12 Number Patterns

12.3 Extending Number Patterns

1. Find the 10th and 20th terms of each of the following sequences:

   (a) 3, 6, 9, 12, 15, . . .
   (b) 4, 8, 12, 16, 20, . . .
   (c) 100, 98, 96, 94, 92, . . .
   (d) 12, 9, 6, 3, 0, . . .
   (e) 2, 3, 5, 8, 12, . . .
   (f) 3, 4, 7, 12, 19, . . .
   (g) 1, 4, 9, 16, 25, . . .
   (h) 2, 4, 8, 16, 32, . . .
   (i) 10, 12, 16, 24, 40, . . .

2. Look at this number pattern:

Line 1	1	×	1	=	1
Line 2	11	×	11	=	121
Line 3	111	×	111	=	12321
Line 4	1111	×	1111	=	1234321
Line 5	11111	×	11111	=	123454321

   (a) Write down the complete Line 6 of this pattern.

   (b) Use the pattern to help you find the value of 111111111 × 111111111.

(MEG)

3. Ranjit is doing an investigation into powers. He begins to make a table as follows.

	Column 1	*Column 2*	*Column 3*	*Column 4*	. .	. .
Row 1	1	1	1	1	. .	. .
Row 2	2	4	8	16	. .	. .
Row 3	3	9	27	. .	. .	. .
Row 4	4	16	64	. .	. .	. .
Row 5	. .	. .	. .	. .	. .	. .
. .	. .	. .	. .	. .	. .	. .
. .	. .	. .	. .	. .	. .	. .

   (a) What is the 6th number in Row 3?

   (b) What is the 10th number in Column 2?

   (c) The number 49 appears in Column 2. In which row is it?

   (d) The number 6561 appears in Row 3. In which column is it?

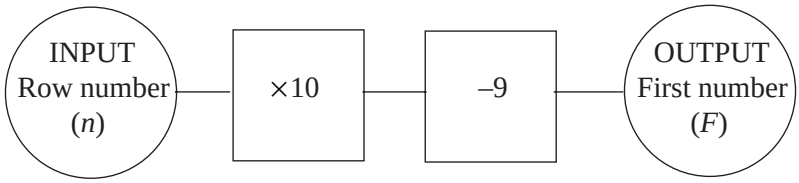
   (e) The number 576 appears in Column 2. In which row is it?

   (f) What is the 20th number in Row 2? *(SEG)*

4. The odd numbers are arranged in rows of five, as follows:

Row number (n)	*First number in row (F)*				*Last number in row (L)*
1	1	3	5	7	9
2	11	13	15	17	19
3	21	23	25	27	29
	and so on.				

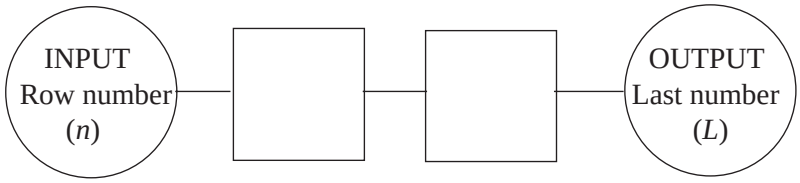
(a)



This number machine can be used to find the first number (F) in row n.

The machine uses the rule $F = 10n - 9$.

(i) Copy and complete the following number machine to give the last number (L) in row number n.



(ii) Write down the rule connecting L and n.

(b) The numbers in Row 1 add up to 25.

(i) Copy and complete the following table.

Row	1	2	3	4	5
Sum of numbers in the row	25				

(ii) Work out which row has a sum of 875.

(MEG)

12.4 Formulae and Number Patterns

1. Use the formulae below to find the first 6 terms of each sequence.

(a) $u_n = 1 + 2n$ (b) $u_n = 5n - 2$ (c) $u_n = 2n^2 - 1$

(d) $u_n = 3^n - 1$ (e) $u_n = (n + 1)^2$ (f) $u_n = (n + 2)(n - 3)$

2. Find (i) the 10th term and (ii) the 20th term of each sequence below:

(a) $u_n = 5n$ (b) $u_n = 3 + 4n$ (c) $u_n = 20 - 2n$

(d) $u_n = 5 + n^2$ (e) $u_n = n^2 + 4n + 4$ (f) $u_n = \dfrac{1}{(n+1)}$

3. Find the formula for u_n, the n th term, for each of the sequences given below:

(a) 4, 7, 10, 13, 16, . . . (b) 2, 6, 10, 14, 18, . . .

(c) 50, 43, 36, 29, 22, . . . (d) 5, 2, –1, –4, –7, . . .

(e) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ (f) 7, 15, 23, 31, 39, . . .

4. The n th term of each of the sequences below can be written in the form $u_n = an + b$. For each sequence, find the constants a and b.

(a) 3, 5, 7, 9, 11, . . . (b) 4, 3, 2, 1, 0, . . .

(c) 4, 11, 18, 25, 32, . . (d) 100, 95, 90, 85, 80, . . .

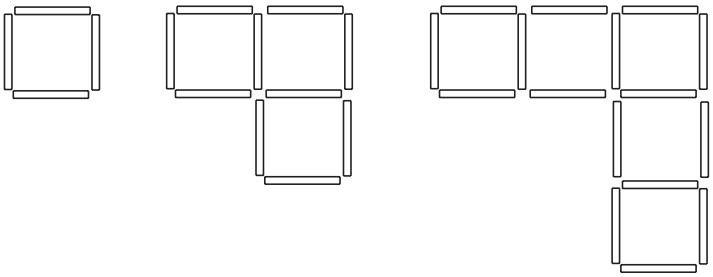
5. Write down the sequence (i) $u_n = n^2$ and (ii) $u_n = n^3$. Use them to find the formula for the n th of the following sequences.

(a) 2, 5, 10, 17, 26, . . . (b) 2, 8, 18, 32, . . .

(c) 2, 9, 28, 65, 126, . . . (d) –1, –7, –17, –31, . . .

(e) 2, 16, 54, 128, 250, . . . (f) 0, 4, 18, 48, 100, . . .

6. The three patterns below are made out of matchsticks.



Pattern 1 *Pattern 2* *Pattern 3*

(a) Draw the next pattern in the sequence.

(b) Copy and complete this table to show the number of matchsticks used for each pattern.

Pattern number	1	2	3	4	5	6
Number of matchsticks	4	10	16			

(c) How many matchsticks would be needed for the 20th pattern?
Show clearly how you worked out your answer.

(d) Write down an expression for the number of matchsticks in the n th pattern.

(NEAB)

12.5 General Laws

1. Find formulae to generate each of the sequences given below:

   (a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

   (b) $\frac{3}{1}, \frac{5}{2}, \frac{7}{4}, \frac{9}{8}, \frac{11}{16}, \ldots$

   (c) 3, 9, 27, 81, . . .

   (d) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

   (e) 0.8, 0.64, 0.512, 0.4096, . . .

   (f) $\frac{2}{3}, \frac{5}{4}, \frac{10}{5}, \frac{17}{6}, \frac{26}{7}, \ldots$

   Which of the above sequences converge? If it converges, find the value to which it converges.

2. The iterative formula $u_{n+1} = \frac{1}{2}\left(u_n + \frac{7}{u_n}\right)$ is used, with $u_1 = 1$, to define a sequence.

   (a) Find the first 6 terms of this sequence

   (b) Show that the sequence converges to $\sqrt{7}$.

3. (a) Find, in terms of *n*, the *n* th term of the sequence

   $$\frac{1}{3}, \quad \frac{2}{5}, \quad \frac{3}{7}, \quad \frac{4}{9}, \quad \frac{5}{11}, \quad \ldots$$

   (b) A sequence is given by

   $$u_{n+1} = 2u_n - 2u_n^{\,2}$$

   $$u_1 = 0.8$$

   (i) Calculate u_2 .

   (ii) What value does u_n approach as *n* gets very large?

4. Gareth is investigating number patterns. He considers only the numbers 1 to 50. He groups every set of four consecutive numbers which when added will give a multiple of 10. His first two groups are shown.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

   (a) What is the next group of four consecutive numbers which when added will give a multiple of 10?

(b) What is the largest multiple of 10 which can be found in the table by grouping four numbers in this way?

The table is extended to include numbers from 1 to 1000.

(c) x and $x + 1$ are the two middle numbers in a group of four consecutive numbers which have a total which is a multiple of 10.

(i) Write expressions, in terms of x, for the other two numbers.

(ii) Write in its simplest form an expression, in terms of x, for the total of these four numbers.

(d) One group of four consecutive numbers has a total of 590. What are the four numbers?

(SEG)

5. (a) Write down the next term in the series

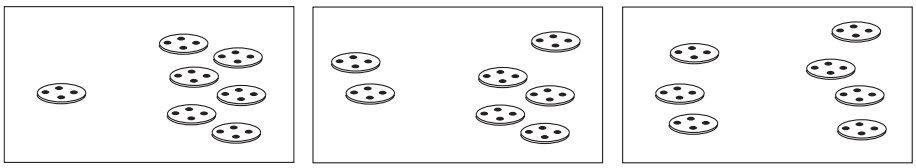
$$x, \quad x^3, \quad x^5, \quad x^7,$$

(b) What is the value of this term when $x = 1$?

6. Rebecca is investigating the number of ways in which different numbers of buttons can be split into two groups.

There must be at least one button in each group.

She finds that seven buttons can be split in three different ways.



1 + 6 2 + 5 3 + 4

She does not count ways which are the same as these, but reversed. For example, $5 + 2$ is not counted because it is the same as $2 + 5$.

(a) Copy and complete the table.

Number of Buttons	*Ways of Splitting into Two Groups*	*Number of Ways*
1	No ways	0
2	1 + 1	1
3	1 + 2	1
4	1 + 3 or 2 + 2	2
5		
6		
7	1 + 6 or 2 + 5 or 3 + 4	3
8		
9		

(b) In how many ways can 15 buttons be split into two groups?

(c) In how many ways can 100 buttons be split into two groups?

(d) What number of buttons can split into two groups in eight ways? There are *two* different answers to this question.

(e) What number of buttons can be split into two groups in 127 ways? There are *two* different answers to this question.

(SEG)

12.6 Quadratic Formulae

1. Show that each of the following sequences has a constant second difference, and use this to find the next 2 terms of the sequence.

   (a) 2, 6, 11, 17, . . . (b) 1, 1, 2, 4, 7, 11, . . .

   (c) 15, 13, 10, 6, . . . (d) –3, –10, –24, –45, . . .

2. The third, fourth and fifth terms of a quadratic sequence are 16, 26 and 38. Find the first, second and sixth terms of the sequence.

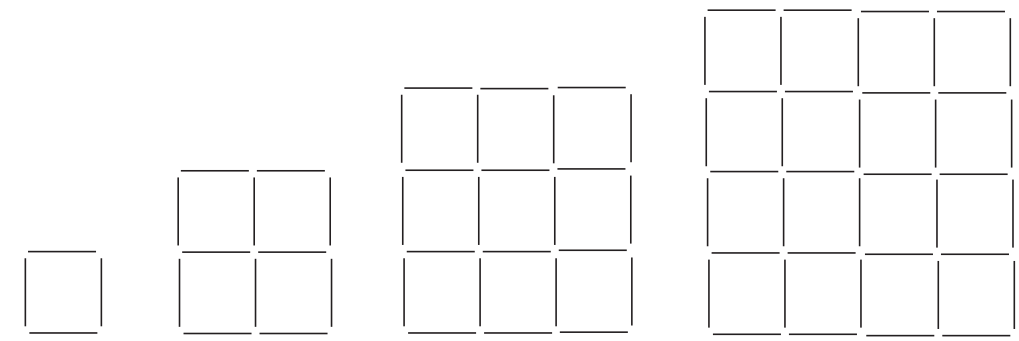
3. Find a quadratic formula which describes each of the following sequences:

   (a) 2, 5, 10, 17, 26, . . .

   (b) 2, 6, 12, 20, 30, . . .

   (c) 7, 11, 16, 22, 29, . . .

4. The 9th, 10th and 11th terms of a quadratic sequence are given by 167, 205 and 249. Find the formula for the *n*th term.

5. 

1st diagram 2nd diagram 3rd diagram 4th diagram

Each diagram consists of squares made from rods. The diagrams form part of a sequence.

(a) Copy and complete the table.

Number of diagram	1	2	3	4	5
Number of rods used to make that diagram	4	12	24		

(b) Write down the answers to these multiplications of consecutive numbers.

$1 \times 2 =$ ..... $2 \times 3 =$ ..... $3 \times 4 =$ .....

(c) How many rods are used to make the 12th diagram in the sequence?

(d) How many rods are used to make the *n* th diagram in the sequence?

(SEG)

6. (a) A number pattern begins 4, 8, 12, 16, 20, 24, ...

Describe this number pattern.

(b) Another number pattern begins 1, 4, 9, 16, 25, 36, ...

(i) Describe this number pattern.

(ii) What is the next number in this pattern?

Each number in this pattern is changed to make a new number pattern.

The new number pattern begins –1, 2, 7, 14, 23, 34, ...

(iii) What is the next number in the new pattern?

Explain how you found your answer.

(SEG)

7. (a) (i) Write down the multiples of 5, from 5 to 40.

(ii) Describe the pattern of the units digits.

(b) SEQUENCE P is 3, 6, 9, 12, 15, 18, 21, ...

Explain how SEQUENCE P is produced.

(c) Copy the table below, and fill in the blanks.

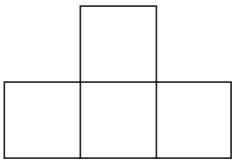
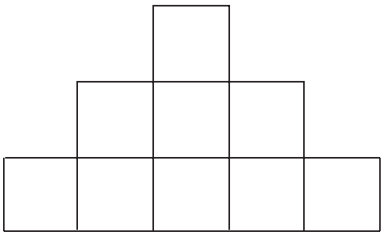
SEQUENCE P	→	Add 1 and then multiply by 2	→	SEQUENCE Q
3	→	$3 + 1 = 4$, $4 \times 2 = 8$	→	8
6	→	$6 + 1 = 7$, $7 \times 2 = 14$	→	14
9	→	.....	→	.....
12	→	.....	→	.....
15	→	.....	→	.....
18	→	.....	→	.....

(d) (i) Find the next two terms in the sequence 1, 4, 10, 19, 31, 46, 64, ...

(ii) Explain how you obtained your answer to part (d) (i).

(MEG)

8. The diagram shows the first 3 members of a sequence of patterns of cm squares.

Pattern Number		*Number of cm squares*
1		1
2		4
3		9

(a) Draw pattern number 4.

(b) Write down the total number of cm squares in pattern number 8.

(c) Express in symbols the number of cm squares in pattern number *n*.

(d) Express in symbols the number of cm squares in the bottom row of pattern number *n*.

(LON)

9. The following diagrams form a sequence.

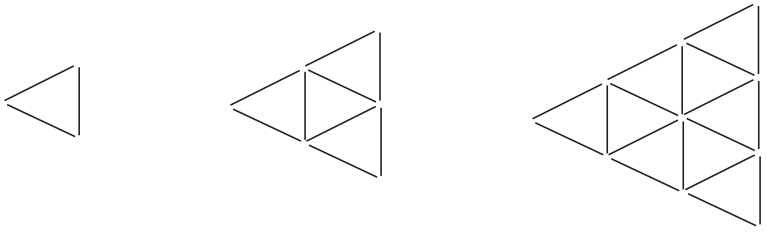


Diagram 1 Diagram 2 Diagram 3

The sequence is continued.

(a) How many lines are needed for diagram 4?

Lines drawn like this | are called *vertical*.

(b) How many non-vertical lines are needed for diagram 5?

(c) How many vertical lines are needed for diagram 6?

(d) Copy and complete the following table.

Diagram	*Number of non-vertical lines*	*Number of vertical lines*	*Total number of lines*
1	2	1	3
2	6	3	9
3	12	6	18
4			
5			
6			

(e) For the n th diagram, write in terms of n,

(i) the number of non-vertical lines,

(ii) the number of vertical lines,

(iii) the total number of lines.

(SEG)