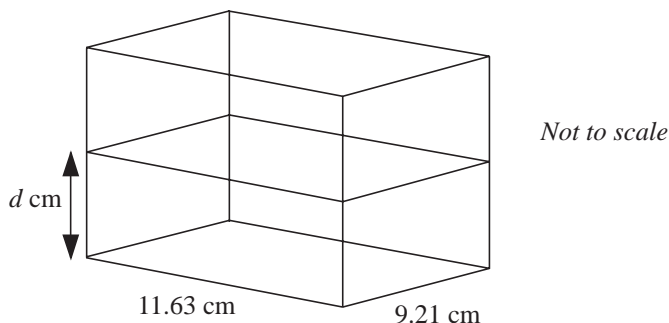
The following table shows their answers. For each calculation, only one of the three obtained the correct answer. By working out an estimate for each question, decide who was correct in each calculation.

	Question	Nigel's answer	Ali's answer	Sue's answer
(a)	1.02×2.9	12.928	2.958	6.438
(b)	0.99×46.7	46.233	32.136	25.633
(c)	4.8×10.4	26.32	49.92	89.42
(d)	$33.264 \div 13.2$	8.42	12.62	2.52
(e)	$35.244 \div 8.01$	4.4	1.4	12.4
(f)	7.1×7.1	50.41	5.41	36.01
(g)	$27.028 \div 4.66$	2.68	11.08	5.8
(h)	$76.16 \div 47.6$	1.6	8.6	12.2
(i)	12.7×8.5	50.85	107.95	204.75
(j)	$8.342 \div 0.97$	2.7	16.16	8.6

5. Without finding the exact answer:
- (a) Which of the following is nearest in value to $6.96 + 7.21 + 7.1 + 6.82$?
21.7, 28.09, 90.73 or 21.826
- (b) Which of the following is nearest in value to $3.14 \times 300 - 34.3$?
57, 87, 870 or 570
- (c) Which of the following is nearest in value to $9 \times \sqrt{7} + 10 \times \sqrt{14}$?
148, 67, 14.8 or 6.7
6. Estimate, correct to 1 significant figure, the value of 2.01×29.2 .
7. Express each number correct to 1 significant figure and work out an *estimate* to

$$\frac{19.7 \times 9.75}{12.4}$$
 Use your calculator to evaluate $\frac{19.7 \times 9.75}{12.4}$ correct to 2 significant figures.
8. (a) Bottles of mineral water cost 39 p each. Estimate the cost of 142 bottles.
Show how you obtained your estimate.
- (b) *Without* using a calculator, work out the exact cost of 142 bottles of mineral water at 39 p each.
(MEG)
9. Charlie has to work out $5.2 \times 3.9 \times 2.1$. He uses a calculator and gets 425.88 for his answer.
Saeeda works out an approximate answer for the question. She knows that Charlie's answer must be wrong.
- (a) (i) Write down approximate values for 5.2, 3.9 and 2.1.
 (ii) Use these approximations to find a rough answer to Charlie's calculation.
- (b) What is the mistake in Charlie's answer?
(SEG)

10. The rectangular glass tank shown in the diagram contains 1 litre (1000 cm^3) of water.



Sanjay wanted to find the depth ($d \text{ cm}$) of the water.

He multiplied 11.63 by 9.21 on his calculator and wrote down the answer.

He then divided 1000 by this answer.

- Explain how you could use your calculator to find the depth without writing down the answer to 11.63×9.21 .
- Work out the depth of the water, and write down all the figures on your calculator display.

(MEG)

6.6 Using Brackets and Memory on a Calculator

1. Use a calculator to evaluate each of the following:

- | | |
|---|--------------------------------|
| (a) $480 - 96 + 15$ | (b) $4059 \div 1353 \times 11$ |
| (c) $533 + 118 - 227$ | (d) $(251 + 696) \times 15$ |
| (e) $(1283 - 694) \div (12 + 19)$ | |
| (f) $241 \times (270 - 121) \div (129 + 112)$ | |
| (g) $77175 \div [(17 + 18) \times (78 - 57)]$ | |
| (h) $[33350 \div (290 \times 115) + 798] \div (869 - 70)$ | |

2. For each of the following expressions,

- evaluate, giving your answer correct to 2 decimal places;
- express each number correct to the nearest whole number and give an estimate to check your calculations.

- | | |
|--|--|
| (i) $4.6 + 3.9 \times 2.2$ | (ii) $(4.6 + 3.9) \times 2.2$ |
| (iii) $3.3 \times 25 \times 0.6125$ | (iv) $4.2 \times 0.8 - 1.6 \times 1.2$ |
| (v) $\frac{1.1 \times 12}{1.8^2 \times 3.1}$ | (vi) $(\sqrt{9.4} + 3.6^2) \div 1.9$ |

3. (a) Use your calculator to work out the value of

$$\frac{6.08 \times (9.72)^2}{581 + 237}$$

Write down the *full* calculator display.

- (b) (i) Write down a calculation that could be done mentally to check the answer to part (a) using numbers rounded to one significant figure.
 (ii) Write down the answer to your calculation in part (b) (i).

(MEG)

4. Work out:

(a) 0.6×2.5 (b) $\frac{78 \times 14}{112 - 86}$ (c) $7^2 - 5^2$.

(MEG)

5. Gabriel buys a packet of 18 biscuits. The packet weighs 285 g.

- (a) Gabriel wants to calculate the weight of one of these biscuits. He presses the following buttons on his calculator.

$$\textcircled{1} \textcircled{8} \textcircled{\div} \textcircled{2} \textcircled{8} \textcircled{5} \textcircled{=}$$

Explain what is wrong with his calculation.

- (b) Calculate the weight of one of these biscuits. Give your answer to the nearest gram.
 (c) Gabriel checks his answer without using a calculator. Show how you can use approximation to check that his answer is of the right order. *You must show all your working.*

(SEG)

6.7 Upper and Lower Bounds

1. Write down the upper and lower bounds for each of the following measurements.

- (a) 56 g (b) 43.0 litres (c) 2.35 metres
 (d) 5.6 km (e) 17.8 metres (f) 8.54 kg
 (g) 17.2 seconds (h) 0.5 mm (i) 1.9 cm

2. Find the upper and lower bounds for each of the calculations shown below, assuming the dimensions given are subject to rounding errors. (Give a *sensible* answer to each calculation.)

- (a) The perimeter of a rectangle 65 cm by 84 cm.
 (b) The area of a rectangle 65 cm by 84 cm.
 (c) The perimeter of an octagon of side 42 mm.
 (d) The volume of a cube of edge length 96 mm.
 (e) The total weight of 54 objects, each weighing 2.62 kg.

3. (a) Angela measures the lengths of some sticks to the nearest centimetre. She arranges them in groups.
- The length of the sticks in the shortest group is 14 cm, to the nearest centimetre.
- (i) What is the smallest possible length for a stick in this group?
- (ii) What is the smallest possible length for a stick which is **not** in this group?
- (b) Angela measures the lengths of some other sticks. She records the length of one of these sticks as 52.2 cm, to the nearest tenth of a centimetre.
- What is the smallest possible length of this stick?
4. Sections of a railway line are measured to the nearest metre as either 200 m or 80 m.
- What are the bounds on the total length of 15 sections, consisting of eight 200 m sections and seven 80 m sections?
5. The area of a rectangle is 54.4 square centimetres, correct to 1 decimal place. The length of this rectangle is 8.3 centimetres, correct to 1 decimal place.
- (a) From this information, write down
- (i) the largest value (ii) the smallest value
- that the length of the rectangle could have.
- (b) Use your answers in (a) to calculate the largest possible width of the rectangle.
- (NEAB)
6. The formula $S = \frac{F}{A}$ is used in engineering.
- $F = 819$, correct to 3 significant figures
- $A = 2.93$, correct to 3 significant figures.
- (a) For the value of F , write down
- (i) the upper bound (ii) the lower bound.
- (b) For the value of A , write down
- (i) the upper bound (ii) the lower bound.
- (c) Calculate
- (i) the upper bound (ii) the lower bound
- for the value of S for these values of F and A . Write down *all* the figures on your calculator display.
- (d) Write down this value of S correct to an appropriate number of significant figures.
- (LON)

6.8 Number System

1. In the following list, which are irrational numbers?

$$0.45, \frac{1}{4}, \sqrt{13}, \sqrt{16}, \pi, 0.\dot{6}, \sqrt[3]{27}, \sqrt[3]{36}$$

2. Express the following fractions as recurring decimals.

$$(a) \frac{2}{3} \quad (b) \frac{5}{6} \quad (c) \frac{5}{11} \quad (d) \frac{29}{33}$$

$$(e) \frac{17}{18} \quad (f) \frac{5}{22} \quad (g) \frac{11}{36} \quad (h) \frac{13}{24}$$

$$(i) \frac{13}{27} \quad (j) \frac{4}{7}$$

3. Express each of the following as a decimal and indicate whether it is recurring or non-recurring.

$$(a) \frac{4}{9} \quad (b) \frac{13}{20} \quad (c) \frac{1}{6} \quad (d) \frac{7}{3}$$

$$(e) \frac{9}{8} \quad (f) \frac{13}{18} \quad (g) \frac{11}{15} \quad (h) \frac{61}{90}$$

$$(i) \frac{7}{11} \quad (j) \frac{59}{99}$$

4. (a) Which of the following numbers are rational?

$$(i) 1 - \sqrt{2} \quad (ii) \pi^2 \quad (iii) 3^0 - 3^{-1} - 3^{-2}$$

- (b) When p and q are two different irrational numbers, $p \times q$ can be rational. Write down one example to show this.

- (c) Write down a fraction which is equal to the recurring decimal 0.03636...
(SEG)

5. (a) Write down a rational number between 1.2 and 1.25.

- (b) Write down an irrational number between 1.2 and 1.25.
(SEG)

6. State whether the following numbers are rational or irrational.

$$(a) -\frac{2}{3} \quad (b) -\sqrt{144} \quad (c) \frac{\pi^2}{8}$$

$$(d) \sqrt{9} + \sqrt{99} \quad (e) \sqrt{100} + \sqrt[3]{1000} \quad (f) \frac{\sqrt[3]{64}}{\sqrt[3]{-8}}$$

If rational, state the value, expressed as simply as possible.

7. (a) The recurring decimal $0.027\ 027\ 027\ \dots$ is a rational number.
The following method is used to convert it to a fraction.
Complete the working.
- (i) Let $x = 0.027\ 027\ 027\ \dots$ (A)
then $1000x = ?$ (B)
- (ii) Take (A) from (B) to give
 $999x = ?$
- (iii) Hence find x as a fraction in its lowest terms.
- (b) Using a similar method, or otherwise, find the fraction equivalent to the recurring decimal
 $0.128\ 712\ 871\ 287\ \dots$
Give your answer in its lowest term. (SEG)
8. Find the fraction equivalent of
(a) $0.428\ 571\ 428\ \dots$ (b) $0.307\ 692\ 30\ \dots$ (c) $0.013\ 698\ 530\ 13\ \dots$

6.9 Surds

1. If $a = 1 + \sqrt{2}$ and $b = 1 - \sqrt{2}$, state whether the following numbers are rational or irrational.
- (a) a^2 (b) $a + b$ (c) ab
(d) $a - b$ (e) b^2 (f) $(a + b)^2$
(g) $(a - b)^2$ (h) $(a - b)(a + b)$ (i) a^2b^2
2. Simplify the expression $\sqrt{12} \times \sqrt{6}$, leaving your answer in surd form. (SEG)
3. (a) What could be added to $2 + \sqrt{3}$ to give an answer which is rational?
(b) State whether the following are rational or irrational.
(i) $\sqrt{3}$ (ii) π (iii) $\frac{22}{7}$ (iv) $1.333\ 3\ \dots$ (NEAB)
4. Using examples or counter examples, decide whether the following statements are true or false.
- (a) When you add two irrational numbers, your answer is still irrational.
(b) When you multiply two irrational numbers, your answer is still irrational.
(c) When you multiply an irrational number by a rational number, your answer is still irrational.