## 18 3-D Geometry

### 18.1 Using Pythagoras' Theorem and Trigonometry in Three Dimensions

1. Find the length of the longest rod that could be placed in each box shown below.
(a)

(b)

(c)

(d)

2. A square-based pyramid is made up of a square and four isosceles triangle with sides of lengths 6,6 and 4 cm . Find the height of the pyramid.
3. The figure shows a triangle ABC , right-angled at B and lying in horizontal plane. P is a point vertically above C . Given that $\mathrm{AB}=7 \mathrm{~cm}, \mathrm{BC}=6 \mathrm{~cm}$ and $\mathrm{AP}=11 \mathrm{~cm}$, calculate
(a) the length of PC,
(b) PA C ,
(c) The angle of elevation of P from B .

4. This shape is a cube with $\mathrm{OG}=\mathrm{OE}=\mathrm{OA}=2$. O is the origin.
(a) Write down the three-dimensional coordinates of point F .
(b) Calculate the distance AC.

5. A rectangular box has a horizontal base EFGH.

The corner D is vertically above H .
Given that $\mathrm{DH}=4 \mathrm{~cm}, \mathrm{HG}=6 \mathrm{~cm}$ and $\mathrm{GF}=8 \mathrm{~cm}$, calculate
(a) $\mathrm{D} \hat{\mathrm{G}} \mathrm{H}$,
(b) the length of HF,
(c) $\mathrm{D} \hat{\mathrm{F}} \mathrm{H}$.

6. VABCD is a pyramid of height 10 cm . Its base is a rectangle with $\mathrm{AB}=6 \mathrm{~cm}$ and $\mathrm{BC}=8 \mathrm{~cm} . \mathrm{V}$ is vertically above $O$, the point of intersection of the diagonals AC and BD. Find
(a) the length of VA,
(b) VÂO.

7. The diagram represents a rectangular box.

Given that $\mathrm{AB}=6 \mathrm{~cm}, \mathrm{BC}=3 \mathrm{~cm}$ and $\mathrm{AE}=2 \mathrm{~cm}$, calculate the length of the diagonal AG.

8. ABCDV is a right square-based pyramid. $M$ is the centre of the square base $A B C D$.
The $(x, y, z)$ coordinates of $\mathrm{A}, \mathrm{B}$, and C are shown on the diagram.
(a) (i) Write down the length of AB .
(ii) Write down the coordinates of D.
(b) Calculate the coordinates of M.
(c) The $z$ coordinate of V is 9 .

What is the height of the pyramid?

(NEAB)

### 18.2 Angles and Planes

1. The cube shown in the figure has edges of length 20 cm . M is the mid-point of AP. Calculate
(a) the length of CM ,
(b) the angle CMR,
(c) $\mathrm{M} \hat{S} P$.

2. The diagram shows a rectangular box in which $\mathrm{AB}=3 \mathrm{~cm}$, $\mathrm{AD}=4 \mathrm{~cm}, \mathrm{BD}=5 \mathrm{~cm}$ and $\mathrm{DH}=12 \mathrm{~cm}$.
Calculate the length of the straight line BH and
(a) BD C
(b) $B \hat{H} C$
(c) $\mathrm{H} \hat{\mathrm{BD}}$

3. The diagram shows a rectangular box which has a horizontal base EFGH where $\mathrm{HG}=15 \mathrm{~cm}$, $\mathrm{GF}=8 \mathrm{~cm}$ and $\mathrm{BF}=7 \mathrm{~cm}$.
$X$ is a point on $A B$ such that $X B=4 \mathrm{~cm}$.
Calculate the angles CEG and GXF.

4. The diagram shows a right triangular prism with $\mathrm{ABP}=90^{\circ}$ and ABCD lying on a horizontal table. If $\mathrm{AB}=6 \mathrm{~cm}, \mathrm{AD}=8 \mathrm{~cm}$ and $\mathrm{AP}=12 \mathrm{~cm}$, calculate
(a) $\mathrm{PA} B$,
(b) the length of PB ,

(c) P̂BB.
5. The diagram shows a right pyramid on a horizontal rectangular base ABCD . Given that $\mathrm{AB}=12 \mathrm{~cm}$, $\mathrm{BC}=16 \mathrm{~cm}$ and $\mathrm{VA}=26 \mathrm{~cm}$, calculate
(a) the length of AX where X is the mid-point of AC,
(b) the vertical height, VX, of the pyramid,
(c) the angle AVC,
(d) the length of VP where P is the mid-point of BC.

6. 



The diagram shows a triangular prism.
The two triangular faces PSY and QRX are vertical. Two of the three rectangular faces PQXY and SRXY are at right angles, i.e. $R \hat{X} Q=90^{\circ}$, while the face PQRS is horizontal.

Given that the angle between the faces $S R X Y$ and $P Q R S$, i.e. $X \hat{R} Q$, is $58^{\circ}$, $\mathrm{XHR}=90^{\circ}, \mathrm{RX}=15 \mathrm{~cm}$ and $\mathrm{PQ}=40 \mathrm{~cm}$, calculate
(a) QX ,
(b) $\mathrm{X} \hat{\mathrm{P}} \mathrm{H}$.
7. ABCDEF is a triangular prism, 10 cm long.

ABC is an equilateral triangle of side 3 cm .
P is the foot of the perpendicular from C to AB .
(a) Calculate the length of PD.

(b) Calculate the size of the angle between CE and PE.
(SEG)

