

15 Variation

15.5 Direct Proportion

1. Use the data given to check whether or not it agrees with the statement given.

(a)

x	1	3	7	11	$y \propto x$
y	4	12	28	44	

(b)

q	0	1	2	3	$p \propto q$
p	0	$\frac{1}{2}$	1	$\frac{3}{2}$	

(c)

x	1	5	10	$y \propto x$
y	0.1	0.5	1	

(d)

t	0.1	1	2	$s \propto t$
s	0.5	5	10	

2. Copy and complete each table using the statement given.

(a) $y \propto x$

x	1	4	9
y	5	?	?

(b) $s \propto t$

t	$\frac{1}{2}$	1	3	5
s	?	2	?	?

(c) $q \propto p$

p	1	3	5	9
q	?	9	?	?

(d) $y \propto x$

x	$\frac{1}{2}$	1	5
y	1	?	?

3. The yield Y of a tomato crop is directly proportional to the quantity of fertiliser F used. 5 kg of fertiliser produces 30 kg of tomatoes.

- Find the relationship between Y and F .
- What is the yield when 12 kg of fertiliser are used?
- How much fertiliser was used to produce a yield of 42 kg?

4. A spring stretches when a mass is attached to one end. The extension x is directly proportional to the magnitude of the mass, m . When a mass of 50 g is attached, the extension is 1 cm.

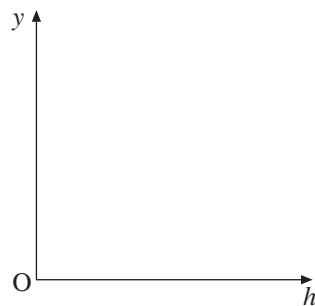
- Find the relationship between x and m .
- When a mass of 120 g is attached, what is the extension?
- What mass will produce an extension of 3.2 cm?

5. A launched rocket travels at constant acceleration.
Its speed $v \text{ ms}^{-1}$, is proportional to the time t , in seconds, since launch.
After 5 seconds its speed is 120 ms^{-1} .
- (a) Find a relationship between v and t
- (b) What is its speed when t equals
(i) 1 second (ii) 10 seconds (iii) 60 seconds?
- (c) How long will it be before its speed is $20\,000 \text{ ms}^{-1}$?
6. A ball is dropped to the floor from a height of h centimetres.
It bounces to a height of y centimetres.
 y is directly proportional to h .

- (a) Sketch a graph to show the relationship between y and h .

When $h = 120$, $y = 80$

- (b) Find y when $h = 150$.



(LON)

15.6 Inverse Proportion

1. For each table of values below, determine whether they agree with the relationship stated.

(a)	x	1	2	4	$y \propto \frac{1}{x}$
	y	12	6	3	

(b)	q	1	2	5	$p \propto \frac{1}{q}$
	p	2	1	0.5	

(c)	r	$\frac{1}{2}$	1	2	$s \propto \frac{1}{r}$
	s	2	1	$\frac{1}{2}$	

(d)	x	$\frac{1}{2}$	1	5	$y \propto \frac{1}{x}$
	y	10	5	1	

2. Copy and complete each of these tables to match the stated relationship.

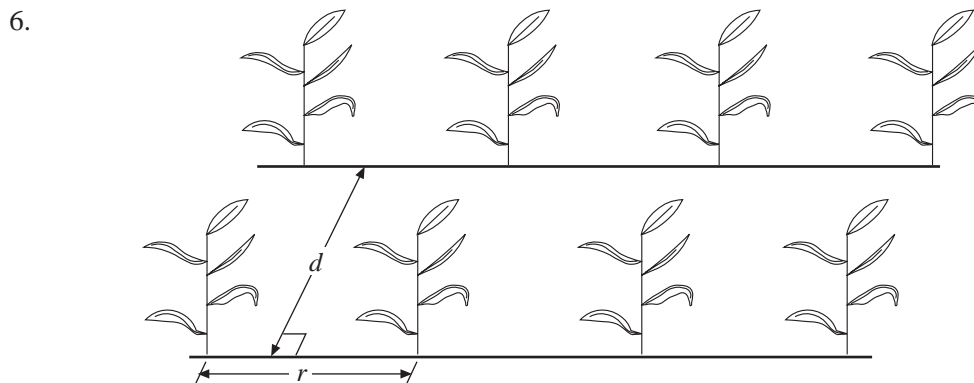
(a)	$y \propto \frac{1}{x}$	x	10	20	40
		y	2	?	?

(b)	$p \propto \frac{1}{q}$	q	1	2	8
		p	?	2	?

(c)	$s \propto \frac{1}{r}$	r	1	2	5
		s	5	?	?

(d)	$v \propto \frac{1}{u}$	u	10	20	100
		v	1	?	?

3. Two quantities, x and y , are such that y is inversely proportional to x . Also note that $y = 4$ when $x = 2$.
- Find the relationship between x and y .
 - What is the value of y when $x = 4$?
4. The value of a TV set is assumed to be inversely proportional to its age. When it is a year old it is sold for £400.
- What will its value be when it is 2 years old?
 - How many years old will it be when its value is first less than £100?
 - Is the assumption made here a reasonable one?
5. The value, v , of a train is assumed to be inversely proportional to its age, x . It was sold for £500 000 when it was 4 years old.
- Find the relationship between v and x .
 - What is its value when it is 10 years old?
 - How many years old is it when its value is first less than £100 000?



The diagram is taken from a book about growing maize.
 The distance between the rows of plants is d metres.
 The spacing between the plants in the rows is r metres.

The number, P , of plants per hectare is given by the formula $P = \frac{10000}{dr}$.

$d = 0.8$ and $r = 0.45$.

- (a) Calculate the value of P .
Give your answer to 2 significant figures.

The value of d is inversely proportional to the value of r and $d = 0.9$ when $r = 0.4$.

- (b) Calculate the value of r when $d = 1.2$.

(LON)

15.7 Functional and Graphical Representation

1. Write down the mathematical relationship between each pair of variables, using the information given.

- (a) y is proportional to x , and when $x = 2$, $y = 5$.
 (b) T is proportional to the square of x , and $T = 4$ when $x = 2$.
 (c) R is inversely proportional to the square of S , and $R = 2$ when $S = 1$.
 (d) q is proportional to the cube of p , and $q = 4$ when $p = 2$.

2. Express each of the following in words

(a) $y \propto \frac{1}{x^2}$ (b) $y \propto x^3$ (c) $y \propto \frac{1}{x^4}$

3. Copy and complete each of the tables below according to the given relationship.

(a) $y \propto x^2$

x	1	2	3	4
y	?	?	27	?

(b) $y \propto x^3$

x	1	3	6
y	?	9	?

(c) $y \propto \frac{1}{x^2}$

x	1	2	4
y	?	1	?

(d) $y \propto \frac{1}{x^3}$

x	1	2	4
y	?	1	?

4. The intensity of illumination, I , at a point varies inversely with the square of the distance, x , of the point from the light.
Express this in mathematical terms, and hence determine the ratio of the intensity of illumination produced by a light 8 m from the point, to the same light, 2 m from the point

(SEG)

5. When a stone is thrown upwards with an initial speed s metres per second, it reaches a maximum height, h metres.

Given that h varies directly as the square of s and that $h = 5$ when $s = 10$,

- (a) work out the formula connecting h and s ,
 (b) calculate the value of s when $h = 20$.

2 stones are thrown up. The ratio of their initial speeds is $3 : 1$.

- (c) Work out the ratio of the maximum heights achieved.

(LON)

15.8 Further Functional Representation

1. For each table of values below, determine they agree with the stated relationship.

(a)

x	1	4	16
y	$\frac{1}{4}$	$\frac{1}{2}$	1

 $y \propto x^{\frac{1}{2}}$

(b)

x	1	4	9
y	1	$\frac{1}{2}$	$\frac{1}{3}$

 $y \propto \frac{1}{x^2}$

(c)

x	1	25	100
y	0.2	25	400

 $y \propto x^{\frac{3}{2}}$

2. Copy and complete each table according to the given relationship.

(a) $y \propto x^{\frac{1}{2}}$

x	1	4	9
y	2	?	?

(b) $y \propto \frac{1}{x^2}$

x	1	4	9
y	?	2	?

(c) $y \propto x^{\frac{3}{2}}$

x	1	4	9	16
y	?	?	?	32

3. The increase in speed, V metres per second, at the lowest point of a 'Big Dipper' ride is proportional to the square root of the vertical height, h metres, dropped.

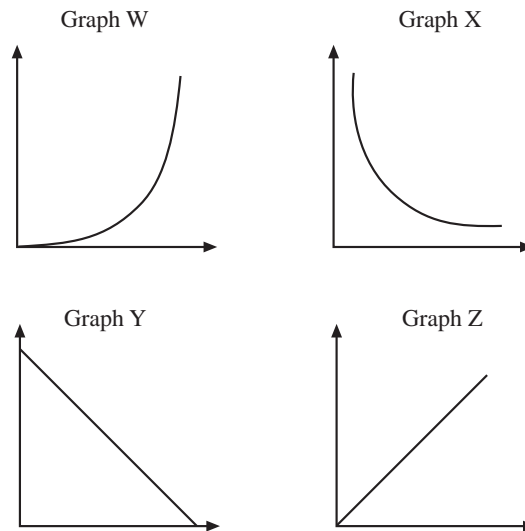
- (a) Write this relationship in mathematical terms.
 (b) Use this relationship to find the ratio of the speeds obtained from heights 100 metres and 25 metres.

(SEG)

4. T is directly proportional to the positive square root of M .
 $T = 32$ when $M = 16$.
- (a) Calculate T when M is 100.
 (b) Calculate M when T is 9.6.

(NEAB)

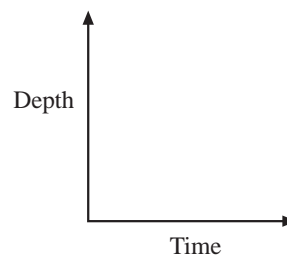
5. Decide which graph matches each relationship.

Graphs**Relationships**

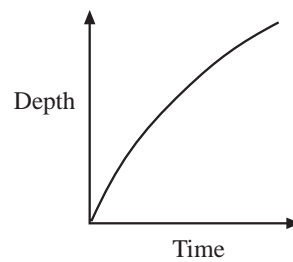
- A: The area of a circle plotted against its radius.
 B: The circumference of a circle plotted against its radius.
 C: The length of a rectangle of area 24 cm^2 plotted against its width.
- (a) Which graph matches relationship A?
 (b) Which graph matches relationship B?
 (c) Which graph matches relationship C?

(SEG)

6. (a) Water flows into a cylinder at a constant rate.
 Sketch the graph of the depth of water against time.



- (b) Water flows into another container at a constant rate.
Sketch the cross-section of the container that generated this graph.



- (c) The values of depth, d , against time, t , for a different container are shown in the table.

Time t (secs)	1	2	3	4	5	6	7	8	9	10
Depth d (cm)	0.1	0.4	0.9	1.6	2.5	3.6	4.9	6.4	8.1	10

Find the equation connecting t and d

(SEG)