16 Inequalities

16.2 Solutions of Linear Inequalities

1. Solve each inequality below and illustrate your solution on a number line.

   (a) $2x + 3 \le 5$ (b) $3x - 4 > 11$ (c) $5x + 3 > 28$

   (d) $5 - 2x \ge 11$ (e) $\frac{3x - 5}{2} < 2$ (f) $3(4x + 1) \ge -9$

2. Solve the following inequalities.

   (a) $3x - 4 < 26$ (b) $6 - 4x > 18$ (c) $7x - 2 \le 12$

   (d) $5x + 7 > -13$ (e) $\frac{1 + 2x}{5} > 3$ (f) $\frac{4 - 5x}{2} \le 7$

3. Solve each of the following inequalities and illustrate each solution on a number line.

   (a) $9 \le 2x - 1 \le 15$ (b) $5 \le 3x + 14 \le 29$

   (c) $13 \le 5 - 4x < 25$ (d) $-2 \le 2x + 1 \le 5$

4. (a) Solve the inequality

   $$7x + 3 > 2x - 15.$$

   (b) Solve the inequality

   $$2(3x - 2) < 11.$$

   (SEG)

5. Find all integer values of *n* which satisfy the inequality

   $$1 \le 2n - 5 < 10.$$

   (SEG)

6. Solve the following inequalities for *x*.

   (a) $1 + 3x < 7$ (b) $4x - 3 > 3x - 2$

   (NEAB)

7. (a) List all the integer values of *n* for which $-4 < n + 1 \le 2$.

   (b) Solve the inequality

   $$3x + 5 < 1 - 2x.$$

   (SEG)

8. *x* is a whole number such that $-5 \le x < 2$.

   (a) (i) Write down all the possible values of *x*.

   (ii) *y* is a whole number such that $-3 < y \le -1$. Write down the greatest possible value of *xy*.

(b) Solve $5n + 6 < 23$.

(NEAB)

9. (a) A sequence is generated as shown.

Term	1st	2nd	3rd	4th	5th
Sequence	3	5	7	9	11

What is the nth term in the sequence?

(b) Another sequence is generated as shown.

Term	1st	2nd	3rd	4th
Sequence	4	7	12	19

What is the nth term in the sequence?

(c) The nth term of a different sequence is $5n + 7$.

Solve the inequality $5n + 7 < 82$.

(SEG)

16.3 Inequalities Involving Quadratic Terms

1. Illustrate the solutions to the following inequalities on a number line.

(a) $x^2 \leq 4$ (b) $x^2 \geq 1$ (c) $x^2 \geq 9$

(d) $x^2 < 36$ (e) $x^2 \leq 2.25$ (f) $x^2 > 0.25$

2. Find the solutions of the following inequalities.

(a) $x^2 + 5 \leq 6$ (b) $2x^2 - 5 \geq 27$ (c) $5x^2 - 4 \leq 16$

(d) $9x^2 \leq 1$ (e) $4x^2 \geq 25$ (f) $16x^2 - 12 \geq 13$

(g) $2(x^2 - 4) < 10$ (h) $\dfrac{x^2 - 3}{2} \geq 23$ (i) $20 - 2x^2 \leq 2$

3. Find the solutions of the following inequalities.

(a) $(x - 1)(x - 2) \geq 0$ (b) $(x + 2)(x - 3) \leq 0$

(c) $(x - 1)(x - 2) < 0$ (d) $(x + 5)(x - 4) > 0$

(e) $x(x + 5) \geq 0$ (f) $(x - 1)x < 0$

4. By factorising, solve each of the following inequalities.

(a) $x^2 + x - 2 \geq 0$ (b) $x^2 - 5x + 6 \leq 0$

(c) $x^2 - 4x < 0$ (d) $2x^2 + 3x - 2 > 0$

(e) $x^2 + 6x + 8 \leq 0$ (f) $5x^2 - 15x \geq 0$

(g) $6x - 2x^2 > 0$ (h) $1 - 5x - 6x^2 \leq 0$

5. The area, A, in cm^2, of a square satisfies the inequality $9 \leq A \leq 36$.

What is the:

(a) maximum (b) minimum

possible length of its sides?

6. (a) Factorise completely $14n - 4n^2$.

(b) Find the integer values of n for which $14n - 4n^2 > 0$.

(MEG)

16.4 Graphical Approach to Inequalities

1. Illustrate on a set of coordinate axes each of the following inequalities.

(a) $y \leq x$ (b) $y > x + 1$ (c) $y < x - 2$

(d) $y \leq x + 4$ (e) $y > 3 - 2x$ (f) $y \leq 3x - 3$

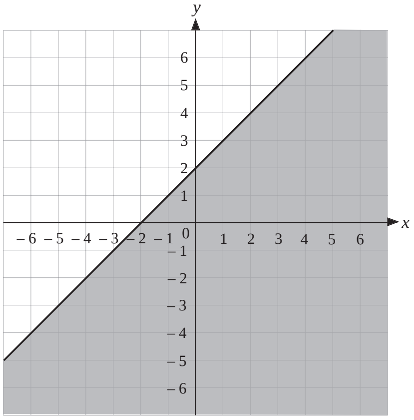
(g) $2x + y \geq 4$ (h) $x - y \geq 2$ (i) $x + 2y < 3$

2. For each region below, find:

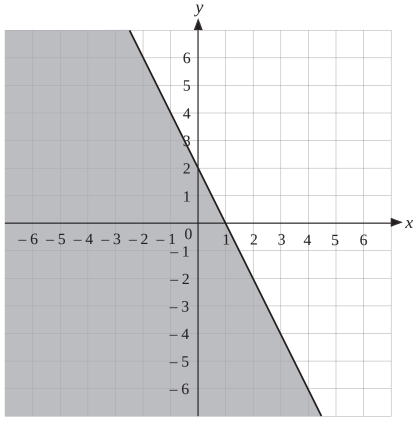
(i) the equation of the line which forms the boundary

(ii) the inequality represented by the shaded region.

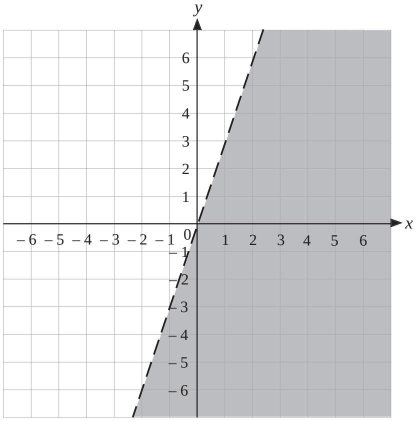
(a)



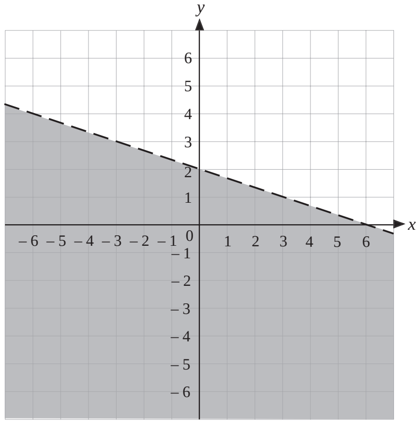
(b)



(c)



(d)



3. On the same set of axes, shade the regions

$$x + y \geq 1, \quad x - y \leq 2.$$

Indicate the region satisfied by both inequalities.

4. Shade the region which satisfies

$$2 \leq x + y \leq 4.$$

5. Shade the region which satisfies

$$-1 \leq 2x + y < 2.$$

16.5 Dealing with More than One Inequality

1. On a suitable set of axes, show by shading the region which satisfies both the inequalities.

(a) $x \geq 2$, $x \leq 4$ (b) $x > 1$, $y \leq 2$ (c) $y \geq x$, $4 \geq x$

(d) $x + y \leq 1$, $y > 2$ (e) $2x + y > 2$, $2x + y < 1$ (f) $x \leq y$, $y \leq 1$

(g) $y \geq 3x$, $x + y < 1$ (h) $y \geq x$, $y \leq 2x$ (i) $y \geq x$, $y \leq x + 2$

2. For each set of inequalities, draw graphs to show the region satisfied by them.

(a) $x \leq 2, \quad x \geq 1, \quad y \geq 4, \quad y \leq 6$

(b) $x \geq -1, \quad x \leq 3, \quad y \leq 2, \quad y \geq -3$

(c) $x \geq 1, \quad y \geq 1, \quad x + y \leq 3$

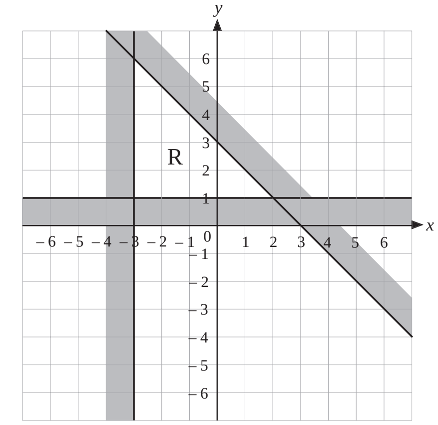
(d) $x - y < 3, \quad x \geq 2, \quad y \leq 2$

(e) $y \leq 2x, \quad y \geq x, \quad x \leq 3$

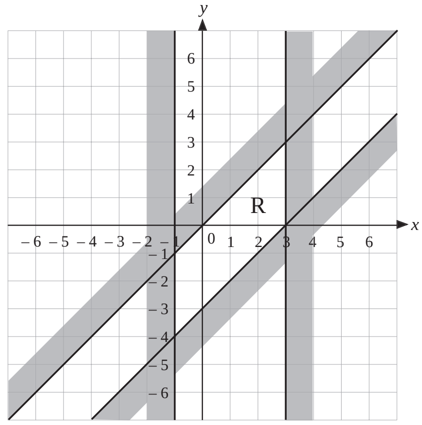
(f) $x + y \geq 2, \quad y \leq x + 2, \quad x \leq 2$

3. Find the inequalities which define each of the regions indicated by the letter R.

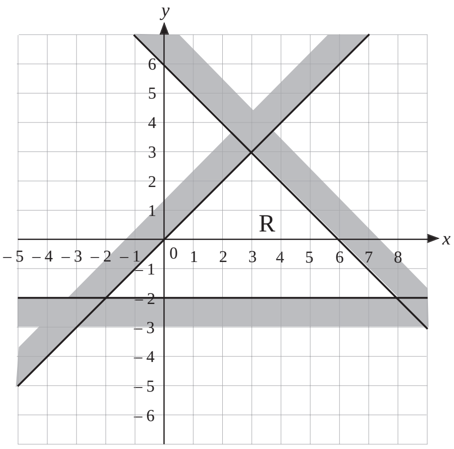
(a)



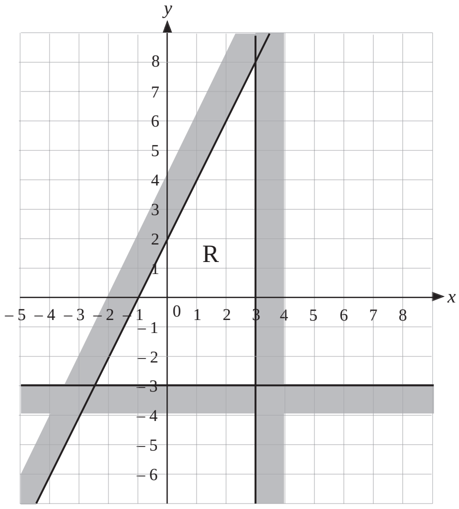
(b)



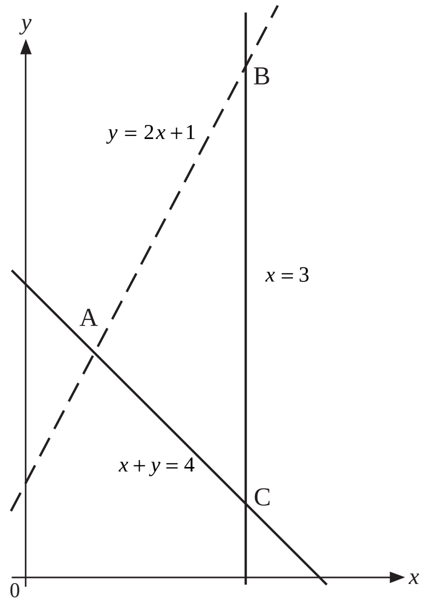
(c)



(d)



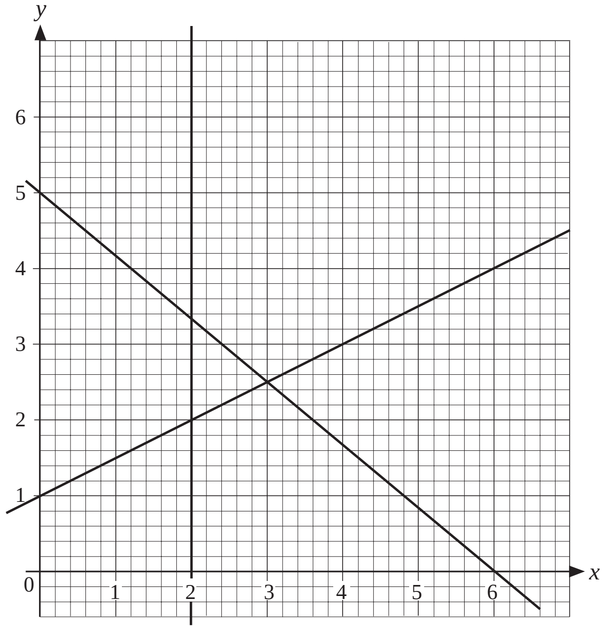
4. Write down the three inequalities which define the triangular region ABC.



(MEG)

5. The diagram shows the graphs of

$$y = \frac{1}{2}x + 1, \quad 5x + 6y = 30 \text{ and } x = 2.$$

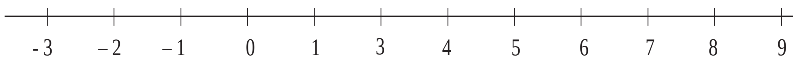


(a) On the diagram, shade, and label with the letter R, the region for which the points (x, y) satisfy the three inequalities

$$y \le \frac{1}{2}x + 1, \quad 5x + 6y \le 30 \text{ and } x \ge 2.$$

(b) (i) Solve the inequality $\frac{1}{2}x + 1 < 3$.

(ii) Represent your answer to part (b) (i) on a copy of this number line.

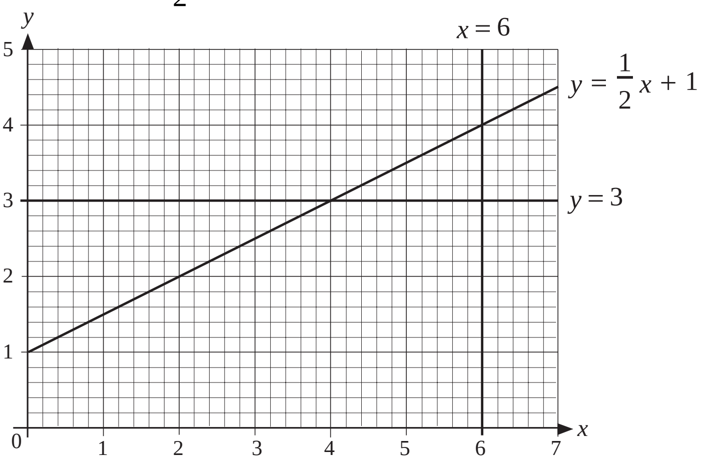


(MEG)

6. (a) Solve the inequality $7x + 3 > 2x - 15$.

(b) Copy the diagram below and label with the letter R the single region which satisfies all of these inequalities:

$$y < \frac{1}{2}x + 1, \quad x > 6, \quad y > 3.$$



(SEG)

7. (a) Using x and y axes from -5 to 5, show the region which satisfies all the inequalities

$$2y \leq x + 2, \quad y \geq 1 - x, \quad y \geq x - 1.$$

Label this region R.

(b) Write down the coordinates of any point (x, y) which has whole number values for x and y and which lies inside the region R.

(SEG)

8. A contractor hiring earth moving equipment has a choice of two machines.

Type A costs £50 per day to hire, needs one person to operate it, and can move 30 tonnes of earth per day.

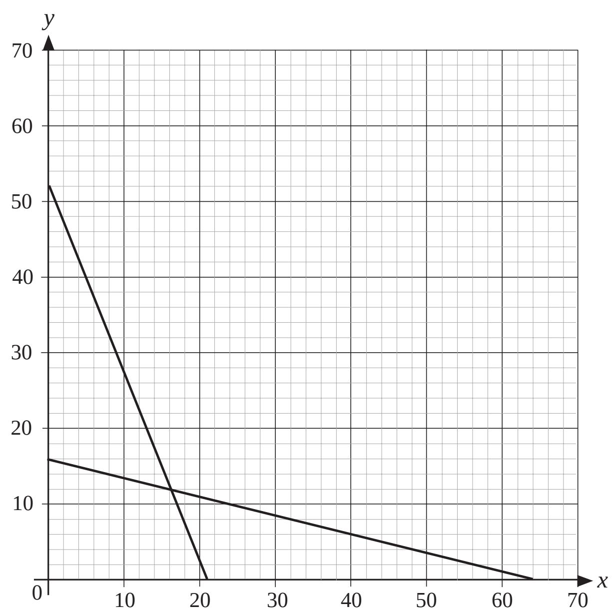
Type B costs £20 per day to hire, needs four people to operate it and can move 70 tonnes of earth per day.

Let x denote the number of *Type A* machines hired and y the number of *Type B* machines hired.

(a) The contractor has a labour force of 64 people. Explain why $x + 4y \leq 64$.

(b) The contractor can spend up to £1040 per day on hiring machines.
Explain why $5x + 2y \leq 104$.

(c) The lines $x + 4y = 64$, $5x + 2y = 104$, $x = 0$ and $y = 0$ are shown on the axes below.



By shading, identify the feasible region:

$$x \geq 0, \quad y \geq 0, \quad x + 4y \leq 64, \quad 5x + 2y \leq 104.$$

(d) The total weight of earth moved is given by $w = 30x + 70y$.

Use your graph to find the values of x and y which satisfy all the inequalities and give a maximum value to w.

(SEG)