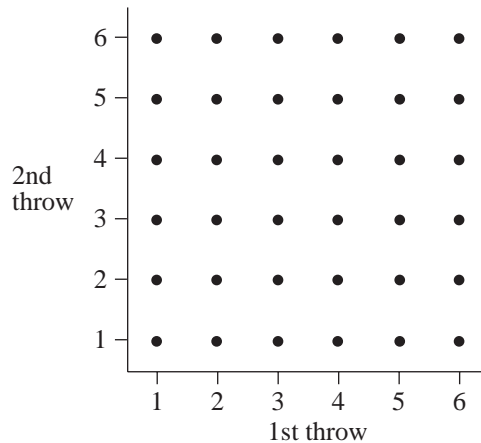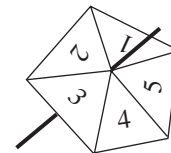# 5 Probability

5.3 Outcome of Two Events

1. A coin is tossed, and a die is thrown. List all the possible outcomes.

2. A die is thrown twice. Copy the diagram below which shows all the possible outcomes.

On your diagram, show outcomes which have

(a)   the same number on both throws,   (b)   a total score of 8.

3. When this spinner is used, the scores 1, 2, 3, 4 and 5 are equally likely.

    (a)   For one spin,

          (i)   what is the probability of scoring a 2,

          (ii) what is the probability of *not* scoring a 2?

    (b)   When playing a game the spinner is spun twice and the scores are added to give a total.
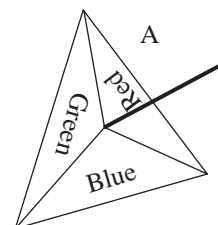
          Write down all the different ways of getting a total of 7.

(SEG)

4. The diagram shows a spinner, labelled A.
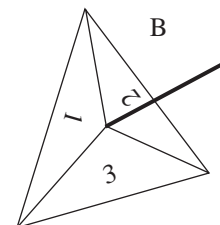   The result shown is Blue.

    Spinner A is a fair spinner.

    (a)   What is the probability of *not* getting Green with spinner A?

    The diagram shows another spinner, labelled B.

    The result shown is 3.

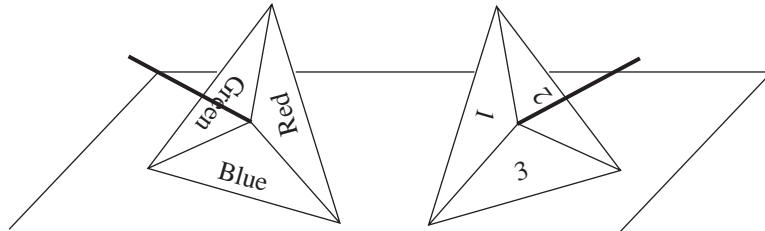   Spinner B is weighted (biased).

60

The probability of getting a 3 is 0.2 and the probability of getting a 1 is 0.1.

(b) What is the probability of getting a 2 with spinner B?

A game is played with the two spinners. They are spun at the same time. The combined result shown in the diagram is Blue 3.

(c) Write down the total number of different possible combined results.

(LON)

5. A coin is tossed 4 times. List all the possible outcomes.

5.4 Finding Probabilities Using Relative Frequency

1. Last year it rained on 150 days out of 365.

    Estimate the probability of it raining on any one day next year.

    How could your estimate be improved?

2. Throw a die 120 times. How many times would you expect to obtain the number 6?

    In an experiment, the following frequencies were obtained.

Number	Frequency
1	31
2	15
3	14
4	16
5	15
6	29

    Do you think that the die is fair? If not, give an explanation why not and estimate what you think are the probabilities of obtaining each number.

3. There are 44 students in a group. Each student plays either hockey or tennis but not both.

	Hockey	Tennis	Total
Girls	8		20
Boys	18		24
Total			44

(a) Complete the table.

(b) A student is chosen at random from the whole group.
     Calculate the probability that this student is a girl.

(c) A girl is chosen at random. Calculate the probability that she plays hockey.

(SEG)

4. John recorded the results of his football team's last 24 matches.

W	W	D	L	W	L	W	D		Key:	W	Win
D	L	L	W	W	W	L	L			D	Draw
D	W	L	W	W	L	W	L			L	Lose

(a) Organise and display this information in a table.

(b) Janet told John that, since there are three possible results of any match, the probability that the next match would be drawn was $\frac{1}{3}$.

    (i) Explain why Janet's argument is wrong.

    (ii) What might John suggest for the probability of a draw, based on the past performance of his team?

(c) Julia estimates that the probability that her hockey team will win their next match is 0.6 and that the probability they will lose is 0.3

    What is the probability that her team will draw?

(MEG)

5. The number of serious accidents on a stretch of motorway in each month of one year are given below.

January	16	July	7
February	12	August	8
March	9	September	6
April	10	October	10
May	6	November	9
June	5	December	12

(a) Estimate the average number of accidents per month over the whole year.

(b)   Estimate the probability of an accident happening on any particular day.
Would your estimate change if you know that the particular day is in
January?

5.5   Determining Probabilities

1.   In a raffle 200 tickets are sold. Peter buys 40 tickets. What is the probability that
he wins first prize? Give your answer as a fraction in its simplest form.

(SEG)

2.   A box contains only blue pencils and red pencils.

6 of the pencils are blue and 5 are red.

A pencil is taken at random from the box.

Write down the probability that

(a)   a blue pencil will be taken,

(b)   a blue pencil will *not* be taken.

(LON)

3.   A bag contains 8 marbles of which 2 are green, 3 are red and the rest yellow.
A marble is taken out at random.

Find the probability that the marble is

(a)   green,   (b)   not yellow.

4.   In an assortment of 36 calculators, 7 have defective switches, 12 have scratched
screens and no calculator has both defects. A calculator is chosen at random for
inspection.

Find the probability that

(a)   it has a defective switch,   (b)   it has no defects.

5.   In a raffle, a winning ticket is to be drawn from 200 tickets numbered 1 to 200.
Yusof holds 1 ticket, Yanling holds 9 tickets and Sam holds 4 tickets. What is the
probability of each of them winning the prize?

6.   Each letter of the word 'PERSPECTIVE' is written on a separate card. The 11
cards are placed face downwards. A card is drawn at random.

What is the probability of picking a card with

(a)   the letter C,   (b)   the letter P,

(c)   a vowel,   (d)   a consonant?

7. One hundred raffle tickets, numbered from 1 to 100 are placed in a drum.

    A ticket is taken from the drum at random.

    (a) What is the probability that the number on the ticket is a multiple of 5?

    (b) What is the probability that the number on the ticket is a square number?

    (SEG)

8. Zaheda conducted a probability experiment using a packet of 20 sweets.
    She counted the number of sweets of each colour.

    Her results are shown in the table.

Red	Green	Orange
12	3	5

    Zaheda is going to take one sweet at random from the packet.

    Write down the probability

    (a) that Zaheda will take a green sweet from the packet,

    (b) that the sweet Zaheda takes will *not* be red.

    (LON)

9. The spinner shown is biased.

    The probabilities of getting a particular
    colour are shown in the table below.

    (a) Complete the table to show the probability of getting GREEN.

Colour	RED	YELLOW	BLUE	GREEN
Probability	0.4	0.1	0.3	

    (b) The spinner is spun once.
         What is the probability of getting either RED or BLUE?

    (c) The spinner is spun 50 times.
         Approximately how many times would you expect to get RED?

    (NEAB)

10. A bag contains 50 discs numbered 1 to 50. A disc is selected at random.
    Find the probability that the number on the disc

    (a) is an even number,   (b) is an odd number,

    (c) has the digit 1.

5.6 Probability of Two Events

1. A fair dice is thrown twice.

    (a) What is the probability of obtaining *two* sixes?

    (b) What is the probability of obtaining exactly *one* six?

2. A coin is biased so that the probability that it lands showing heads is $\frac{2}{3}$. The coin is tossed three times. Find the probability that

    (a) no heads are obtained,

    (b) more heads than tails are obtained.

3. If a coin and a die are tossed together, calculate

    (a) the probability of getting a tail with the coin and an even number with the die,

    (b) the probability of a head with the coin and a number less than three on the die,

    (c) the probability of a head with the coin and a multiple of 3 on the die.

4. A box contains 5 red, 3 yellow and 2 blue discs. Two discs are drawn at random from the box one after another.

    (a) What is the probability that the first disc drawn will be red?

    (b) If the first disc drawn is blue and it is not replaced, what is the probability of drawing a yellow disc on the second draw?

5. Consider the experiment of rolling two dice and noting the two values uppermost. The score is the sum of these two numbers.

    Complete the table of outcomes, as shown below.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4				
3						
4						
5						
6						

    From your table, deduce the probability that the score:

    (a) equals 12,   (b) is less than 12,

    (c) equals 7,   (d) is less than 7.

    Remember that each of the 36 entries in the table is equally likely.

6.   Two bags contain 9 marbles each. In each bag, there are 4 red marbles, 3 white marbles and 2 green marbles.

    (a)   One marble is drawn from the first bag. Find the probability that it is white.

    (b)   One marble is drawn from the second bag. Find the probability that it is either red or green.

These marbles are then returned to their original bags.

    (c)   One marble is drawn from each bag. Calculate the probability that the two marbles are

       (i)   red,   (ii)   of different colours.

7.   When throwing a dice, the possible outcomes are 1, 2, 3, 4, 5 or 6.

A particular dice is biased so that the probability of throwing a 6 is 0.25.

    (a)   What is the probability of *not* throwing a 6?

    (b)   The outcomes 1, 2, 3, 4 and 5 have the same probability as each other.

        What is the probability of throwing a 4?

    (c)   The dice is thrown twice.

       (i)   How many ways are there of reaching a total score of 10?

       (ii)   What is the probability that the total score is 12?

(MEG)

5.7 Use of Tree Diagrams

1.   A fair coin is tossed three times. By drawing a tree diagram, determine the probability of obtaining

    (a)   exactly two heads,   (b)   at least two heads.

2.   George passes three sets of traffic lights on his way to work.

The lights work independently of each other.

The probability that he has to stop at any set of traffic lights is 0.35.

What is the probability that George stops at two or three sets of traffic lights?

(SEG)

3.   The faces of a die are marked with the numbers 2, 2, 4, 4, 6, 6. If the die is rolled twice what is the probability of getting

    (a)   a 4 each time,

    (b)   either a 2 or a 6 each time, or a 2 and a 6?

If the die is rolled three times, what is the probability of getting

    (c)   a 2 each time,

    (d)   either a 4 or a 6 each time, or a combination of 4s and 6s?

4.   There are two spinners, one marked into equal sections numbered 1, 2, 3, 4, 5 and the second spinner marked into equal sections A, B, C.

Calculate the probability of getting

(a)   a 2 and a B,   (b)   a 5 and an A,

(c)   an even number and an A,   (d)   an odd number and either B or C.

5.   Rob has a bag containing 3 blue balls, 4 red balls and 1 green ball.

Sarah has a bag containing 2 blue balls and 3 red balls.

The balls are identical except for colour.

Rob chooses a ball at random from his bag and Sarah chooses a ball at random from her bag.

(a)   Draw a tree diagram and write the probability of each of the events on each of the branches of the tree diagram.

(b)   Calculate the probability that both Rob and Sarah will choose a blue ball.

(c)   Calculate the probability that the ball chosen by Rob will be a different colour from the ball chosen by Sarah,
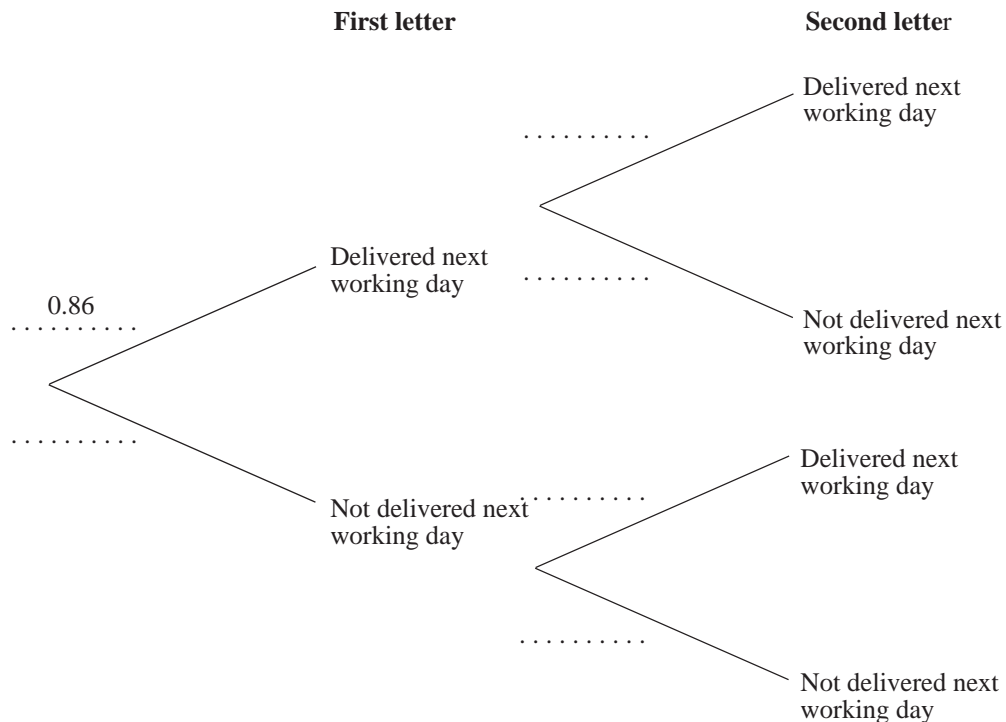
(MEG)

6.   A letter has a first class stamp on it.

The probability that it will be delivered on the next working day is 0.86.

(a)   What is the probability that the letter will *not* be delivered on the next working day?

Sam posts 2 letters with first class stamps.

(b)   Copy and complete the tree diagram.

Write all the missing probabilities on the appropriate branches.

First letter   **Second lette**r

. . . .   Delivered next working day

Delivered next working day   . . . .   Not delivered next working day

0.86
. . . .

. . . .

Not delivered next working day   . . . .   Delivered next working day

. . . .   Not delivered next working day

(c)   Calculate the probability that both letters will be delivered on the next working day.

<div align="right">*(LON)*</div>

5.8 Multiplication for Independent Events

1.   A die is thrown and a coin is tossed. What is the probability of obtaining an even number on the die and a Head on the coin?

2.   Three dice are thrown and their scores are added.

   What is the probability of scoring in total

   (a)   18   (b)   17   (c)   16?

3.   A day which is fine has probability $\frac{3}{4}$ of being followed by another fine day.

   A day which is wet has a probability $\frac{2}{3}$ of being followed by another wet day.

   Given that days are classified either fine or wet, and that June 6th is fine, set out a tree diagram for June 7th, 8th and 9th.

   Calculate the probability that at least two of the three days are fine.

4.   On a stretch of main road there are 4 independent sets of traffic lights, each phased for 120 seconds red, 60 seconds green.

   What is the probability that a motorist arriving at random will have to stop at least once?

5.   Four balls are drawn at random, one after the other and without replacement, from a bag containing

   <div align="center">5 Red, 4 White , 8 Blue and 3 Purple balls.</div>

   Find the probability that you obtain one ball of each colour.

6.   A fair dice is thrown three times.

   (a)   What is the probability of throwing 3 sixes?

   (b)   What is the probability of throwing a six on the first throw, a six on the second throw but not a six on the third throw?

   (c)   What is the probability of throwing exactly two sixes in the three throws?

   (d)   What is the probability of throwing at least two sixes in the three throws?
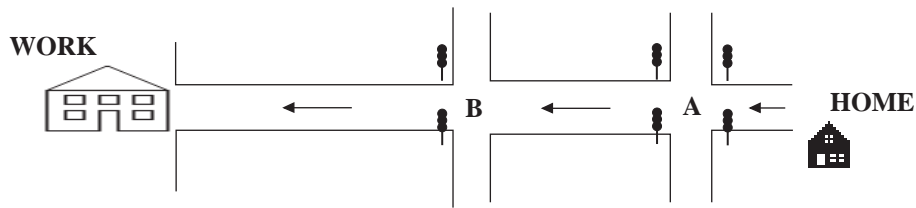
<div align="right">*(SEG)*</div>

7.   The diagrams show two fair spinners. Both spinners are spun and the scores are added together.

   What is the probability that the sum of the scores is at least 5?
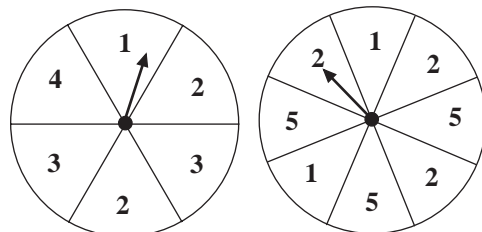
8. Mrs Collins drives to work. On her way to work she has to cross two sets of traffic lights marked A and B in the diagram. The probability of having to stop at the traffic lights is shown in the table.

Traffic	Probability of having to stop
A	0.3
B	0.6

On Monday Mrs Collins drives to work.

(a) What is the probability that she will *not* have to stop at traffic lights A?

(b) What is the probability that she will *not* have to stop at either set of traffic lights?

(c) What is the probability that she will have to stop at only *one* set of traffic lights?

(SEG)

9. A car driver has 4 keys, only one of which will open the car door. Given that the keys are otherwise indistinguishable, find the probability (before he starts trying them) that the door will open on the first, second, third and fourth attempts.

(a) Consider two cases where

(i) he discards each key which fails to open the door,

(ii) he returns each key to the collection before choosing the next one at random.

(b) Consider the cumulative probabilities with each strategy. i.e. the probability that he will have succeeded by the first, second, third and fourth attempts.

10. A company secretary carries out a survey of incoming post to compare the delivery times of 1st and 2nd class letters. His results are shown below.

Days to deliver	1	2	3	4
1st class letter	92%	7%	1%	0%
2nd class letter	5%	55%	34%	6%

Use the information in the table to find the probability of

(a) a 2nd class letter taking more than two days to deliver,

(b) two 1st class letters taking two days to deliver,

(c) a 1st and a 2nd class letter taking the same number of days to deliver. *(SEG)*

11. At the village fete, Susan helps on a stall where radios can be won. She makes the following poster explaining the rules.

WIN A RADIO

Throw 3 dice

Score a total of 18 and the radio is yours!

(a) The first person to try their luck was told that they must throw a six with each dice to win. Calculate the probability of this person winning the radio.

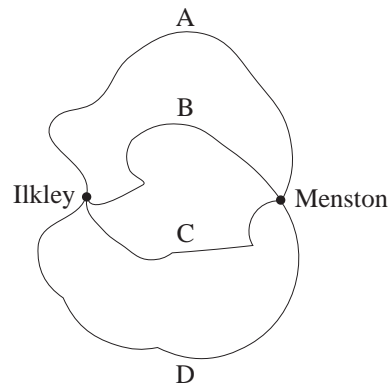(b) During the day 648 people tried to win a radio. How many radios would you expect to be won during the day of the fete?

(SEG)

12. Helen lives in Ilkley.

She cycles to work in Menston.

Peter lives in Menston.

He cycles to work in Ilkley.

Ilkley and Menston are connected by four roads, A, B, C and D.

(a) Make a list of all the possible combinations of roads which they can take to go to work.

Write them in pairs with the road Helen takes written down first.

For example, A, C means that Helen goes along road A, and Peter goes along road C.

(b) Each day, Helen chooses the road she takes to go to work at random. So too does Peter. All four roads are equally likely to be chosen.

Calculate the probability that on any given day both of them will go to work on the same road.

(NEAB)

13.

START→	1	2	3	
	8	7		5
	9		11	12
	16	15	14	13

'SWEET SIXTEEN' is a game for any number of players. To play the game, players take it in turns to throw a fair die and then move their counter the number of places shown uppermost on the die. If a player lands on one of the shaded squares the player must start again. The first player to *land on a square 16* is the winner. If a player would move past square 16 on a throw, the player is not allowed to move and misses that turn.

(a)   What is the probability that a player lands on a shaded square on the first throw?

(b)   A player moves to square 3 on the first throw. What is the probability that the player lands on a shaded square on the second throw?

(c)   (i)   A player is on square 12 after three turns. Write, in the order thrown, three scores the player could have had.

        (ii)   In how many different ways could a player have reached square 12 with three throws? Show working to support your answer.

(d)   (i)   What is the minimum number of turns necessary to complete the game?

        (ii)   What is the probability of this happening?

(SEG)

5.9 Mutually Exclusive Events

1.   A man throws a die and a coin. Find the probability that he will get

       (a)   the number 3 followed by a head,

       (b)   an even number followed by a tail.

2.   In an experiment, a card is drawn from a pack of playing cards and a coin is tossed. Find the probability of obtaining

       (a)   a card which is a king and a head on the coin,

       (b)   the ace of diamonds and a tail on the coin.

3. In an experiment consisting of throwing a die followed by drawing a card from a pack of playing cards, find the probability of obtaining

    (a) an odd number on the die and a card which is an ace,

    (b) a six on the die and a picture card,

    (c) a six on the die and a club.

4. In a certain class, $\frac{1}{3}$ of the pupils read the local newspaper and $\frac{2}{3}$ watch the local news on television. None of these pupils read the local newspaper and also watch the local news on television. What is the probability that a pupil chosen at random reads the local newspaper or watch television?

5. In an inter-school mathematics quiz, the probability of school A winning the competition is $\frac{1}{2}$, the probability of school B winning is $\frac{1}{6}$ and the probability of school C winning is $\frac{1}{10}$.

    Find the probability that

    (a) B or C wins the competition,

    (b) A, B or C wins the competition,

    (c) none of these wins the competition.

6. A box contains buttons of various colours. The probability of drawing a red button at random is $\frac{1}{5}$ and the probability of drawing a white button at random in $\frac{2}{7}$.

    What is the probability of drawing neither a red nor a white button?

7. A box contains eight marbles: 1 is red, 2 are blue and 5 are green,

    One marble is drawn at random from the box. A second marble is drawn at random from the remaining seven marbles in the box.

    (a) Find the probability that both marbles are green.

    (b) If the first marble is red, find the probability that the second marble is blue.

8. Nine slips of paper are numbered 1 to 9. A slip is drawn at random. This is replaced before a second slip is drawn. Find the probability that one is an odd number and the other is an even number.

5.10 Tree Diagrams and Conditional Probability

1. A bag contains 7 red counters, 8 green counters and 5 blue counters.

    Anna takes one counter at random from the bag and, without replacing it, takes a second counter at random.

    What is the probability that Anna

    (a)   (i)   has *two* red counters,

           (ii)   has exactly *one* red counter,

    (b)   has *two* counters of the same colour?

    (SEG)

2. Three cards are drawn at random from a pack of playing cards. Find the probability of obtaining

    (a)   three picture cards (ace is not a picture card)

    (b)   two picture cards

    if each card chosen is not replaced.

3. Bag A contains 3 white counters and 2 black counters whilst bag B contains 2 white and 3 black. One counter is removed from bag A and placed in bag B without its colour being seen.

    What is the probability that a counter removed from bag B will be white?

4. A box of 24 eggs is known to contain 4 old and 20 new eggs. If 3 eggs are picked at random determine the probability that

    (a)   2 are new and the other old,

    (b)   they are all new.

5. Calculate the probability of obtaining 3 picture cards of the same suit when dealt a hand of 3 cards.

6. Terry has a box of chocolates.

   The box contains six milk chocolates and five plain chocolates.

   Terry chooses two chocolates at random and eats them.

   (a) Copy and complete the tree diagram showing all the probabilities.

   First choice   **Second choice**

   $\frac{6}{11}$ ——— Milk
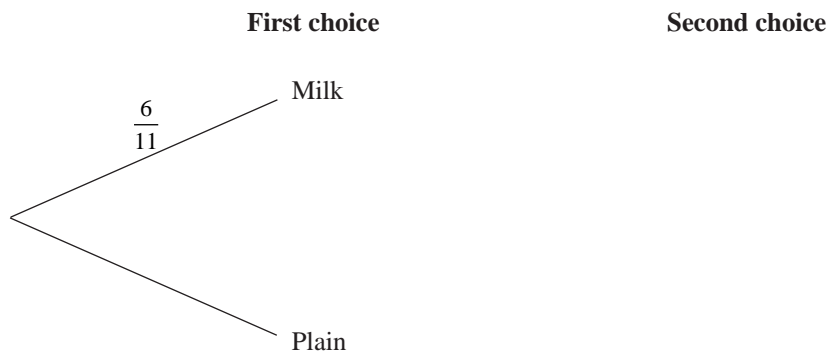
   ——— Plain

   (b) Calculate the probability that when Terry eats two chocolates, he eats either two milk chocolates *or* two plain chocolates.

   (SEG)

7. Sanjay has four possible ways home from school.

   From school he takes either a bus or a train.

   The probability that he will go by train is $\frac{3}{5}$.

   If he goes by train, he complete the journey by walking or by getting a lift.

   The probability that he gets a lift is $\frac{1}{5}$.

   If he catches a bus, the second part of his journey can be complete by catching another bus or he can walk.

   The probability that he will walk is $\frac{7}{8}$.

   What is the probability that Sanjay

   (a) catches a bus from school and then walks,

   (b) walks for part of his journey home?

   (SEG)

8. Magic matches all look the same but when they are struck they burn red, white or blue. Each box contains 24 matches. In every box $\frac{1}{4}$ will burn red, 10 will burn blue and the rest will burn white.

    (a) What is the probability that the first match taken from a box will burn blue?

    (b) How many matches in a box will burn white?

    (c) The first match taken from a box burns red. What is the probability that the second match taken from the box will also burn red?

    (SEG)

9. During a word game the following 27 letter tiles remain to be taken at random from a bag. Some are vowels and some are consonants.

    VOWELS

A	A	E	E	E	I	I	O	U

    CONSONANTS

B	C	D	D	F	G	J	K	K

L	N	P	Q	R	S	S	T	Z

    (a) Zoe wants to choose first. What is the probability that her tile would be,

    (i) a vowel,   (b) a letter S?

    (b) David actually chooses a tile first. The letter is a vowel. What is the probability that this vowel will be an **E**?

    (c) John is another player. If he had started first and taken three tiles, what is the probability that he chose the letters **SEG** in that order?

    (SEG)

10. There are 8 balls in a box. 7 of the balls are yellow and 1 ball is red.

    Jean selects balls at random, without replacement, from the box until she obtains the red ball.

    When she obtains the red ball, then she stops selecting.

    By extending a copy of the tree diagram shown below, or otherwise, calculate the probability that Jean selects the red ball on one of her first three selections.

    (LON)

    Start
    $\frac{1}{8}$ ● Red
    $\frac{7}{8}$ ● Yellow
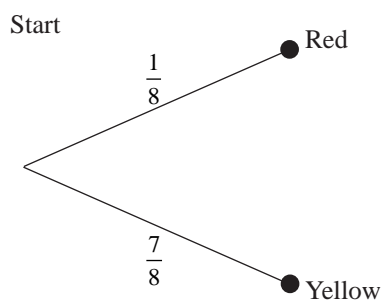
11. A bag contains 5 red, 4 orange and 3 yellow sweets. One after another, 3 children select and eat one sweet each. What are the probabilities that

   (a) they all choose red sweets,

   (b) at least one orange sweet is chosen,

   (c) each chooses a different colour,

   (d) all choose the same colour?

   Answers may be left as fractions in their lowest terms.

12. A sailing competition between two boats, A and B, consists of a series of independent races.

   Every race is won by either A or B, and their respective probabilities of winning are influenced by the weather. In rough weather the probability that A will win is 0.9; in fine weather the probability that A will win is 0.4. For each race the weather is either rough or fine, the probability of rough weather being 0.2.

   Show that the probability that A will win the first race is 0.5.

# 5.11 Using Venn Diagrams to find Probabilities

1. 80 pupils in a certain school may choose one, two or three optional subjects: History (H), Geography (G) and Biology (B). The numbers in the Venn diagram represent the number of pupils in each subset.

   (a) If a pupil is chosen at random from the group, find the probability that

      (i) he studies Geography,

      (ii) he studies one optional subject only.

   (b) If two pupils are chosen at random from the group, find the probability that

      (i) both study all three optional subjects,

      (ii) neither study History.
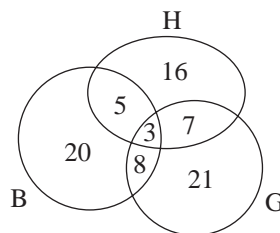
2. A school offers 3 racket games: squash (S), badminton (B) and tennis (T). 70 pupils play one or more of these games. The figures in the Venn diagram represent the number of players in each subset. Two pupils are chosen at random.

   Find the probability that

   (a) both play only squash,

   (b) both play 2 of the 3 racket games,

   (c) neither plays tennis,

   (d) one plays all 3 games and the other plays only one game.
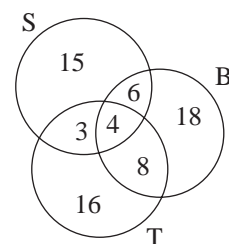
3.   A group of people apply for work in either one or two of the three firms, L, M and N.

In the Venn diagram the numbers represent the numbers of people who apply for jobs in the three firms.

(a)   A person is chosen at random from the group.
Calculate the probability that the person applies for L and M.

(b)   A person is chosen at random from those who apply for N. Calculate the probability that this person also applies for L.

(c)   Two people are chosen at random from the group.
Calculate the probability that

(i)   they both apply for only one firm,

(ii)   they both apply for M.