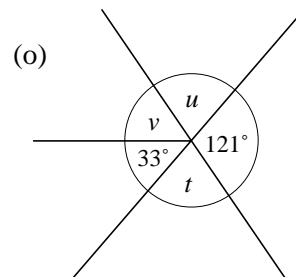
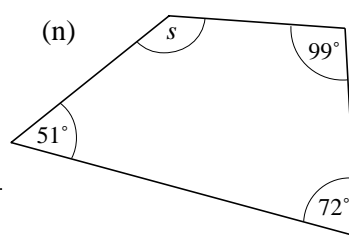
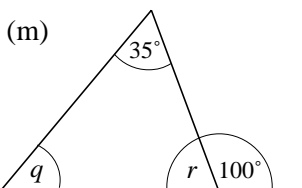
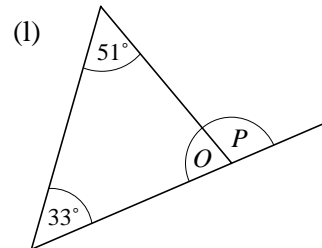
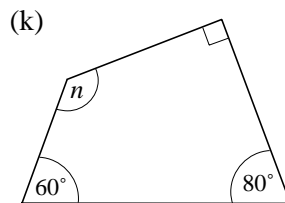
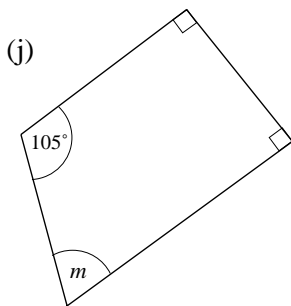
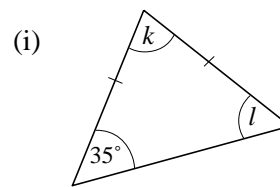
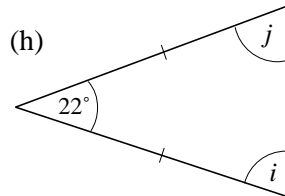
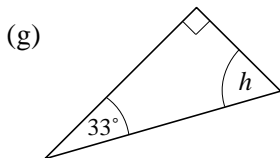
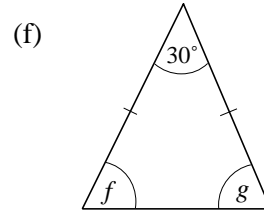
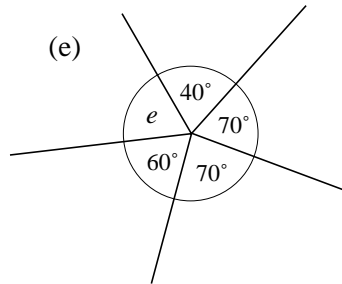
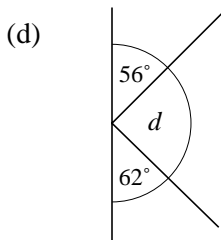
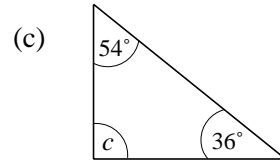
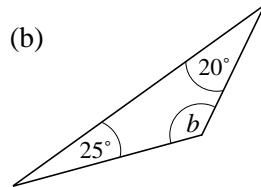
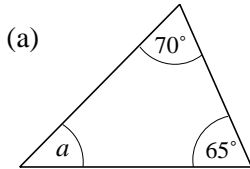


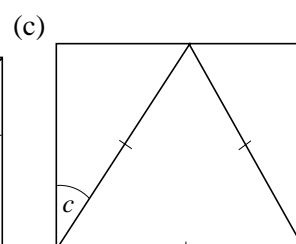
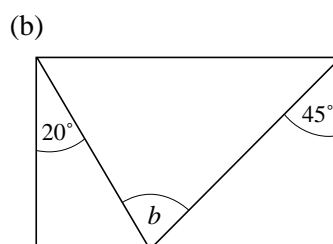
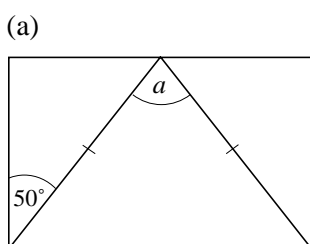
# 3 Angle Geometry

## 3.3 Angle Geometry

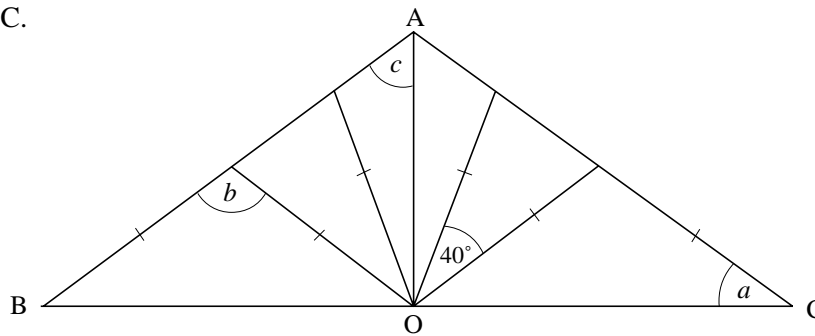
1. Calculate the size of the angles marked with a letter in each diagram. *None to scale*



2. Find the angles marked with a letter in each rectangle below.

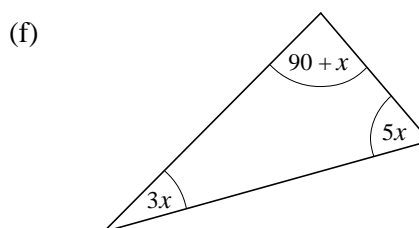
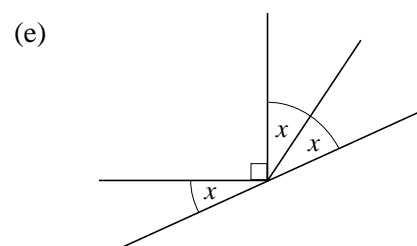
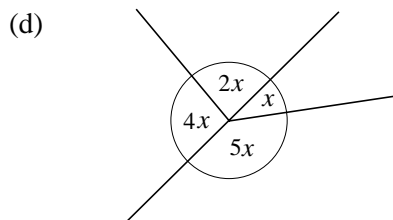
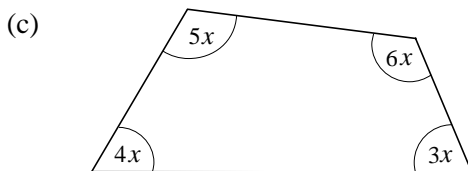
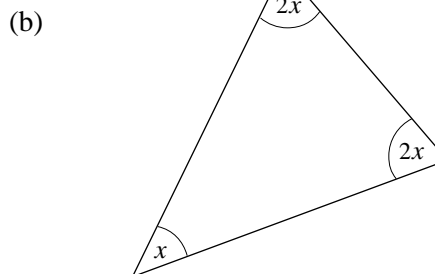
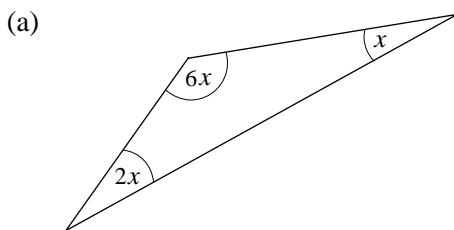


3. The framework of a symmetrical roof is illustrated below. OA is perpendicular to BOC.



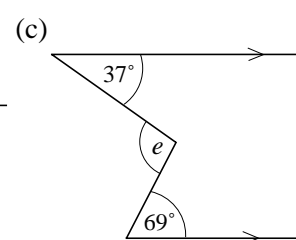
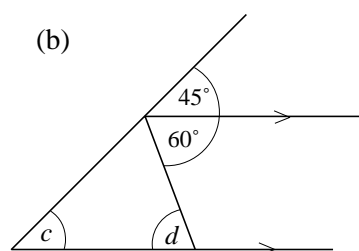
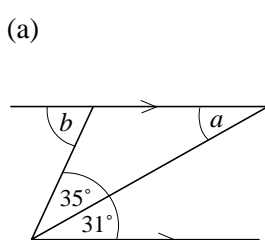
Find the size of the angles marked  $a$ ,  $b$  and  $c$ .

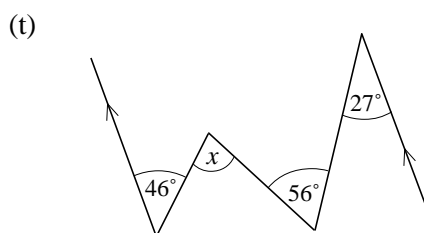
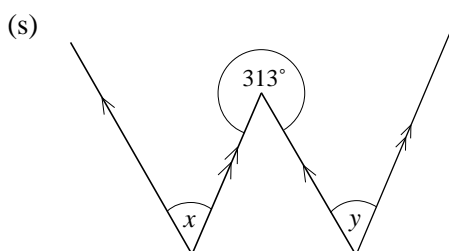
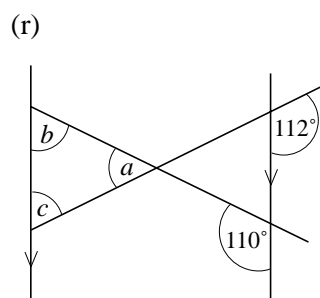
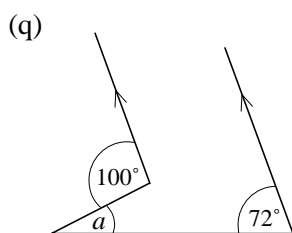
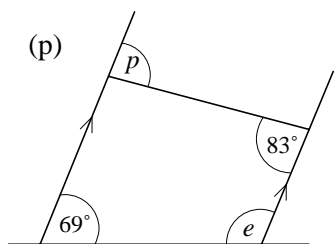
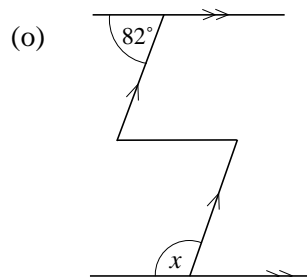
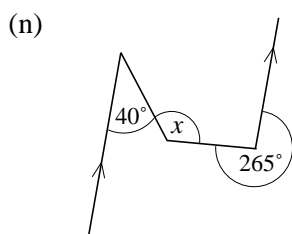
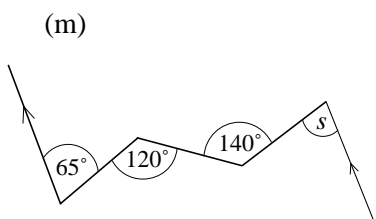
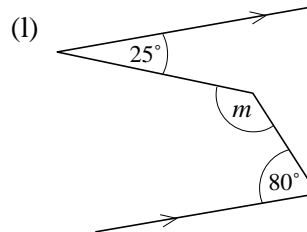
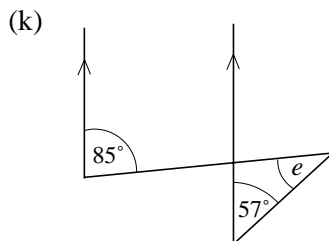
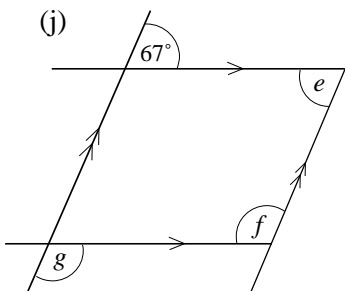
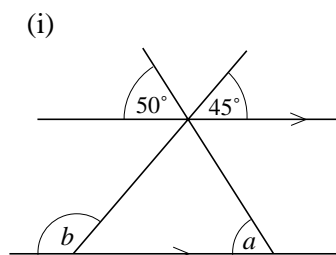
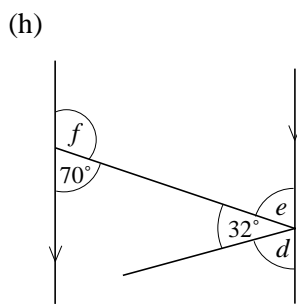
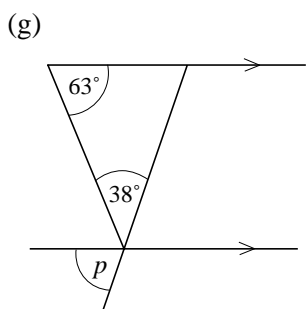
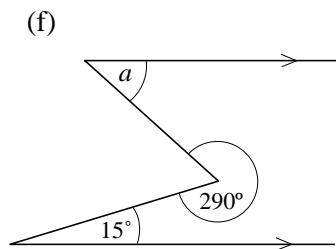
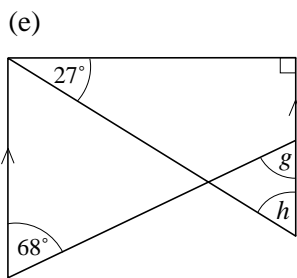
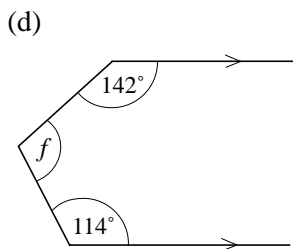
4. Write down an equation that is satisfied in each of the following diagrams. In each case, solve the equation for  $x$ .



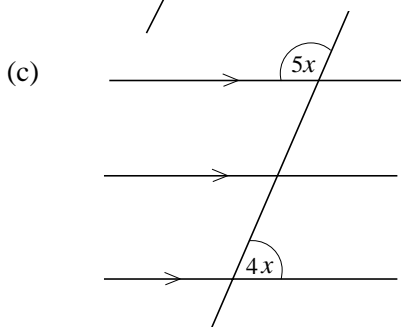
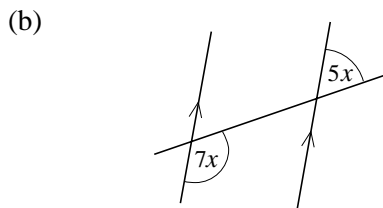
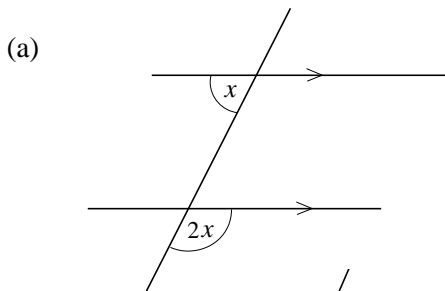
### 3.4 Angles with Parallel and Intersecting Lines

1. Calculate the unknown angles in the following diagrams.

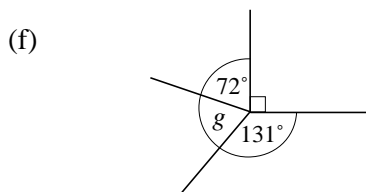
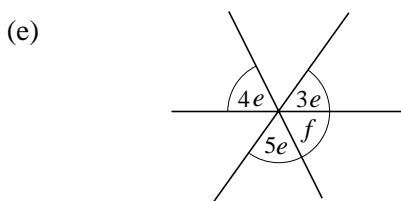
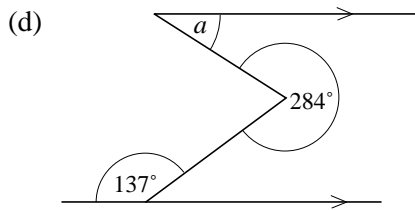
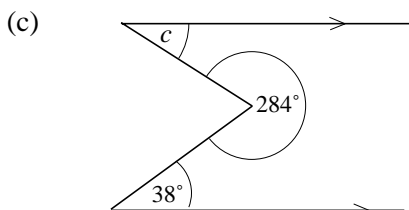
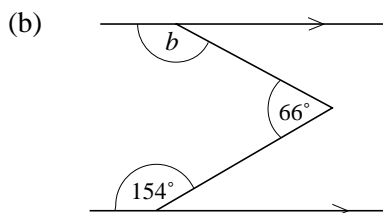
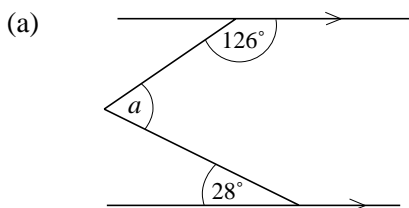




2. For each diagram, find an equation in  $x$ , and hence solve for  $x$ .



3. Find the values of the unknown angles in each of the following.

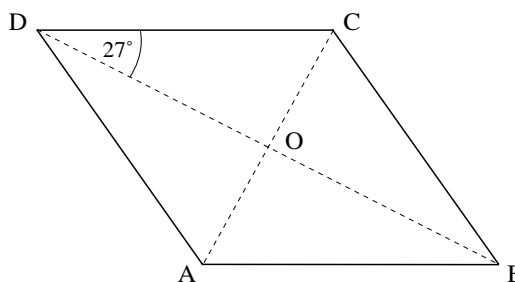


4. ABCD is a rhombus.

Angle BDC =  $27^\circ$

The diagonals AC and BD cross at O.

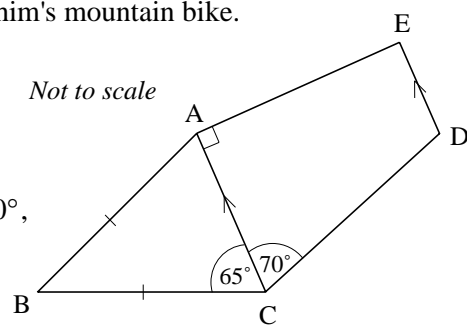
Calculate the size of the angle ADC.



5. The pentagon ABCDE is the frame for Ibrahim's mountain bike.

ABC is an isosceles triangle in which  
 $AB = BC$  and angle  $BCA = 65^\circ$ .

In the quadrilateral ACDE angle  $ACD = 70^\circ$ ,  
 angle  $CAE = 90^\circ$  and AC is parallel to ED.

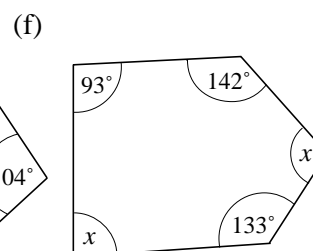
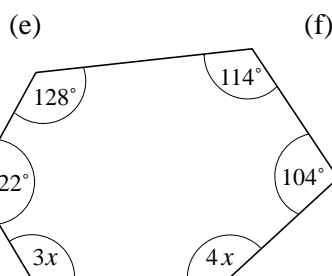
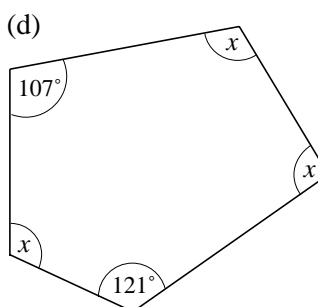
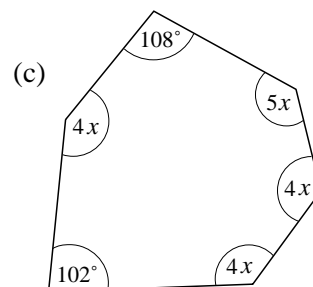
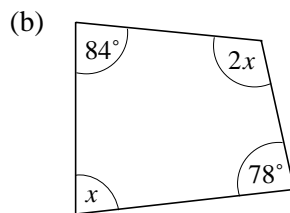
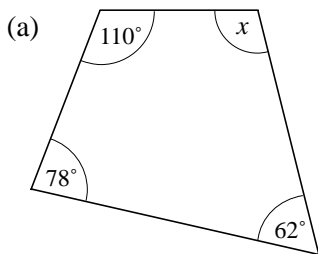


- (a) (i) Calculate the size of angle ABC.  
 (ii) What facts about the angles of a triangle did you use in your calculation?
- (b) Calculate the size of the angle CDE.

(MEG)

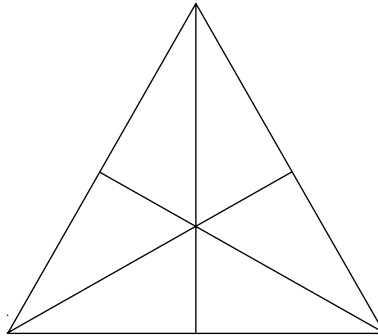
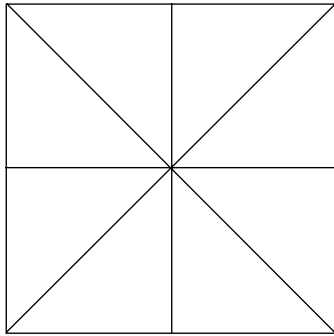
## 3.5 Angle Symmetry in Polygons

- Find the sum of the interior angles of
  - a quadrilateral
  - a pentagon.
- Find the size of each interior angle of
  - a regular hexagon
  - a regular nonagon.
- Find the number of sides of a polygon if the sum of its interior angles is
  - $1800^\circ$
  - $1080^\circ$ .
- Each interior angle of a regular polygon is  $140^\circ$ . Find the number of sides of the polygon.
- Each interior angle of a regular  $n$ -gon is  $168^\circ$ . What is the value of  $n$ ?
- Find the value of  $x$  in each of the following diagrams.

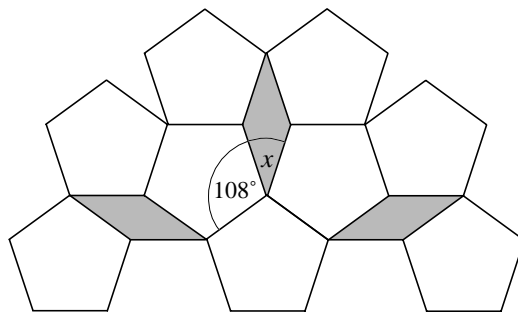


7. The angles of a quadrilateral are  $3x$ ,  $4x$ ,  $5x$  and  $6x$ .  
 (a) Find  $x$ .      (b) What are the angles in degrees?

8.

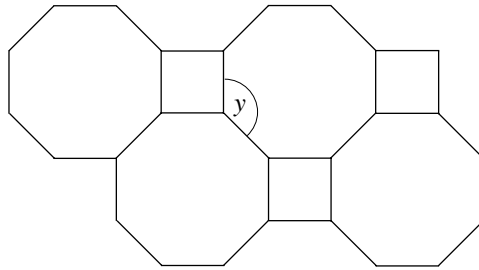


- (a) For each diagram above, show three different ways of shading parts of the shapes so that they have line symmetry but no rotational symmetry.  
 (b) Shade sections of one shape so that it has rotational symmetry of order 2 but no lines of symmetry. Is it possible to do this for both shapes?  
 (c) Repeat (b) for rotational symmetry of order 3.  
 (d) Repeat (b) for rotational symmetry of order 4.
9. (a) A regular polygon has an interior angle of  $175^\circ$ .  
 How many sides does it have?  
 (b) A second regular polygon has an interior angle which is  $1^\circ$  smaller.  
 How many sides does it have?  
 (c) Is it possible for a regular polygon to have an interior angle of  $173^\circ$ ?
10. (a) The diagram shows part of a tiling pattern of regular pentagons and another shape.



- (i) Which of the following correctly describes the shaded shape:  
 square, rhombus, trapezium, rectangle, parallelogram, kite?  
 (ii) Calculate the size of the angle marked  $x$ .  
 (iii) A regular pentagon has rotational symmetry. What is the order of rotational symmetry of a regular pentagon?

- (b) Another tiling pattern is formed using regular octagons and squares, as shown.



Calculate the size of the angle marked  $y$ .

- (c) Draw a tiling pattern using regular hexagons only. You must draw at least five hexagons.

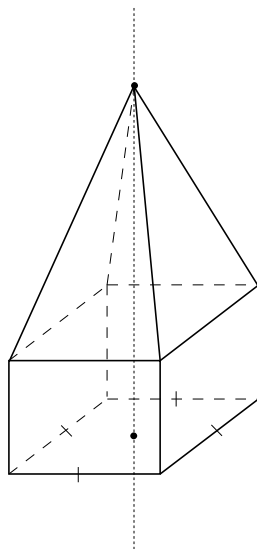
(SEG)

## 3.6 Symmetry Properties of 3D Shapes

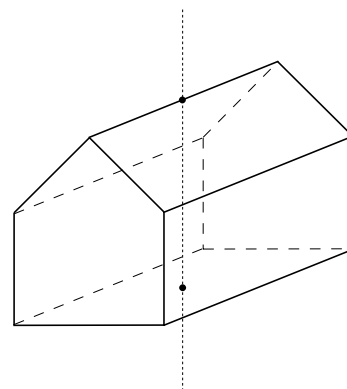
1. The following solids have rotational symmetry.

For each of them, state the order of rotational symmetry about the given axis.

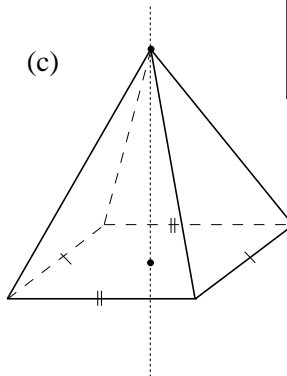
(a)



(b)

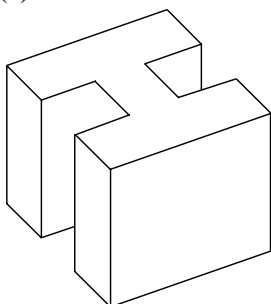


(c)

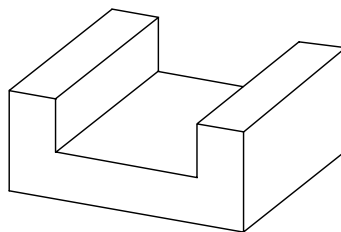


2. For each of the following prisms, draw an axis so that the order of rotational symmetry about that axis is 2.

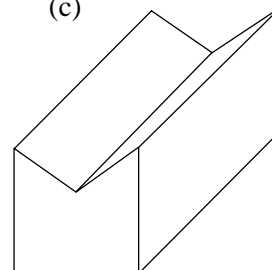
(a)



(b)

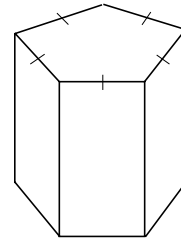


(c)

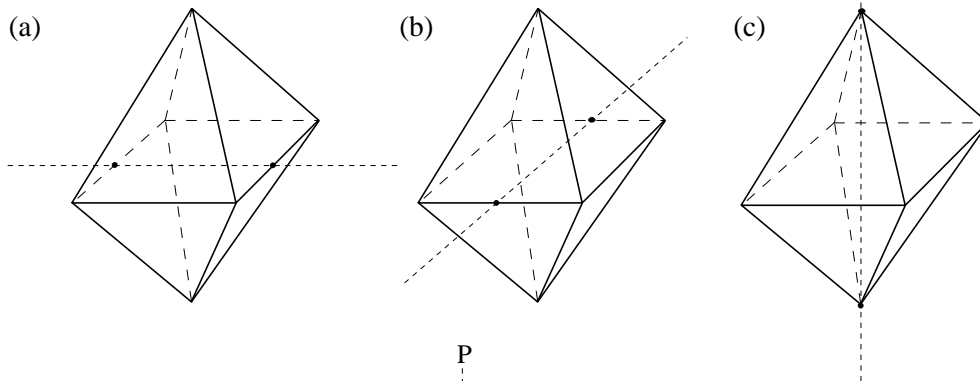


3. In the given prism, the cross-section is in the shape of a regular pentagon. Draw

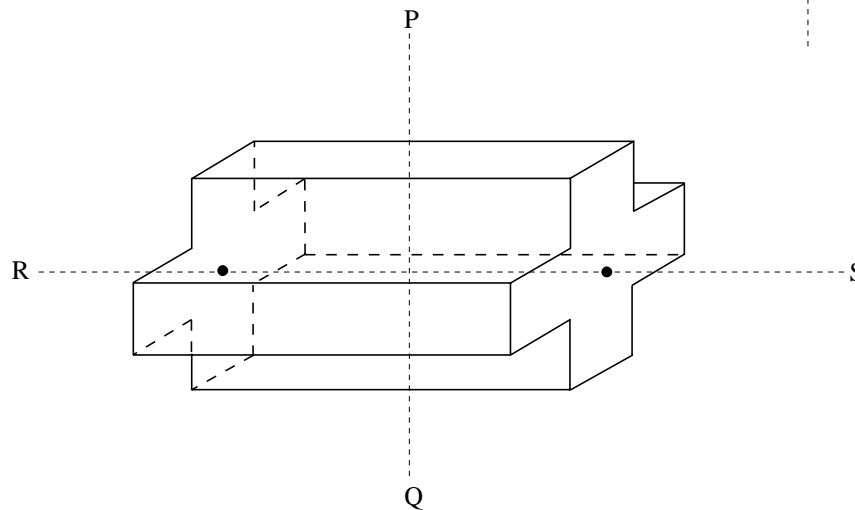
- (a) an axis  $ST$  so that the order of rotational symmetry about  $ST$  is 2;
- (b) an axis  $XY$  so that the order of rotational symmetry about  $XY$  is 5.



4. State the order of rotational symmetry about each of the axes shown. All the 12 edges of the solid are equal in length.



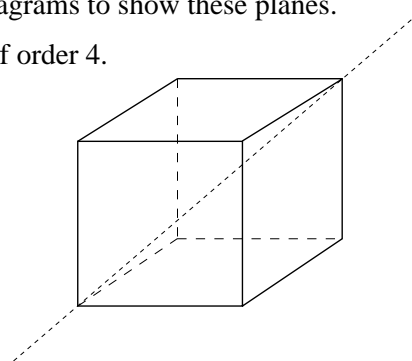
5.



For the solid above, find the order of its rotational symmetry about

- (a)  $PQ$
- (b)  $RS$ .

- 6. (a) A cube has 9 planes of symmetry. Draw diagrams to show these planes.
- (b) A cube has 3 axes of rotational symmetry of order 4. Draw diagrams to show these axes.
- (c) The diagram of a cube opposite shows one axis of rotational symmetry of order 3. There are 3 other axes with the same order. Draw diagrams to show these axes.
- (d) There are 6 axes with symmetry of order 2. Draw diagrams to show these axes.

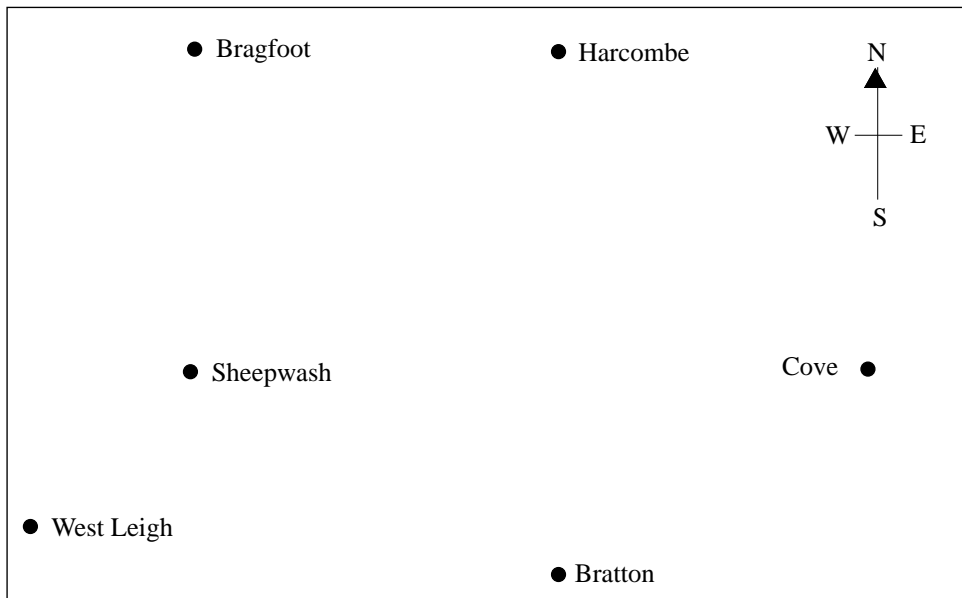


7. Draw a solid that has one axis of symmetry and rotational symmetry of order 5 about the axis.



# 3.7 Compass Bearings

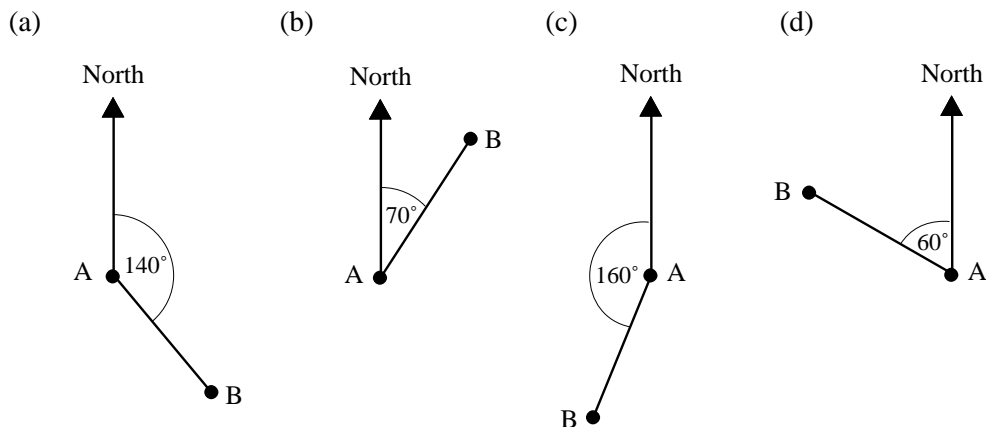
1. The map below shows the positions of some villages.



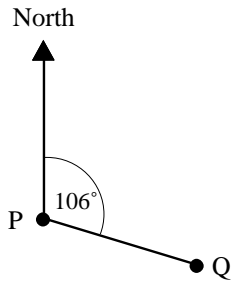
Scale: 2 miles to 1 cm

- (a) Which village is due north of *Sheepwash*?
- (b) Which village is due west of *Cove*?
- (c) What is the compass direction of *Sheepwash* from *West Leigh*?
- (d) How many miles is
  - (i) *Bratton* from *Cove*
  - (ii) *Harcombe* from *Bragfoot*?
- (e) Make a tracing of the map and mark the positions of
  - (i) *Darley*, which is 3 miles due south of *Harcombe*,
  - (ii) *Lee*, which is 4 miles south east of *Bragfoot*.

2. For each of the following, write down the bearing of B from A.



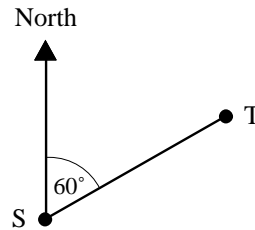
3.



- What is the bearing of
- (a) Q from P
  - (b) P from Q?

4.

- What is the bearing of
- (a) T from S?
  - (b) S from T?

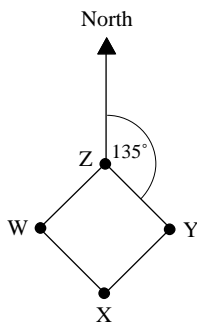


5.

Draw a diagram with 4 towns marked, so that that three of the towns are equidistant from the fourth town, P, and have bearings from P of

- (a)  $036^\circ$
- (b)  $132^\circ$
- (c)  $265^\circ$ .

6.



A field is in the shape of a square, with corners W, X, Y and Z.

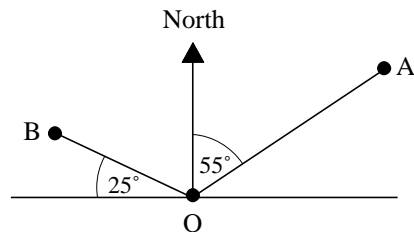
The bearing of Y from Z is  $135^\circ$ .

- Find the bearing of
- (a) Y from X
  - (b) W from Z.

7.

What is the bearing of

- (a) Q from O
- (b) B from O
- (c) O from A
- (d) O from B?

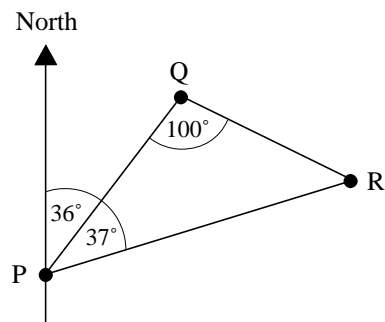


8.

The figure shows the positions of P, Q and R.

What is the bearing of

- (a) Q from P
- (b) P from Q
- (c) R from P
- (d) P from R
- (e) Q from R
- (f) R from Q?



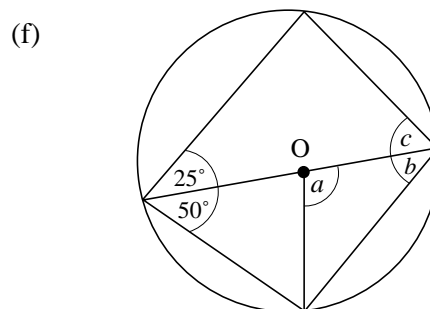
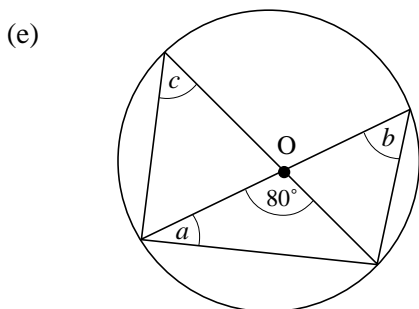
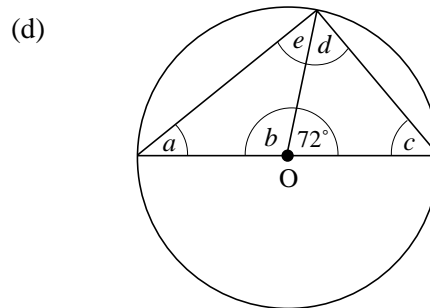
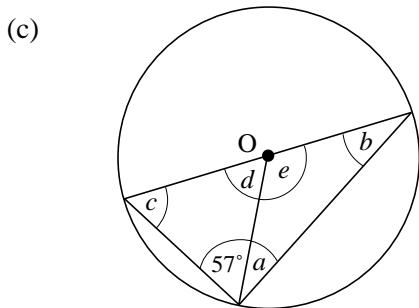
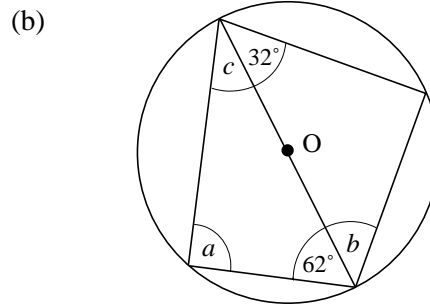
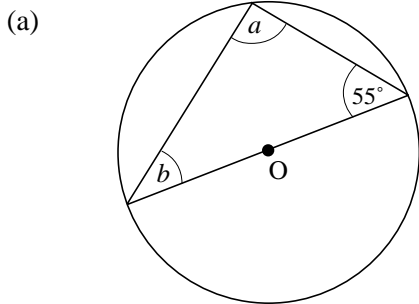
9.

A point B is 280 m due North of the point A. A man walks from A in the direction  $050^\circ$ . Calculate how far he walks before he is

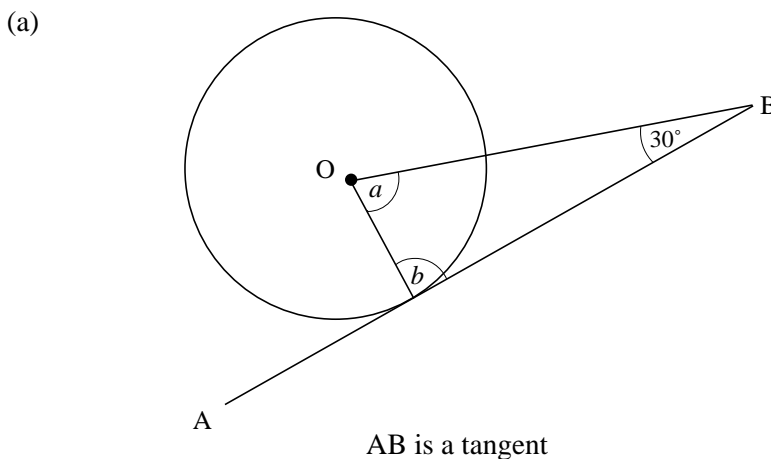
- (a) equidistant from A and B,
- (b) as close as possible to B,
- (c) due east of B.

# 3.8 Angles and Circles 1

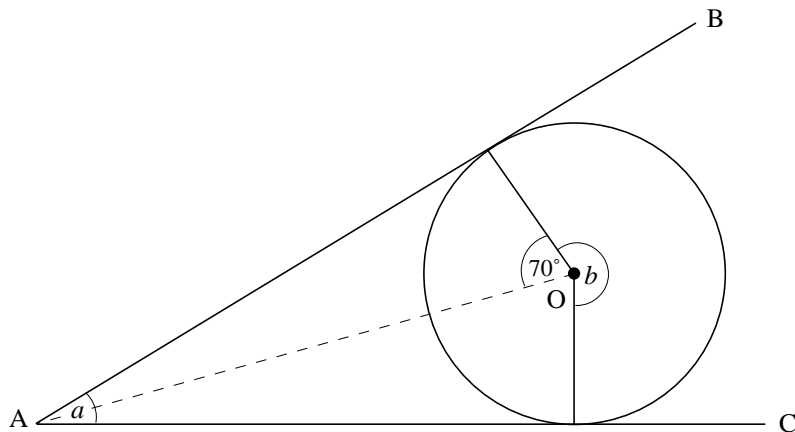
1. Find the angles marked with a letter in each of the following diagrams. (In each case O is the centre of the circle.)



2. Find the angles marked with a letter in each diagram below. (In each case O is the centre of the circle.)



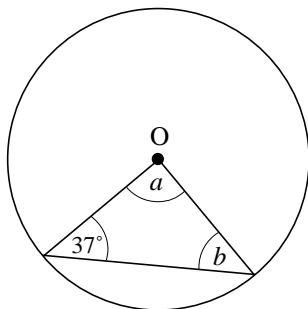
(b)



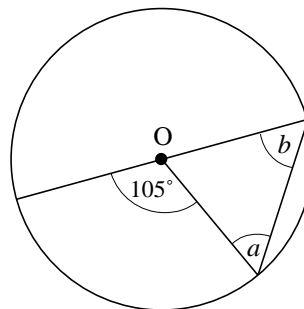
AB and AC are tangents

3. Find the angles marked with letters in each of the following diagrams. (In each case O is the centre of the circle.)

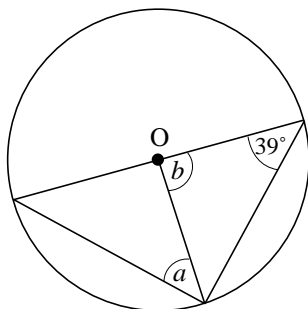
(a)



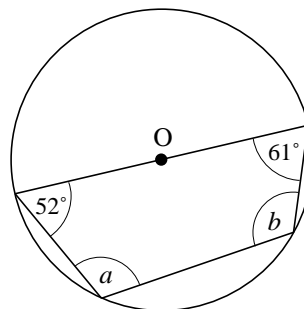
(b)



(c)

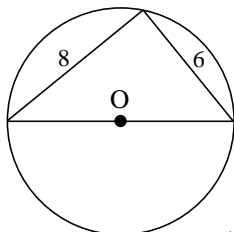


(d)

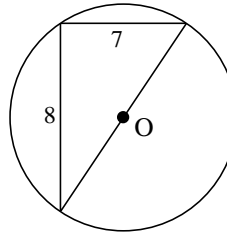


4. Find the diameter of each circle below. (In each case O is the centre of the circle.)

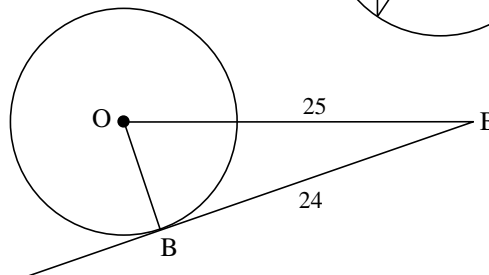
(a)



(b)

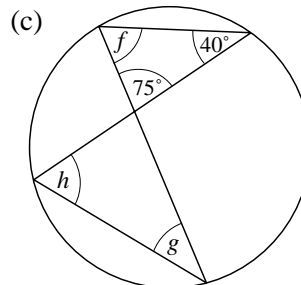
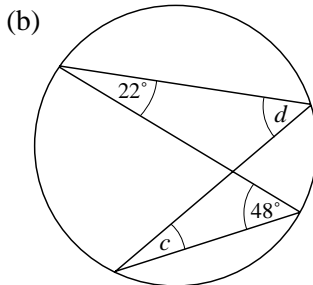
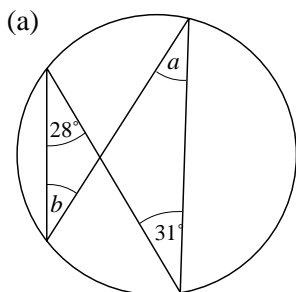


(c)

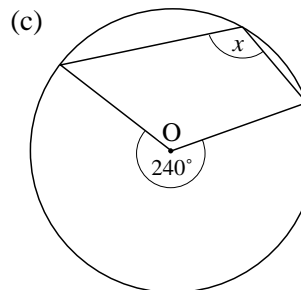
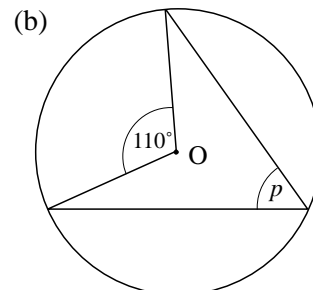
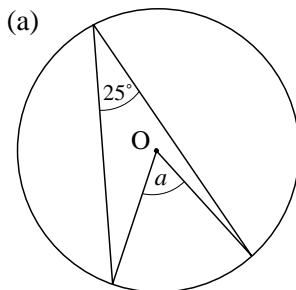


## 3.9 Angles and Circles 2

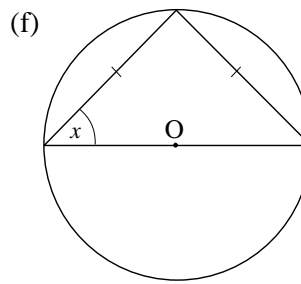
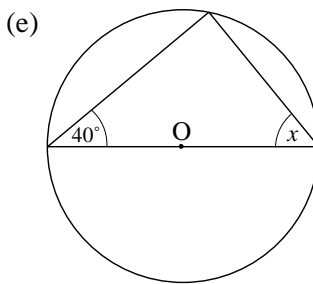
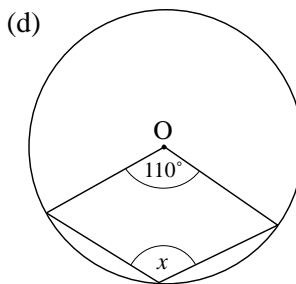
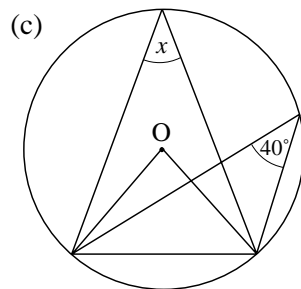
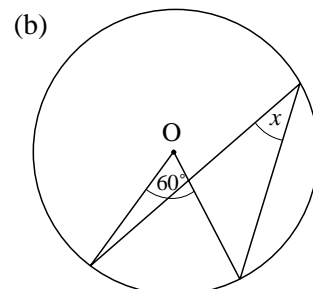
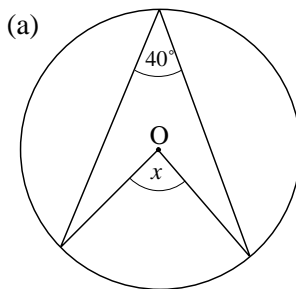
1. In each of the following circles, find the angles marked with letters.

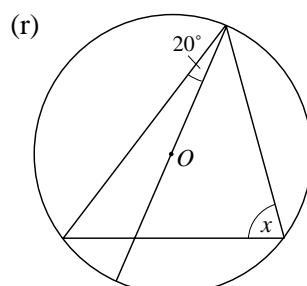
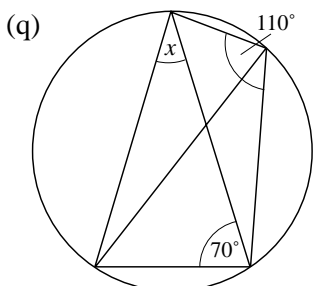
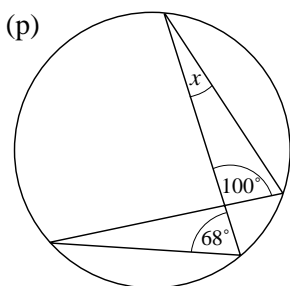
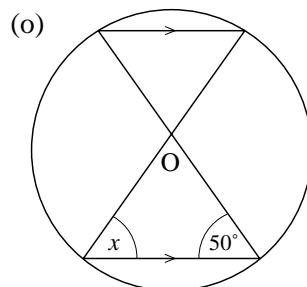
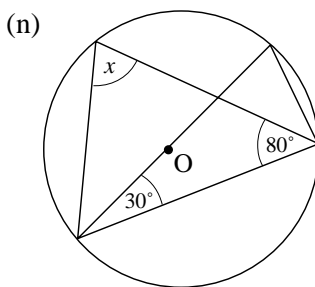
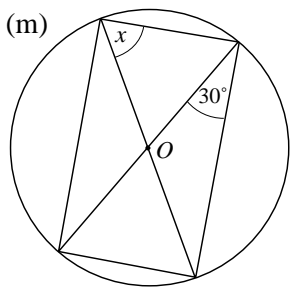
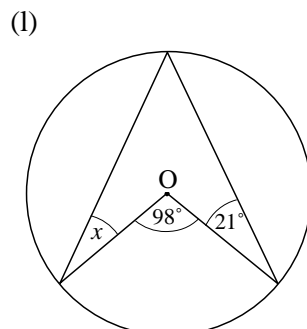
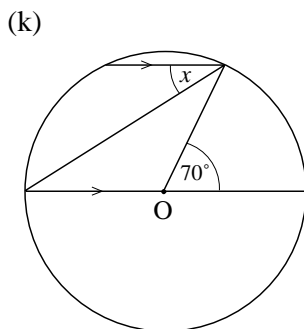
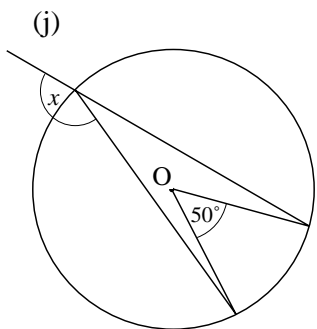
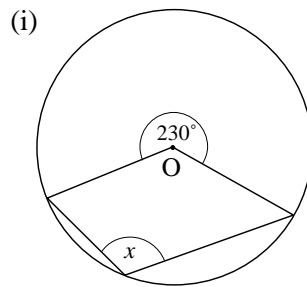
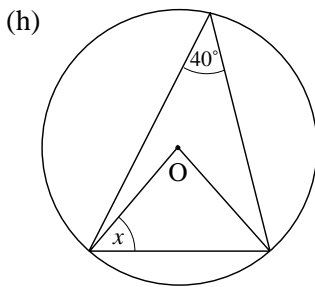
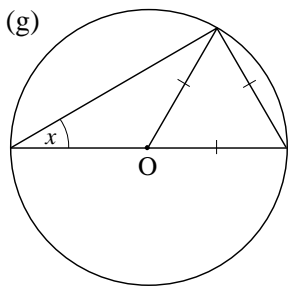


2. In each of the following circles, O is the centre. Find the angles marked with a letter.

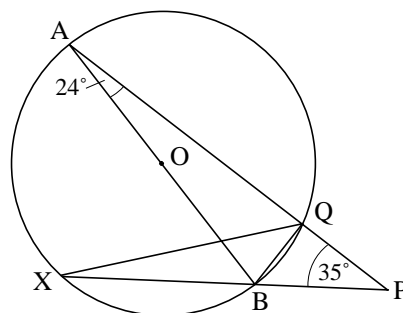


3. In each of the following figures, find the value of  $x$ . In each case, O is the centre of the circle.





4. In the diagram, AB is a diameter of the circle.  
 Given that  
     angle BAP =  $24^\circ$   
 and      angle BPA =  $35^\circ$ ,  
 find angle BQX.

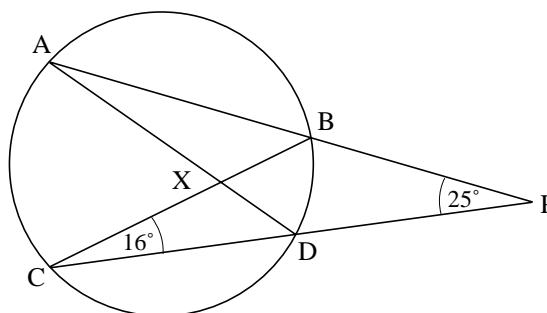


5. In the diagram,

$$\hat{A}PC = 25^\circ$$

and  $\hat{B}CD = 16^\circ$ .

Find  $\hat{A}XB$ .



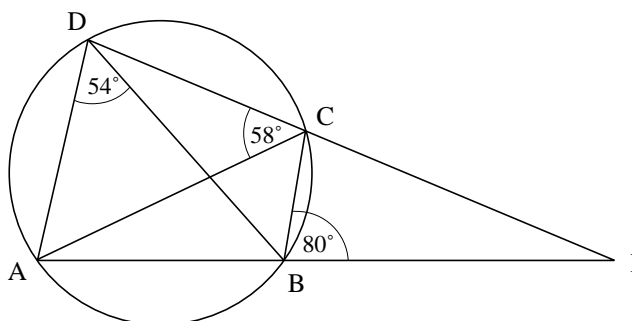
6. In the diagram,

$$\hat{A}DB = 54^\circ$$

$$\hat{A}CD = 58^\circ$$

and  $\hat{C}BP = 80^\circ$ .

Find  $\hat{A}PD$ .



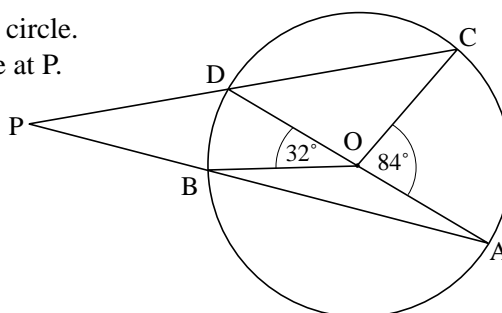
7. In the diagram, O is the centre of the circle.  
AB and CD intersect outside the circle at P.

Given that

$$\hat{A}OC = 84^\circ$$

and  $\hat{B}OD = 32^\circ$ ,

find  $\hat{A}PC$ .

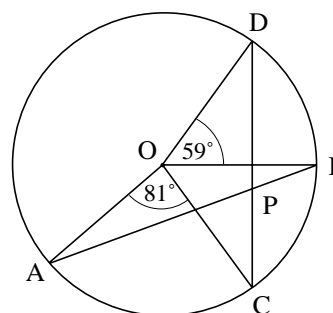


8. In the diagram, O is the centre of the circle.  
Chords AB and CD intersect inside the circle at P.  
OB is perpendicular to CD.

Given that

$$\hat{A}OC = 81^\circ \text{ and } \hat{B}OD = 59^\circ,$$

find  $\hat{A}PC$ .



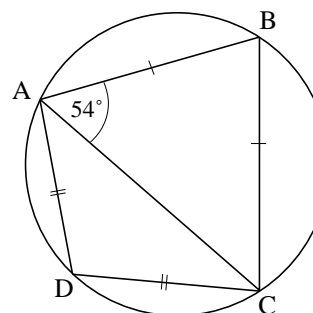
9. In the diagram,

$$AB = BC$$

$$CD = DA$$

and  $\hat{B}AC = 54^\circ$ .

Find the value of  $\hat{A}CD$ .



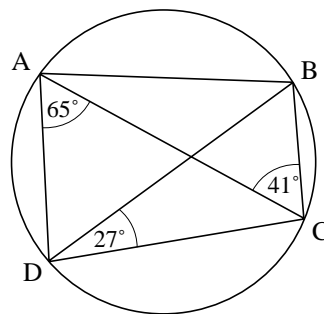
10. In the diagram,

$$\hat{D}AC = 65^\circ$$

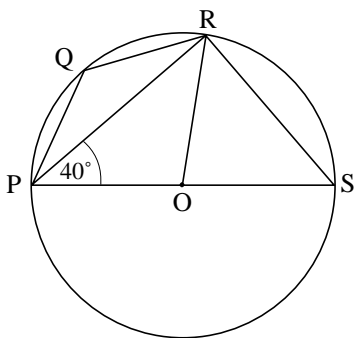
$$\hat{A}CB = 41^\circ$$

and  $\hat{B}DC = 27^\circ$ .

Find  $\hat{A}BD$ .



11.



In the diagram, O is the centre of the circle and  $\hat{R}PS = 40^\circ$ .

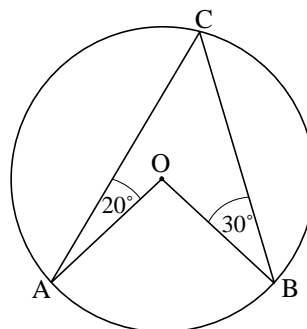
Calculate  $\hat{P}QR$  and  $\hat{O}RS$ .

12. In the figure, O is the centre of the circle, ABC.

Given that

$$\hat{C}AO = 20^\circ \text{ and } \hat{C}BO = 30^\circ,$$

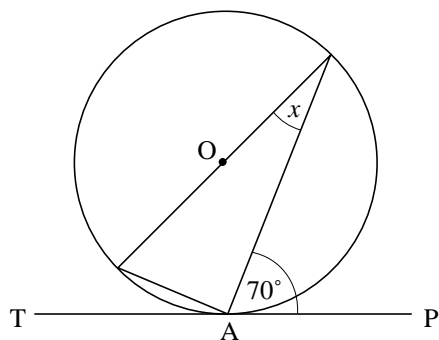
find  $\hat{A}CB$ .



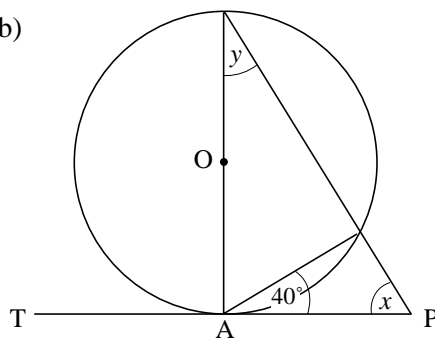
## 3.10 Circles and Tangents

1. Given that PAT is a tangent to the circle with centre O, find the values of x, y and z.

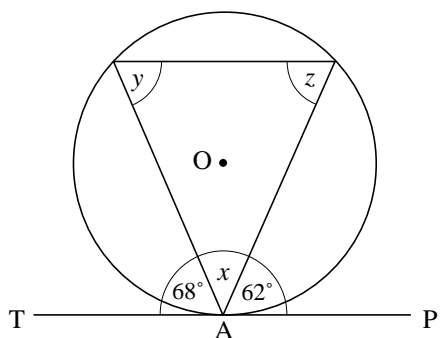
(a)



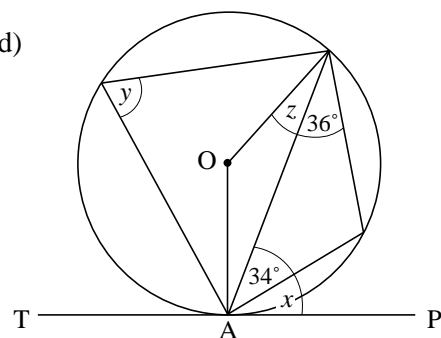
(b)



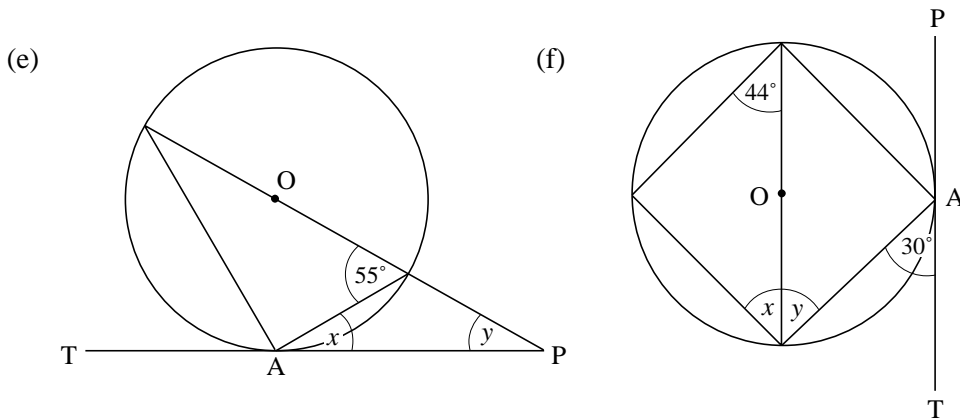
(c)



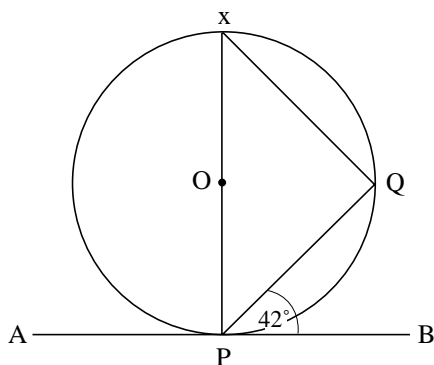
(d)



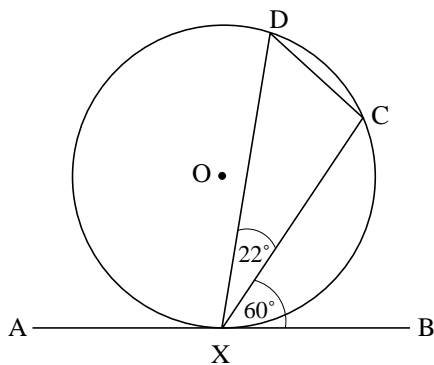




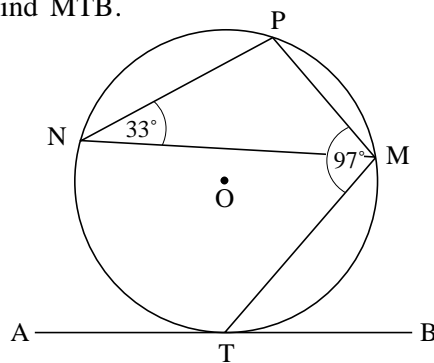
2. In the diagram, AB is the tangent to the circle at P and PX is a diameter. Given that  $\hat{BPQ} = 42^\circ$ , find  $\hat{PQX}$ ,  $\hat{PXQ}$  and  $\hat{XPQ}$ .



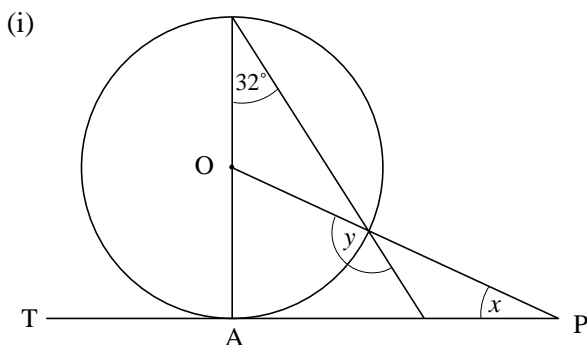
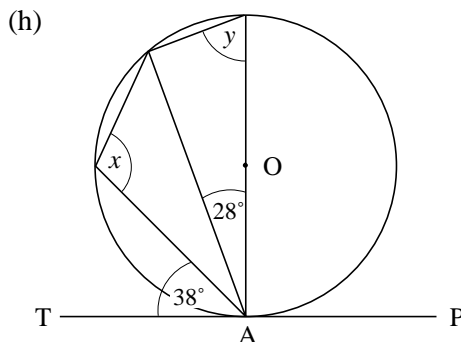
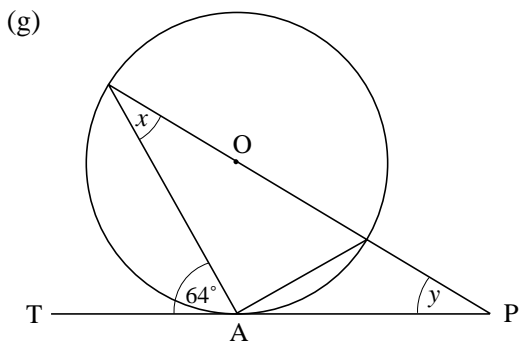
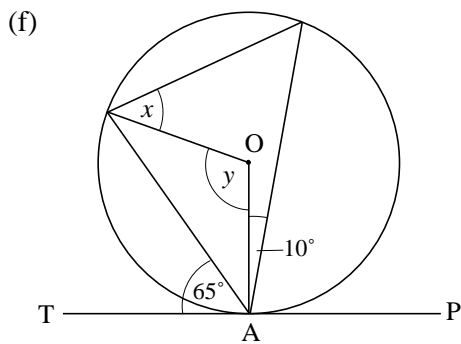
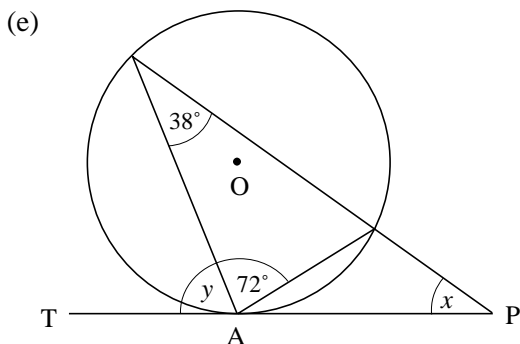
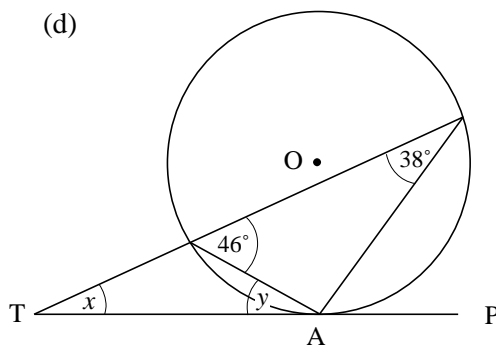
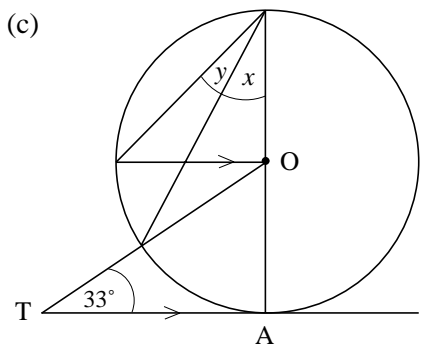
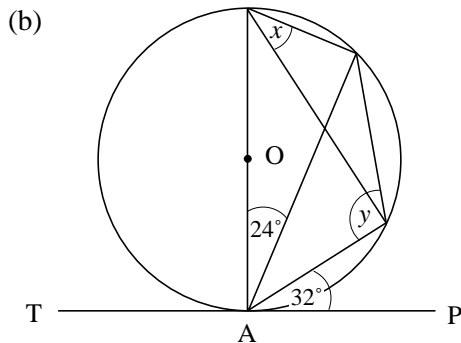
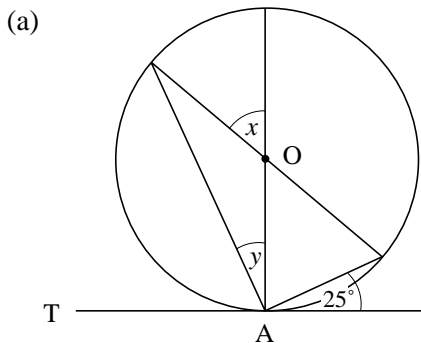
3. In the diagram, O is the centre of the circle. AB is the tangent to the circle at X,  $\hat{CXB} = 60^\circ$  and  $\hat{CXD} = 22^\circ$ . What is the size of  $\hat{XCD}$ ?



4. In the diagram, ATB is the tangent to the circle at point T. Given that  $\hat{PNM} = 30^\circ$  and  $\hat{TMP} = 97^\circ$ , find  $\hat{MTB}$ .

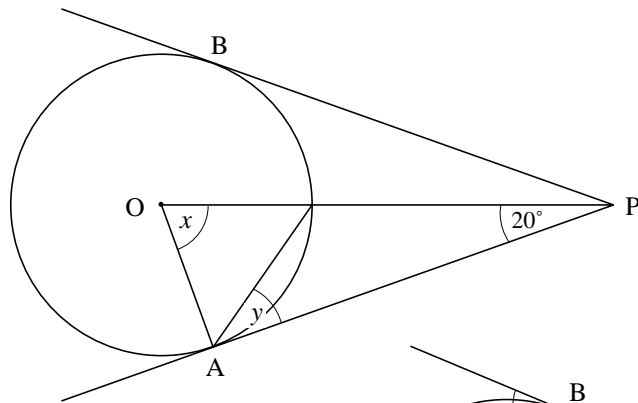


5. Given that  $PAT$  is a tangent at  $A$  to the circle with the centre  $O$ , find the value of  $x$  and of  $y$  in each case.

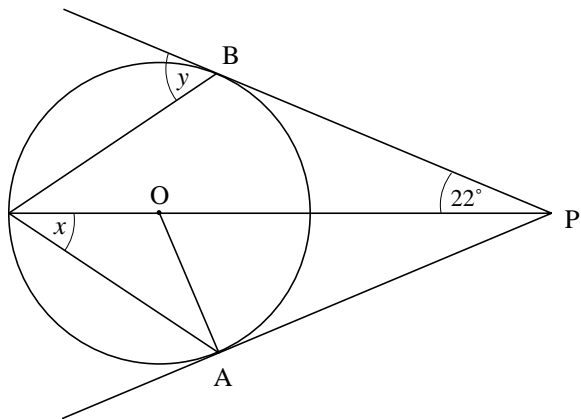


6. Given that PA and PB are tangents to the circle with centre O, find the value of  $x$  and of  $y$  in each case.

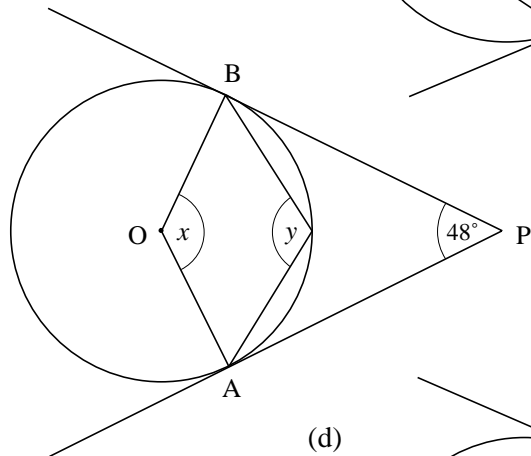
(a)



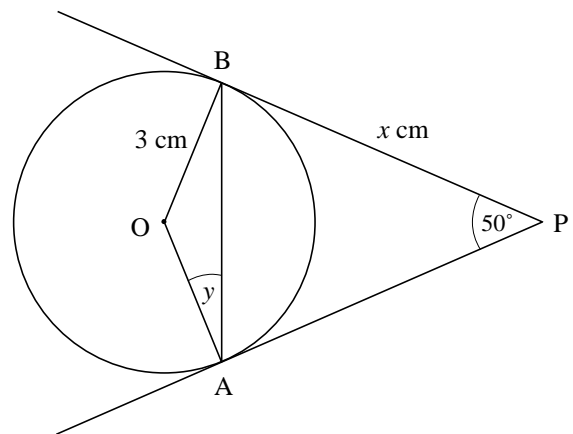
(b)



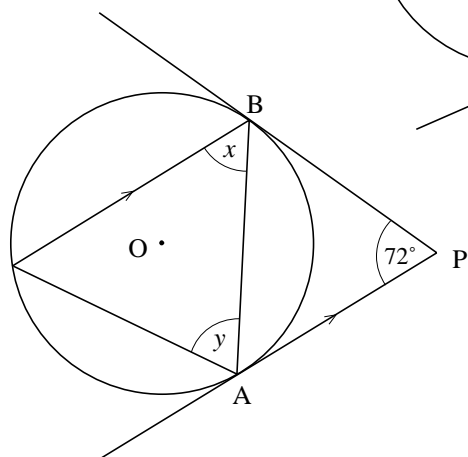
(c)

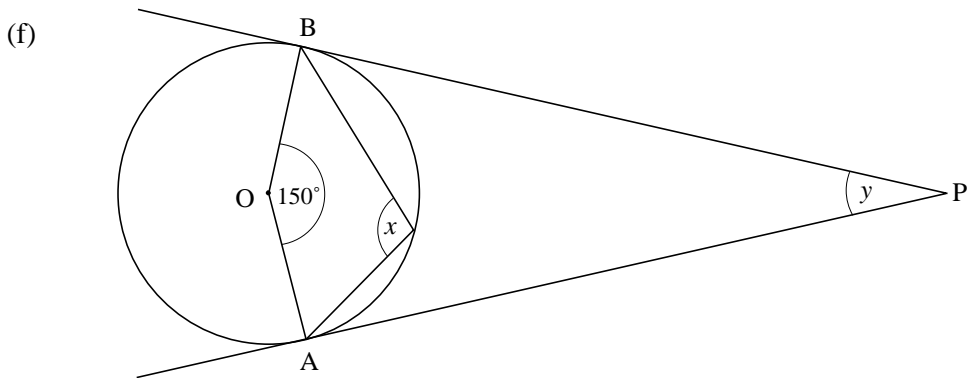


(d)

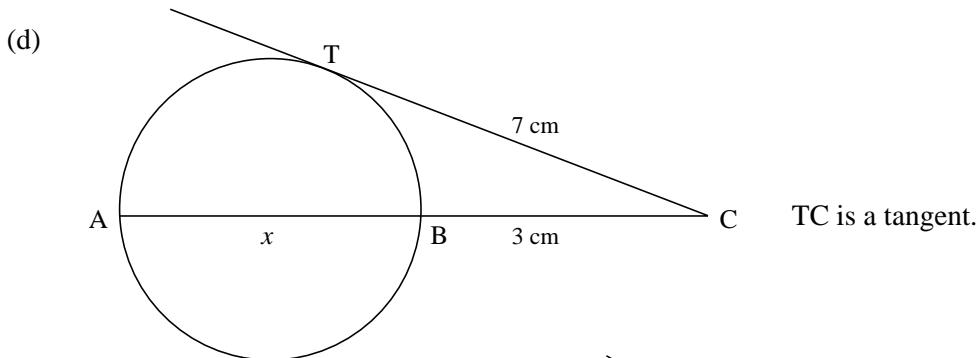
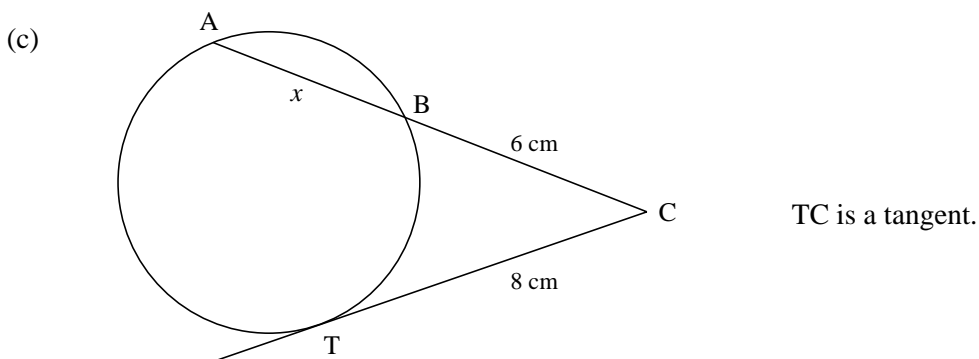
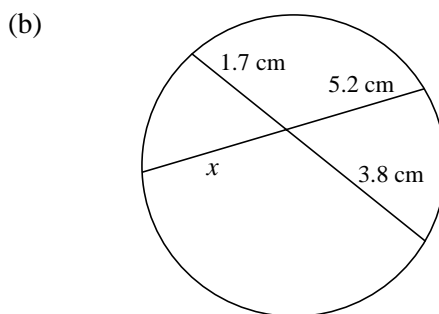
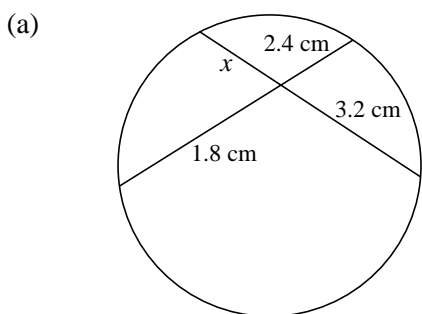


(e)





7. Find the length  $x$  in each case.



8. If  $BP = 8$  cm,  $DC = 7$  cm and  $CP = 9$  cm, calculate the lengths of

- (a) chord AB
- (b) tangent PT.

