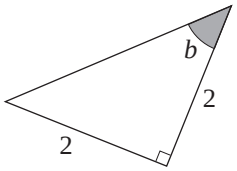
4 Trigonometry

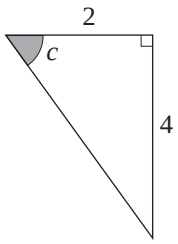
4.4 Sine, Cosine and Tangent

1. For each of the following triangles, all dimensions are in cm. Find the tangent ratio of the shaded angle.

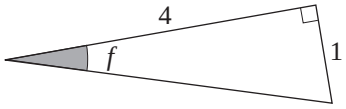
(a)



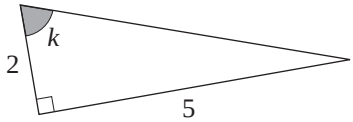
(b)



(c)



(d)



2. Find each of the following, giving your answer correct to 3 decimal places.

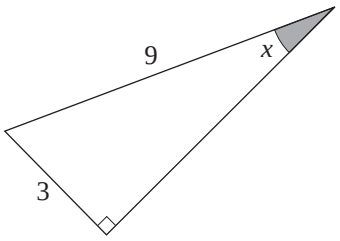
(a)	$\tan 36°$	(b)	$\tan 42°$	(c)	$\tan 55°$
(d)	$\tan 17°$	(e)	$\tan 68°$	(f)	$\tan 73°$
(g)	$\tan 67.4°$	(h)	$\tan 75.5°$	(i)	$\tan 81.2°$
(j)	$\tan 89.3°$	(k)	$\tan 16.9°$	(l)	$\tan 26.2°$

3. Find the size of angle x in each of the following. Give your answer correct to 1 decimal place.

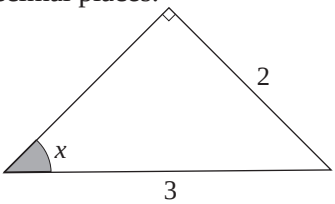
(a)	$\tan x = 0.3$	(b)	$\tan x = 0.4$	(c)	$\tan x = 0.8$
(d)	$\tan x = 1.3$	(e)	$\tan x = 1.5$	(f)	$\tan x = 2$
(g)	$\tan x = 2.5$	(h)	$\tan x = 3.3$	(i)	$\tan x = 4.5$
(j)	$\tan x = 5.8$	(k)	$\tan x = 100.4$	(l)	$\tan x = 233.5$

4. For each of the following triangles, all dimensions are in cm. Find the sine ratio of the shaded angle. Give your answer correct to 2 decimal places.

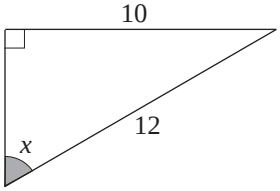
(a)



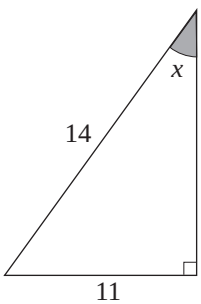
(b)



(c)



(d)



5. Find the value of each of the following. Give your answer correct to 3 decimal places.

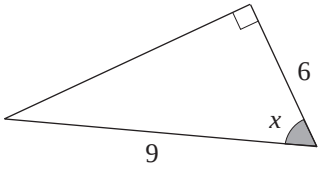
(a)	$\sin 22°$	(b)	$\sin 76°$	(c)	$\sin 19.6°$
(d)	$\sin 39.2°$	(e)	$\sin 61.3°$	(f)	$\sin 85.7°$
(g)	$\sin 44.9°$	(h)	$\sin 50.4°$	(i)	$\sin 67.1°$
(j)	$\sin 79.3°$	(k)	$\sin 81.2°$	(l)	$\sin 29.6°$

6. Find the size of angle x in each of the following. Give your answer correct to 1 decimal place.

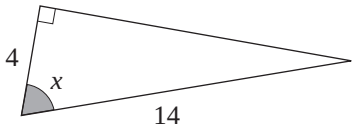
(a)	$\sin x = 0.31$	(b)	$\sin x = 0.27$	(c)	$\sin x = 0.46$
(d)	$\sin x = 0.64$	(e)	$\sin x = 0.189$	(f)	$\sin x = 0.986$
(g)	$\sin x = 0.497$	(h)	$\sin x = 0.721$	(i)	$\sin x = 0.584$
(j)	$\sin x = 0.842$	(k)	$\sin x = 0.992$	(l)	$\sin x = 0.999$

7. For each of the following triangles, all dimensions are in cm. Find the cosine ratio of the shaded angle. Give your answer correct to 2 decimal places.

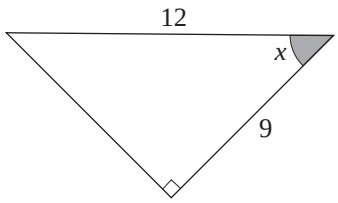
(a)



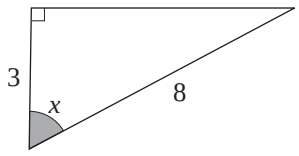
(b)



(c)



(d)



8. Find the value of each of the following. Give your answer correct to 3 decimal places.

(a)	$\cos 29°$	(b)	$\cos 48°$	(c)	$\cos 30°$
(d)	$\cos 69°$	(e)	$\cos 80.2°$	(f)	$\cos 54.7°$
(g)	$\cos 79.3°$	(h)	$\cos 35.5°$	(i)	$\cos 43.8°$
(j)	$\cos 56.2°$	(k)	$\cos 61.2°$	(l)	$\cos 83.8°$

9. Find the size of angle x in each of the following. Give your answer correct to 1 decimal place.

(a)	$\cos x = 0.33$	(b)	$\cos x = 0.26$	(c)	$\cos x = 0.51$
(d)	$\cos x = 0.37$	(e)	$\cos x = 0.016$	(f)	$\cos x = 0.998$
(g)	$\cos x = 0.305$	(h)	$\cos x = 0.816$	(i)	$\cos x = 0.538$
(j)	$\cos x = 0.276$	(k)	$\cos x = 0.171$	(l)	$\cos x = 0.662$

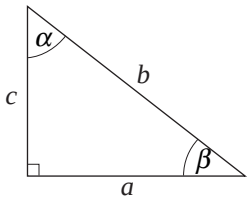
10. Write expressions for

$$\sin\alpha,\ \cos\alpha,\ \tan\alpha$$

and

$$\sin\beta,\ \cos\beta,\ \tan\beta$$

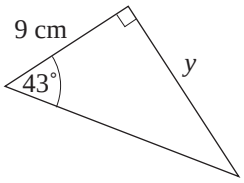
in terms of a, b and c. What do you notice about the results?



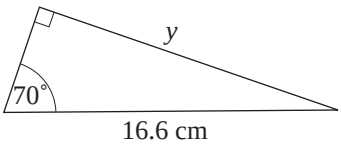
4.5 Finding Lengths in Right Angled Triangles

1. In each of the following find the length of y, giving your answer correct to 2 decimal places.

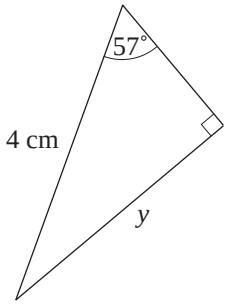
(a)



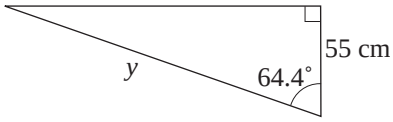
(b)



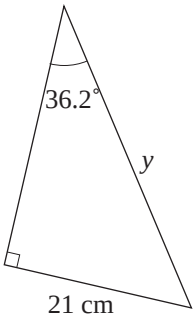
(c)



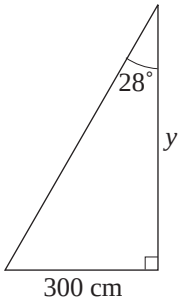
(d)



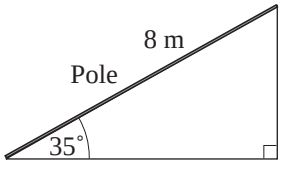
(e)



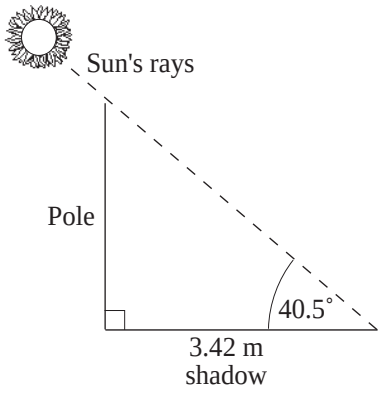
(f)



2. One end of a pole, 8 metres long, reaches a corner of the ceiling of a room. If the angle made by the pole with the horizontal is $35°$, what is the height of the ceiling? Give your answer correct to 2 significant figures.



3. The length of the shadow of a vertical pole is 3.42 metres long when the rays of the sun are inclined at an angle of $40.5°$ to the horizontal. What is the height of the pole? Give your answer correct to 2 decimal places.

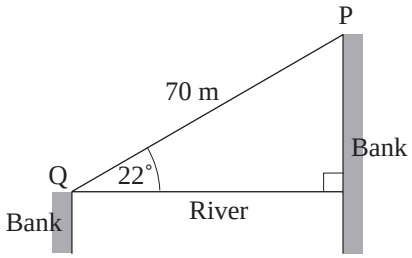


4. The diagram shows two banks of a river which are at different levels. Points P and Q are on opposite sides of the river such that a rope attached from P to Q makes an angle of $22°$ to the horizontal. If $PQ = 70$ m, calculate

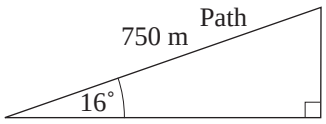
   (a) the width of the river,

   (b) the difference in heights of the two banks.

   Give your answers correct to the nearest metre.



5. A path, 750 metres long, runs straight up the slope of a hill. If the angle made by the path with the horizontal is $16°$, find the height of the point at the top end of the path. Give your answer correct to 3 significant figures.

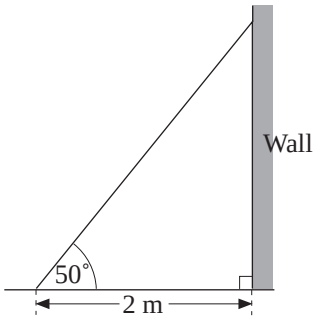


6. A ladder is placed on horizontal ground with its foot 2 metres from a vertical wall. If the ladder makes an angle of $50°$ with the ground, find

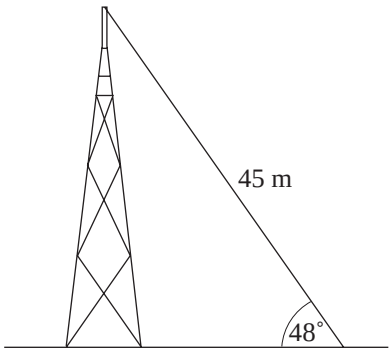
   (a) the length of the ladder,

   (b) how far up the wall it reaches.

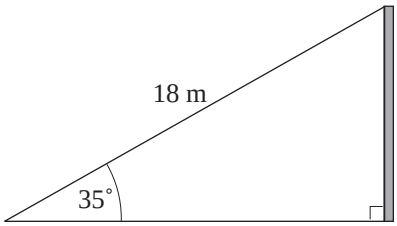
   Give your answers correct to 1 decimal place.



7. One end of a rope of length 45 metres is tied to a point on the ground and the other end to the top of an antenna. When the rope is taut, its inclination to the horizontal is $48°$. Find, correct to 3 significant figures, the distance of the top of the antenna from the ground.



8. A wire 18 metres long runs from the top of a pole to the ground as shown in the diagram. The wire makes an angle of $35°$ with the ground.



Calculate the height of the pole.

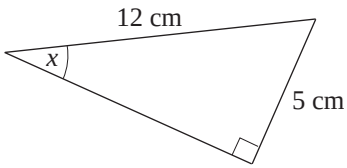
Give your answer to a reasonable degree of accuracy.

(NEAB)

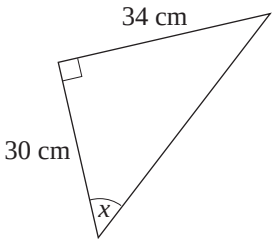
4.6 Finding Angles in Right Angled Triangles

1. In each of the following find angle x, giving your answer correct to 1 decimal place.

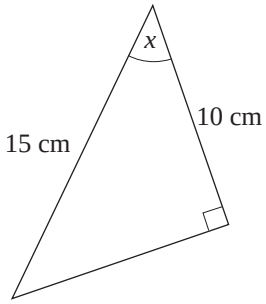
(a)



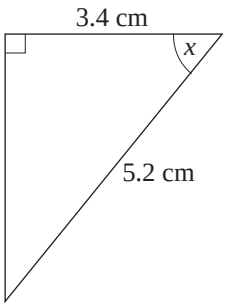
(b)



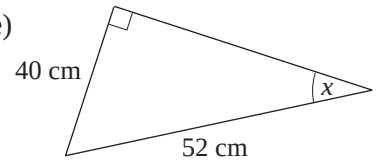
(c)



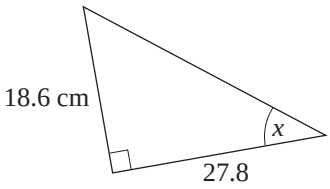
(d)



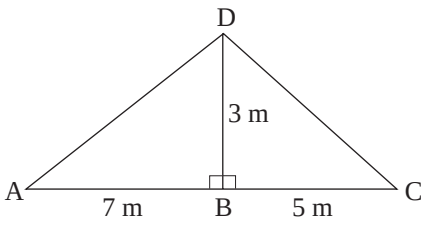
(e)



(f)



2. The diagram shows a roofing frame ABCD.
AB = 7 m, BC = 5 m, DB = 3 m, angle ABD = angle DBC = 90°.

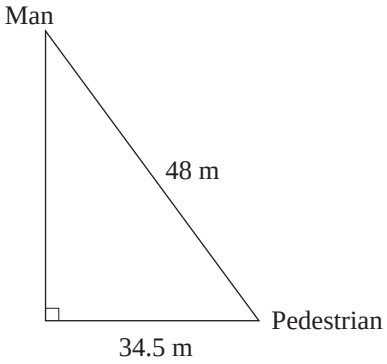


(a) Calculate the length of AD.

(b) Calculate the size of angle DCB.

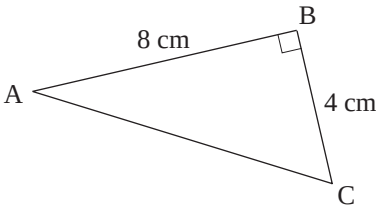
(MEG)

3. From the top of a building a man sights a pedestrian on the street below at a distance of 48 metres away. The pedestrian is 34.5 metres away from the foot of the building. Find the angle of depression of the pedestrian from the man, correct to the nearest degree.

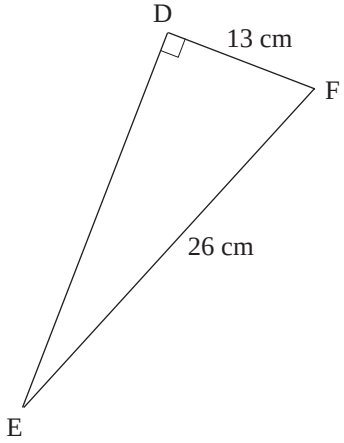


4. Find all unknown angles and lengths for each triangle. Give your answers correct to the nearest cm or degree.

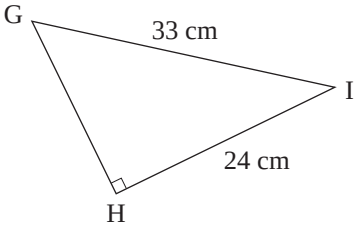
(a)



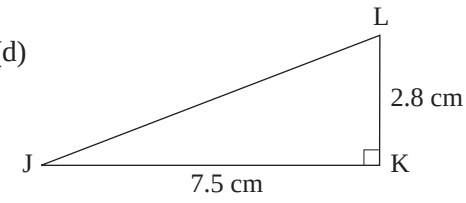
(b)



(c)



(d)

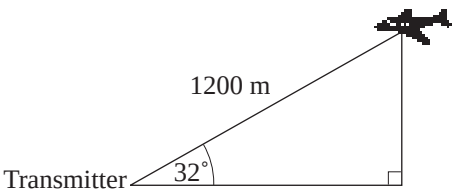


4.7 Mixed Problems with Trigonometry

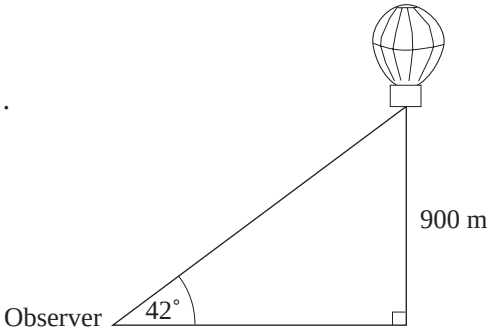
1. The angle of elevation of a radio-controlled model aeroplane from the transmitter on the ground is $32°$. If the aeroplane is 1200 metres from the transmitter, find the

   (a) height of the aeroplane from the ground,

   (b) horizontal distance of the aeroplane from the transmitter.

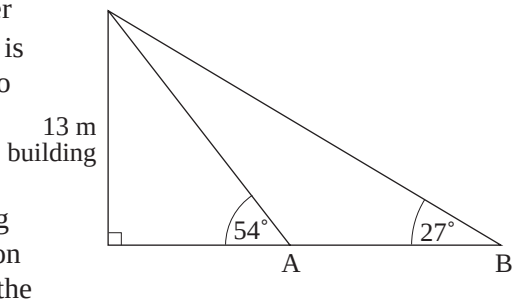


2. A weather balloon is at a height of 900 metres. The angle of elevation of the balloon from an observer on the field is $42°$. What is the distance of the balloon from the observer, correct to the nearest metre?



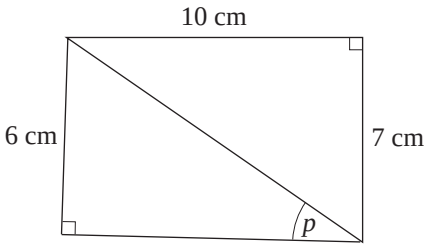
3. The angle of elevation of the top of a building, 13 metres high, from an observer at point A on the ground is $54°$. How far is he from the base of the building, correct to the nearest metre?

If he walks further away from the building to a point B such that the angle of elevation is halved, find the distance AB correct to the nearest metre.

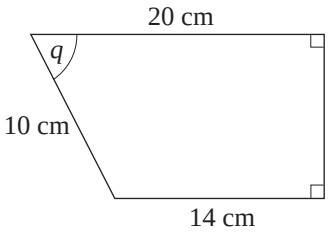


4. In each of the following cases, find the labelled side or angle. Give each answer correct to the nearest cm or degree.

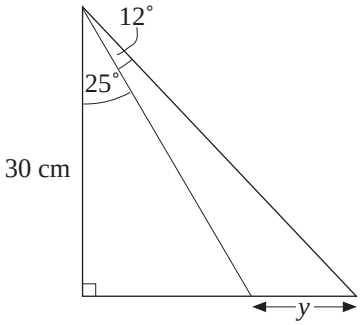
(a)



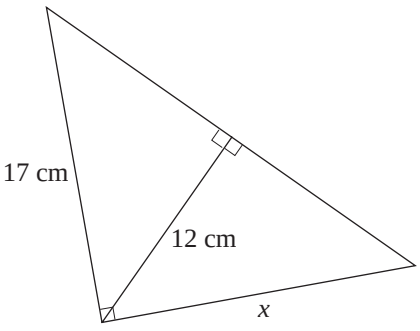
(b)



(c)



(d)

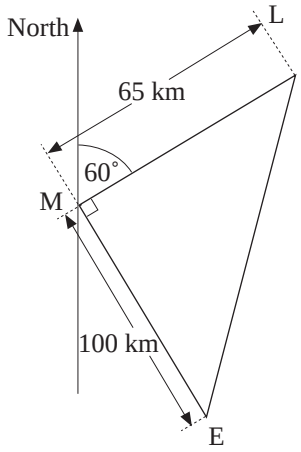


5. The diagram shows the positions of three airports:

E (East Midlands)

M (Manchester) and

L (Leeds).



The distance from M to L is 65 km on a bearing of $060°$.

Angle $LME = 90°$ and $ME = 100$ km.

(a) Calculate, correct to three significant figures, the distance LE.

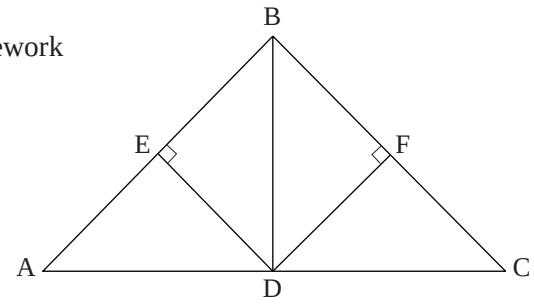
(b) Calculate, to the nearest degree, the bearing of E from L.

(c) An aircraft leaves M at 10.45 a.m. and flies direct to E, arriving at 11.03 a.m.

Calculate the average speed of the aircraft in kilometres per hour. Give your answer correct to the appropriate number of significant figures.

(MEG)

6. The diagram shows a symmetrical framework for a bridge.



$AC = 100$ m, $AB = BC = 70$ m.

(a) (i) Calculate the angle BAD.

(ii) Calculate the length ED.

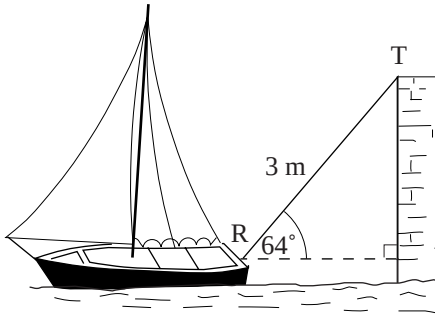
A *similar* framework is made with the length corresponding to $AC = 180$ m.

(b) (i) Calculate the length corresponding to AB.

(ii) What is the size of the angle corresponding to angle BAD?

7. A yacht is moored to the side of a quay by a rope 3 metres long. The rope is tied to the yacht at R and the quay at T.

At low tide, when the yacht is as far away from the quay as possible, the rope makes an angle of $64°$ with the horizontal, as shown.



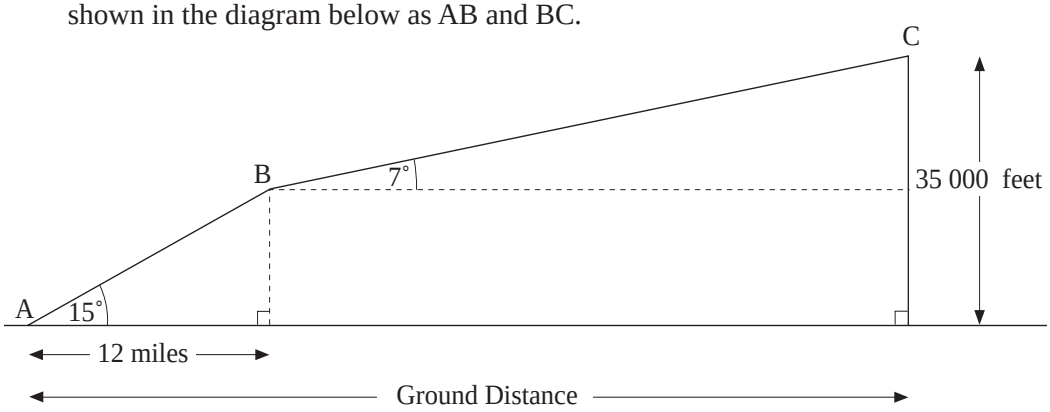
(a) Calculate the vertical distance of R below T when the yacht is in this position. Give your answer correct to one decimal place.

At high tide, the water level at the quay is 1.6 metres higher than at low tide.

(b) At high tide, when the yacht is as far away from the quay as possible, calculate the distance from R to the side of the quay.

(SEG)

8. When an aeroplane takes off, its ascent is in two stages. These two stages are shown in the diagram below as AB and BC.



1 mile = 5280 feet

(a) In the first stage the aeroplane climbs at an angle of 15° to the horizontal. Calculate the height it has reached when it has covered a ground distance of 12 miles. Give your answer correct to the nearest thousand feet.

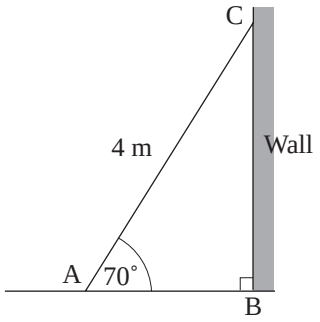
(b) In the second stage the aeroplane climbs at an angle of 7° to the horizontal. At the end of its ascent it has reached a height of 35 000 feet above the ground. Calculate the total distance it has covered. Give your answer to a reasonable degree of accuracy.

(NEAB)

9. Paul's ladder is 4 metres long.

(a) Paul leans his ladder against a vertical wall, with the end, A, on horizontal ground. The angle between the ladder and the ground is 70°.

Calculate the distance of A from the wall.

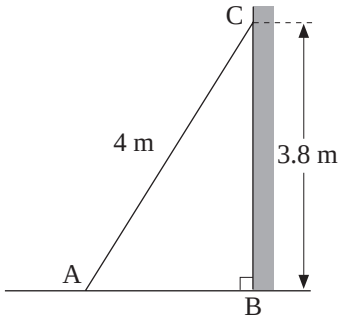


(b) Pamela moves the ladder and uses it to reach a windowsill which is 3.8 metres above the ground.

For safely, the angle between the ladder and the ground should be within 2° of 70°.

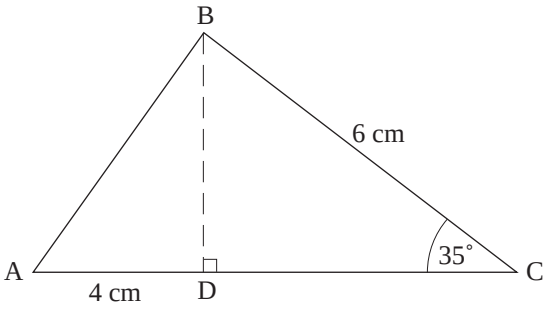
Is the ladder safely placed?

You must show some calculation to explain your answer.



10. AD = 4 cm, BC = 6 cm, angle BCD = 35°.

BD is perpendicular to AC.



(a) Calculate BD.

(b) Calculate angle BAC.

(c) Triangle A′B′C′ is similar to triangle ABC.

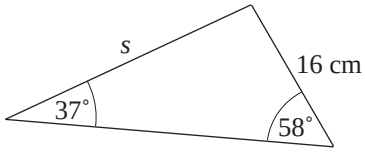
The area of triangle A′B′C′ is nine times the area of triangle ABC.

(i) What is the size of angle A′B′C′? (ii) Work out the length of B′C′.

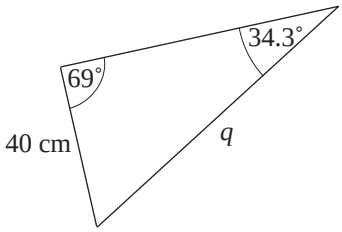
4.8 Sine and Cosine Rules

1. Find the side marked with a letter in each triangle below.

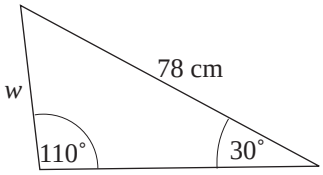
(a)



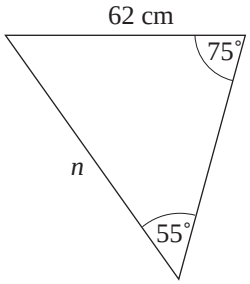
(b)



(c)

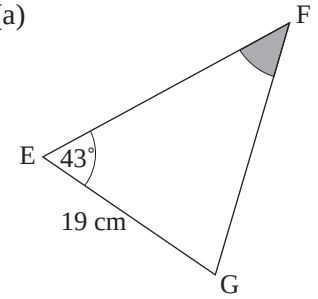


(d)

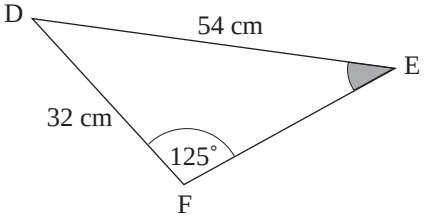


2. Find the shaded angles in the triangles shown.

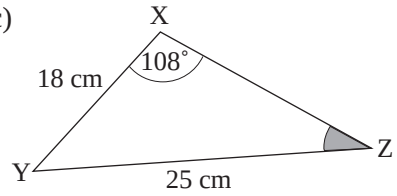
(a)



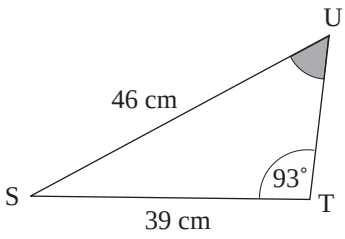
(b)



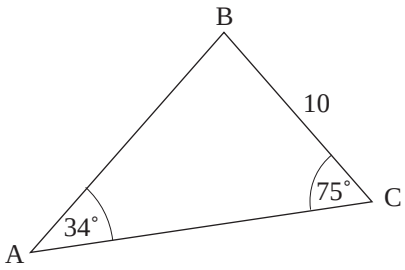
(c)



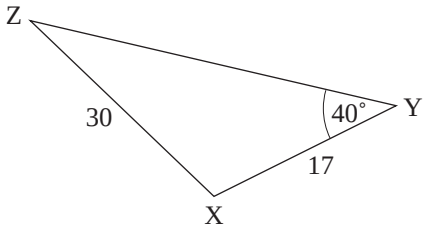
(d)



3. Find AC in triangle ABC, given that angle BAC = 34°, angle BCA = 75° and BC = 10 cm.



4. Find angle ZXY in triangle XYZ, given that XY = 17 cm, XZ = 30 cm and angle XYZ = 40°.

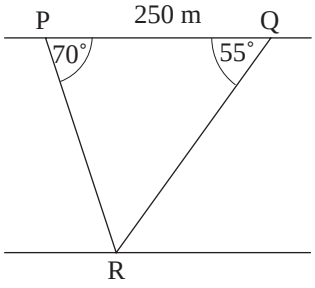


5. The figure shows two trees, P and Q, on a bank of a river. R is another tree on the opposite bank.

Calculate

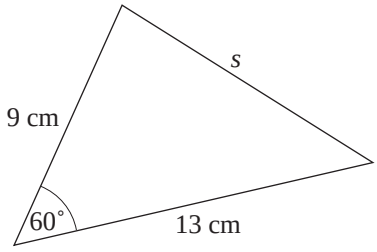
(a) angle PRQ,

(b) RQ.

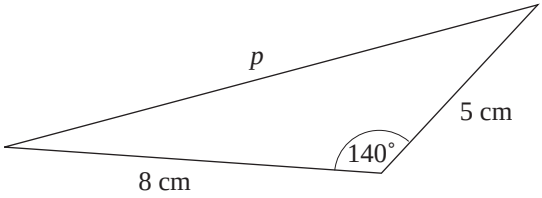


6. Find the side marked with a letter in each triangle below.

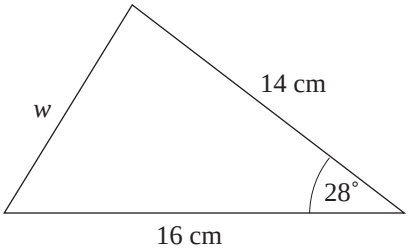
(a)



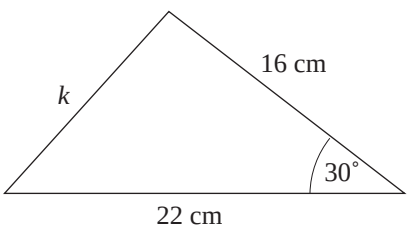
(b)



(c)

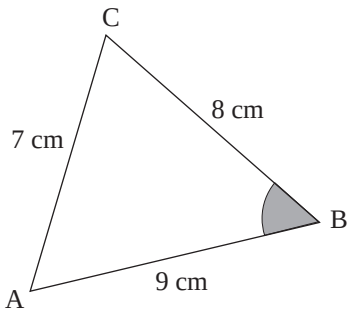


(d)

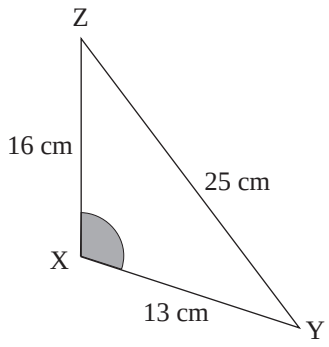


7. Find the shaded angles in the triangles shown.

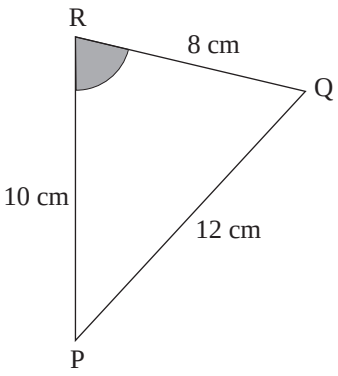
(a)



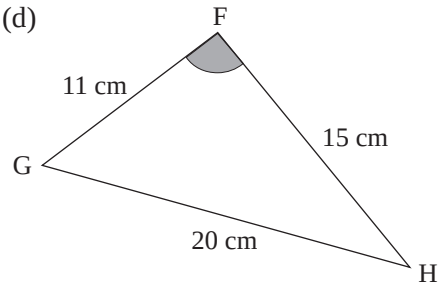
(b)



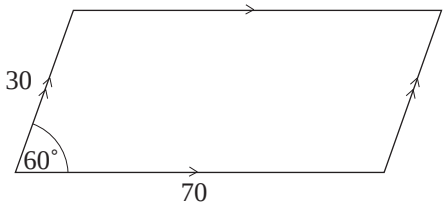
(c)



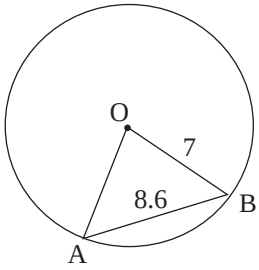
(d)



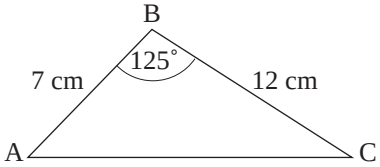
8. A parallelogram has sides of lengths 30 cm and 70 cm. One of its angles is $60°$ as shown. Find the lengths of its diagonals.



9. AB is a chord of a circle, centre O and radius 7 cm. If $AB = 8.6\text{ cm}$, calculate angle AOB.



10. In triangle ABC, $AB = 7\text{ cm}$, $BC = 12\text{ cm}$ and angle $ABC = 125°$.

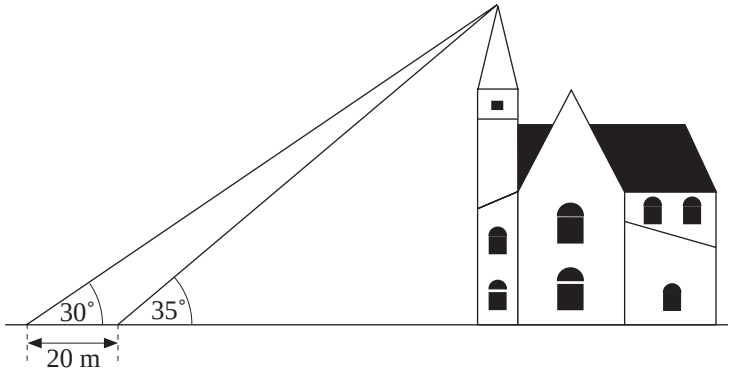


Calculate the length of AC. Give your answer correct to 3 significant figures.

(LON)

11. A surveyor wishes to measure the height of a church.

Measuring the angle of elevation, she finds that the angle increases from $30°$ to $35°$ after walking 20 metres towards the church.



What is the height of the church?

(SEG)

12. Two ships, A and B, leave Dover Docks at the same time.

Ship A travels at 25 km/h on a bearing of $120°$.

Ship B travels at 30 km/h on a bearing of $130°$.

Calculate how far apart the two ships are after 1 hour.

(SEG)

4.9 Angles Larger than 90°

1. Using the values

$$\cos 45° = \sin 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = 1$$

$$\cos 60° = \sin 30° = \frac{1}{2} \qquad \tan 60° = \sqrt{3}$$

$$\cos 30° = \sin 60° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}}$$

find, without using a calculator, the value of

(a)	$\sin 135°$	(b)	$\sin 180°$	(c)	$\sin 120°$
(d)	$\cos 180°$	(e)	$\cos 135°$	(f)	$\cos 210°$
(g)	$\sin 270°$	(h)	$\sin 240°$	(i)	$\tan 135°$
(j)	$\tan 240°$	(k)	$\tan 150°$	(l)	$\sin 480°$
(m)	$\cos 405°$	(n)	$\cos 315°$	(o)	$\sin 315°$

2. Use a calculator, if needed, and a sketch to find all solutions in the range $-360° \le \theta \le 360°$ of the following equations.

(a)	$\cos\theta = -1$	(b)	$\sin\theta = \frac{1}{2}$	(c)	$\sin\theta = 0$
(d)	$\sin\theta = 0.3$	(e)	$\cos\theta = -0.2$	(f)	$\sin\theta = -0.4$
(g)	$\tan\theta = 1$	(h)	$\tan\theta = 2$	(i)	$\cos\theta = 0.8$
(j)	$\sin\theta = 0.6$	(k)	$\cos\theta = -0.8$	(l)	$\sin\theta = 1$

3. Use a sketch to find how many solutions, in the range $0 \le \theta \le 720°$ exist for the equation $\sin\theta = 0.7$. Evaluate each solution, to the nearest degree.

4. The depth of water in a harbour *t* hours after midday, *d* metres, is given by

$$d = 10 + 7\cos(30t)°$$

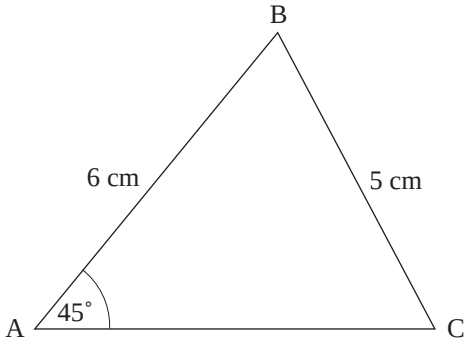
(a) Draw a graph of depth against time for a 24-hour period.

(b) When is high tide and low tide?

(c) A ship needs a minimum depth of 11.5 metres to berth in the harbour. For how long during the 24-hour period can the ship remain in the harbour?

5. In the triangle ABC, $AB = 6$ cm, $BC = 5$ cm, and angle $BAC = 45°$.

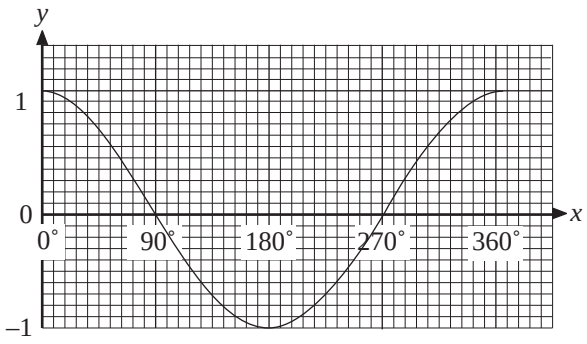
There are two possible triangles that can be constructed.



Calculate the *two* possible values of the angle BCA.

(SEG)

6. (a) The diagram shows the graph of $y = \cos x$ for $0° \le x \le 360°$.



(i) Copy the diagram and show the location of the two solutions of the equation $\cos x = -0.5$.

(ii) The angle x is between $0°$ and $360°$.

Work out accurately the two solutions of the equation $\cos x = -0.5$.

(b) On a particular day the height, h metres, of the tide at Weymouth, relative to a certain point, can be modelled by the equation

$$h = 5\sin(30t)°$$

where t is the time in hours after midnight.

(i) Sketch the graph h against t for $0 \le t \le 12$.

(ii) Estimate the height of the tide, relative to the same point, at 2 pm that day.

(SEG)