19 Vectors

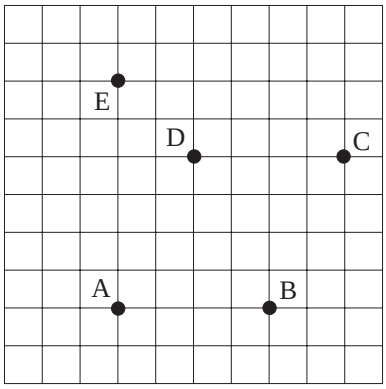
19.1 Vectors and Scalars

1. Which of the following are vectors and which are scalars?

   (a) Speed (b) Acceleration (c) Mass

   (d) Velocity (e) Weight (f) Time

2. Use the points in the grid below to find the vectors



   (i) (a) $\vec{AB}$ (b) $\vec{BC}$ (c) $\vec{AC}$

   (d) $\vec{CD}$ (e) $\vec{DA}$ (f) $\vec{DE}$

   (g) $\vec{AE}$ (h) $\vec{DB}$ (i) $\vec{EC}$

   (ii) What is the relationship between

   $\vec{BC}$ and $\vec{AD}$

   (iii) What is

   $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA}$

   equal to?

3. Plot the positions of the points, A, B, C, D, E and F relative to O if

   (a) $\vec{OA} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (b) $\vec{OB} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (c) $\vec{BC} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

   (d) $\vec{AD} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ (e) $\vec{EB} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ (f) $\vec{EF} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

4. If $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, find

   (a) $\mathbf{a} + \mathbf{b}$ (b) $\mathbf{a} - \mathbf{b}$ (c) $\mathbf{a} + \mathbf{c}$

   (d) $2\mathbf{a}$ (e) $3\mathbf{a} - 2\mathbf{b}$ (f) $\mathbf{a} + \mathbf{b} + \mathbf{c}$

   (g) $2\mathbf{b} + 3\mathbf{c}$ (h) $-3\mathbf{a}$ (i) $4\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$

5. If $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, solve each of the equations below to find the unknown vector, $\mathbf{x}$.

(a)	$\mathbf{a} + \mathbf{x} = \mathbf{b}$	(b)	$\mathbf{x} - \mathbf{c} = \mathbf{a}$	(c)	$\mathbf{x} + \mathbf{b} = \mathbf{a}$
(d)	$2\mathbf{x} + \mathbf{a} = \mathbf{c}$	(e)	$2\mathbf{b} - \mathbf{x} = 3\mathbf{c}$	(f)	$2\mathbf{a} + \mathbf{x} = \mathbf{b} + \mathbf{c}$
(g)	$2\mathbf{b} + \mathbf{c} + \mathbf{x} = \mathbf{o}$	(h)	$3\mathbf{a} + 2\mathbf{x} = \mathbf{b}$	(i)	$4\mathbf{a} - \mathbf{x} = 2\mathbf{c}$

19.2 Applications of Vectors

1. The water in a river flows at a speed of 1 m/s.

   Ann swims at a speed of 2 m/s at right angles to the bank.

   The river is 10 m wide. Find

   (a) the time taken to reach the other side of the river,

   (b) the distance she travels downstream.

2. (a) An aeroplane travels NW at a speed of 400 m/s in still air. If there is a wind of speed 50 m/s heading directly west, find the resultant speed and direction of travel of the plane.

   (b) If the wind continues to head west at a speed of 50 m/s, find the required speed and direction if the resultant speed is 400 m/s in a NW direction.

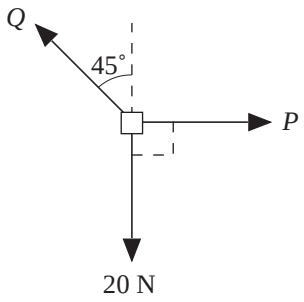
3. A boy rows at 4 m/s across a river of width 27 m. The river flows at a speed of 2.4 m/s. Find the direction in which the boy must row if he is to land on the other side directly opposite his starting point. Also find the time it takes for him to reach the other side.

4. A small block is held in position by 2 cables that exert forces of magnitude P and Q.

   Gravity exert a downward force of 20 N.

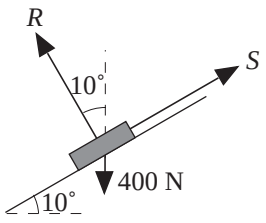
   The block remains at rest.

   Find P and Q by using trigonometry.



5. The diagram shows the forces acting on a car, at rest, on a slope.

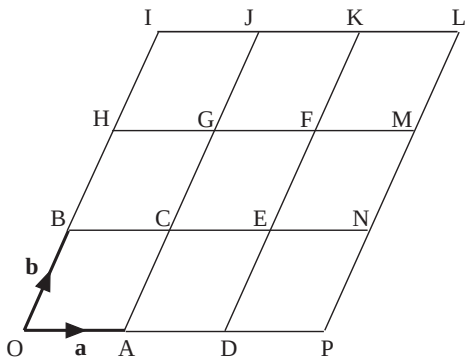
   Find the magnitude of R and S.



19.3 Vectors and Geometry

1. The diagram shows a grid made up of sets of equally spaced parallel lines.

The vectors $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$ are shown. Write each of the following in terms of **a** and **b**.



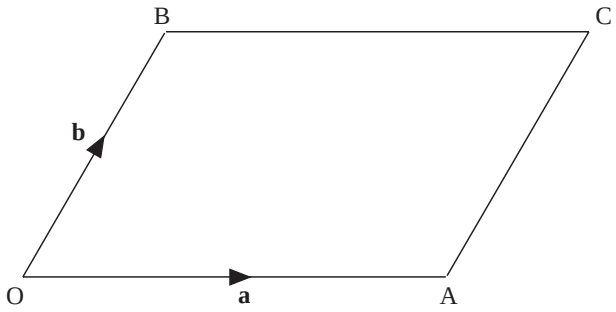
(a) $\vec{OD}$ (b) $\vec{OI}$ (c) $\vec{CE}$

(d) $\vec{GM}$ (e) $\vec{CJ}$ (f) $\vec{MN}$

(g) $\vec{BK}$ (h) $\vec{DC}$ (i) $\vec{FI}$

(j) $\vec{CL}$ (k) $\vec{PI}$ (l) $\vec{EH}$

2. The diagram shows the parallelogram OABC in which $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$.



(a) Write the following vectors in terms of **a** and **b**.

(i) $\vec{BC}$ (ii) $\vec{CA}$ (iii) $\vec{AB}$ (iv) $\vec{OC}$

(b) X is the midpoint of OA and Y is the midpoint of OB.

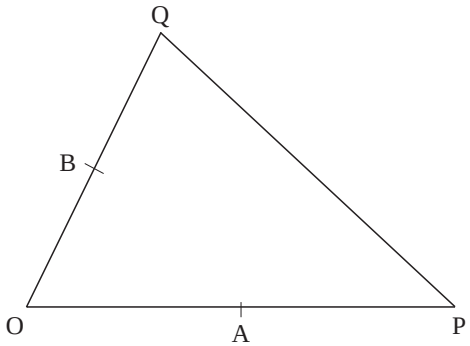
Find, in terms of **a** and **b**,

(i) $\vec{OX}$ (ii) $\vec{OY}$ (iii) $\vec{XY}$

What is the relationship between $\vec{XY}$ and $\vec{AB}$?

3. In the diagram $\vec{OA} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

The points A and B are the mid-points of OP and OQ respectively.



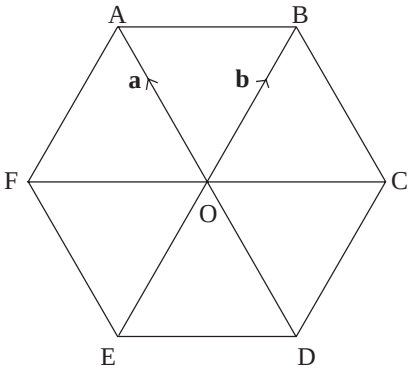
(a) Calculate the vector $\vec{OP}$.

(b) Calculate the vectors $\vec{AB}$ and $\vec{PQ}$

(c) Use your answers to part (b) to give two facts about the lines AB and PQ.

(SEG)

4.



ABCDEF is a regular hexagon

$$\vec{OA} = \mathbf{a}, \quad \vec{OB} = \mathbf{b}$$

(a) Write down, in terms of **a** and **b**, the vectors

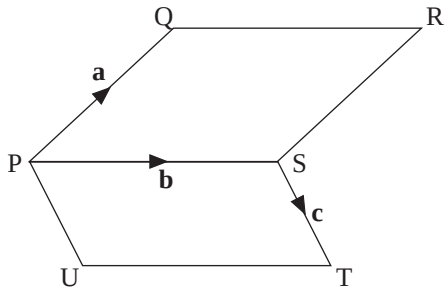
(i) $\vec{AB}$

(ii) $\vec{FC}$

(b) Write down one geometrical fact about AB and FC which could be deduced from your answers to part (a).

(LON)

5.



PQRS and PSTU are parallelograms.

$\vec{PQ}$ is **a**, $\vec{PS}$ is **b**, $\vec{ST}$ is **c**.

Find, in terms of **a**, **b** and **c** expressions in their simplest forms for

(a) $\vec{PT}$

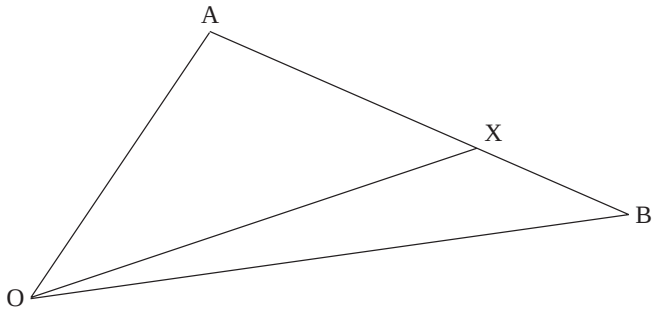
(b) $\vec{US}$

(c) $\vec{PX}$, where X is the midpoint of QT.

(d) $\frac{1}{2}\left(\vec{PQ} + \vec{PT}\right)$

(LON)

6.



In the diagram X is the point on AB such that $AX = 3XB$.

Given that $\vec{OA} = 8\mathbf{a}$ and $\vec{OB} = 4\mathbf{b}$, express in terms of **a** and/or **b**

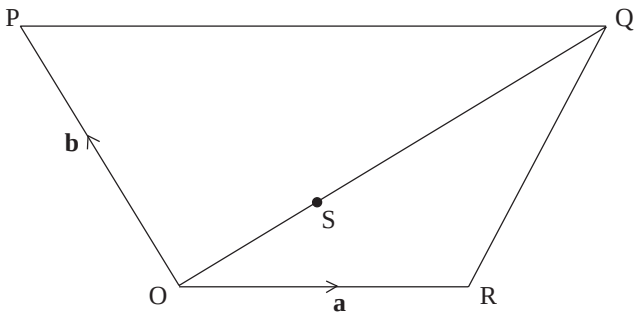
(a) (i) $\vec{AB}$

(ii) $\vec{AX}$

(b) $\vec{OX}$

(MEG)

7.



The diagram shows the position of points O, P, Q and R with vectors **a** and **b** acting along OR and OP, respectively.

$\vec{OR} = \mathbf{a}$

$\vec{OP} = \mathbf{b}$

$\vec{OS} = \frac{1}{3}\vec{OQ}$

$\vec{PQ} = 2\vec{OR}$

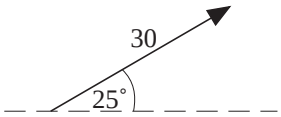
By expressing $\vec{PS}$ and $\vec{RS}$ in terms of the vectors **a** and **b** find the ratio PS : SR and explain the relationship between the points P, S and R.

(SEG)

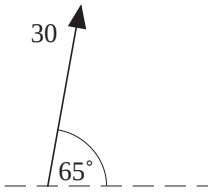
19.4 Further Work with Vectors

1. Write each vector shown below in component form, using components parallel and perpendicular to the dashed line.

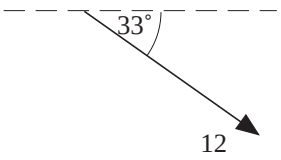
(a)



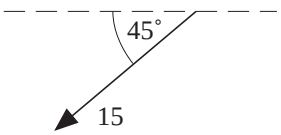
(b)



(c)



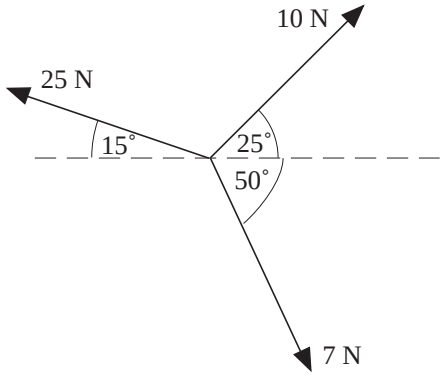
(d)



2. For each column vector below, find its magnitude and draw a diagram to show its direction, indicating the size of the relevant angles.

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (d) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

3. The three forces shown in the diagram act at a point. Find the magnitude and direction of its resultant.



4. The forces shown in the diagram are in equilibrium, so that their resultant is the zero vector.

Find F and the angle θ.