11 Fractions and Percentages

11.4 More Complex Percentages

1. In a constituency, there are 12 000 eligible voters. In a particular election, the following results were obtained by three of the candidates:

Candidate	*Percentage of votes*
A	7%
B	39%
C	42%

Find the actual number of votes for each candidate, given that 12% of eligible voters did not vote.

2. A factory has 1600 workers and the percentages of workers absent from work from Monday to Friday in a certain week are given in the table.

Find the number of workers who turn up for work on each day.

Day	*Percentage of absentees*
Monday	15%
Tuesday	1.5%
Wednesday	10%
Thursday	5%
Friday	7%

3. The Smith family's expenses for a particular month are shown below.

Item	*Expenditure*
Rent	£169
Food	£273
Clothing	£52
Travel	£65
Miscellaneous	£91

Calculate each expenditure as a percentage of the total expenditure.

4. Kathy earned £30 000 in 1991. Her tax allowance was £3295. She did not pay tax on this amount of her income.

On a further £2570 of her income she did not pay tax, because she paid this amount into a pension scheme.

She paid tax on the rest of her income.

(a) How much of her income was taxable?

She paid tax at 25% on the first £23 700 of her taxable income.

She paid tax at 40% on the rest of her taxable income.

(b) Calculate the total amount of tax that she paid in 1991.

(SEG)

5. A shopkeeper buys a washing machine for £480. Find the selling price if the shop keeper is to make a profit of

(a) 5% (b) $9\frac{1}{2}\%$ (c) $12\frac{1}{3}\%$ (d) 15% (e) $33\frac{1}{3}\%$.

6. A supermarket sells 4 brands of detergent, A, B, C and D. On a particular day, 15% of the total number of boxes sold was brand A and 45% was brand C.

(a) Find the ratio of the number of boxes of brand A sold to the total number of boxes sold. Give your answer as a fraction.

(b) Given that 60 boxes of brand A were sold, calculate the number of boxes of brand C that were sold.

(c) Given that the number of boxes of brand D sold is one third the number of boxes of brand B that were sold, what percentage of the detergent sold was brand D?

7. Find (a) the discount, (b) the actual amount of money paid, in the following cases.

(a) A watch is priced in a catalogue at £198 but the dealer offers a 15% discount to the purchaser.

(b) Luggage which has a catalogue price of £595 but is sold at a discount of 20% during a sale.

(c) A cabinet which has a marked price of £1400 but is sold at a discount of 8% to a customer who pays for it in cash.

(d) A sofa-bed priced at £500 but is sold at a discount of 16% to a customer who arranges for its delivery.

(e) An air ionizer, with a marked price of £600, is offered for sale at a discount of 9% to a customer who pays in cash.

11.5 Percentage Increase and Decrease

1. Ten years ago, a town had a population of 12 250. Now, the population of the town is 13 965. Find the percentage increase in the population of the town.

2. The ABC Dress Company determines the selling price of its dresses by adding 32% to the cost. Calculate the selling price of a garment that costs £25.

3. A dealer sells cloth at £4.20 a metre, which he bought at £80 for 20 metres. Find the percentage profit or loss.

4. A carpenter made a dozen chairs at a cost of £420. She sold each of them for £40. Find her percentage gain.

5. A trader mixes 2 kg of butter which costs £8 per kg with 3 kg of butter which costs £6 per kg. He sells the mixture at £2.55 per 250 g. Find his percentage gain.

6. Calculate the percentage decrease for each of the following, correct to the nearest 1%.

   (a) From £124 to £100.

   (b) From 1.49 to 0.37.

   (c) From $56\frac{1}{2}$ kg to 50 kg.

   (d) From 300 km to 250 km.

7. Calculate the percentage increase for each of the following correct to the nearest 1%.

   (a) From £1250 to £1448.

   (b) From 51.4 to 70.4.

   (c) From 35.3 to 60.5.

   (d) From 12 h to 13 h.

8. (a) Decrease 246 by 20%.

   (b) Decrease £1270 by 25%.

   (c) Increase 40 kg by 10%.

   (d) Increase 1.65 m by 10%.

9. A bookshop sells its books at 10% less than the marked price. If a book is marked at £8, at what price will the shop sell it?

10. A long distance call costs £46.00. If a 2.5% service charge is added to it, what will be the total cost of this long distance call?

11. Between 1989 and 1990, the enrolment of a school fell from 2001 to 1500. What is the percentage decrease in the enrolment of the school from 1989 to 1990? Give your answer correct to the nearest 1%.

12. Calculate the percentage increase in each of the following cases:

   (a) A bus fare of 40p is now 50p.

   (b) An train fare of 50p is now 60p.

13. The breakdown for different races for the population of Singapore in 1985 and 1988 is given in the table below. For each race, calculate the percentage increase from 1985 to 1988, giving your answers correct to 1 decimal place.

	Race	*Population (1985)*	*Population (1988)*
(a)	Chinese	1 953 900	2 011 300
(b)	Malay	380 800	401 200
(c)	Indian	164 700	171 800
(d)	Others	58 600	62 800

14. In 1990, a charity sold $2\frac{1}{4}$ million lottery tickets at 25p each.

   80% of the money obtained was kept by the charity.

   (a) Calculate the amount of money kept by the charity.

In 1991, the price of a lottery ticket fell by 20%. Sales of lottery tickets increased by 20%. 80% of the money obtained was kept by the charity.

(b) Calculate the percentage change in the amount of money kept by the charity.

(LON)

15. Janet invests £50 in a building society for one year.
The interest rate is 6% per year.

(a) How much interest, in pounds, does Janet get?

Nisha invests £60 in a different building society. She gets £3 interest after one year.

(b) Work out the percentage interest rate that Nisha gets.

(LON)

16. If the price of a watch is increased by 15% from £p, give the new price in terms of p.

11.6 Addition and Subtraction of Fractions

1. Evaluate the following, expressing your answers in the simplest form.

(a) $\frac{1}{9}+\frac{5}{9}$ (b) $\frac{7}{12}+\frac{11}{12}$ (c) $\frac{5}{8}-\frac{3}{8}$

(d) $\frac{3}{4}+\frac{5}{12}$ (e) $\frac{3}{8}+\frac{1}{6}$ (f) $\frac{7}{8}-\frac{5}{6}$

(g) $\frac{9}{10}-\frac{11}{15}$ (h) $6\frac{2}{3}+5\frac{7}{12}$ (i) $5\frac{7}{12}-3\frac{4}{9}$

2. Evaluate the following:

(a) $\frac{2}{9}-\frac{1}{18}$ (b) $\frac{4}{15}-\frac{9}{30}$ (c) $\frac{1}{15}+\frac{5}{12}+\frac{1}{6}$

(d) $\frac{1}{4}-\frac{1}{3}-\frac{1}{2}$ (e) $\frac{23}{30}-\frac{5}{12}-\frac{1}{6}$ (f) $\frac{5}{8}+\frac{7}{12}+\frac{7}{16}$

(g) $\frac{4}{27}-\frac{5}{18}+\frac{7}{36}$ (h) $\frac{1}{2}+\frac{2}{3}-\frac{1}{6}+\frac{2}{9}$

3. Arrange the following in ascending order:

(a) $\frac{7}{10}, \frac{13}{20}, \frac{2}{3}$ (b) $\frac{13}{20}, \frac{11}{15}, \frac{3}{4}$ (c) $\frac{13}{15}, \frac{5}{6}, \frac{37}{45}$

(d) $\frac{5}{12}, \frac{7}{18}, \frac{11}{27}$ (e) $\frac{7}{8}, \frac{5}{6}, \frac{13}{16}$

4. Jane used $\frac{1}{2}$ of a piece of ribbon and her sister used $\frac{1}{3}$ of it. What fraction of the ribbon was used?

5. Joe painted $\frac{2}{5}$ of a fence and Bill painted $\frac{1}{2}$ of it. What fraction of the fence did the boys paint?

6. Mr Smith had $15\frac{1}{2}$ m of wire. He cut off a piece of wire $2\frac{3}{4}$ m long. How many metres of wire did he have left?

7. Mrs Bell made 40 cookies. Her son ate $\frac{1}{5}$ of them. How many cookies did he eat?

8. Harban was given £15 allowance each week. He spent $\frac{3}{5}$ of it. What fraction did he save? How much did he save in pounds.

9. Sue bought a record with $\frac{1}{4}$ of her allowance. She spent another $\frac{1}{8}$ to see a movie. What part of her allowance did she spend?

10. At a sale, some shirts are sold at $\frac{1}{2}$ their original price. If the original price of these shirts is £30, what is the sale price?

11. I have one whole candy bar. I give $\frac{1}{2}$ of it to my brother and $\frac{1}{4}$ of it to my friend. What fraction of the candy bar do I have left?

12. Khalid spent $\frac{1}{3}$ of his money on a pen, $\frac{1}{4}$ of it on books and $\frac{1}{6}$ of it on a magazine. What fraction of the money is left?

13. Mrs Holland spends $\frac{1}{4}$ of her money in the market and $\frac{1}{3}$ of the remainder in a shop. What fraction of her money is left?

14. Joan earns £1800 a month. She spends $\frac{3}{8}$ of her salary every month.

    She gives her parents $\frac{2}{5}$ of the remainder and saves the rest.
    How much money does she save every month?

11.7 Multiplication and Division of Fractions

1. Evaluate the following:

(a)	$\frac{1}{2}\times\frac{1}{2}$	(b)	$\frac{1}{2}\times\frac{1}{3}$	(c)	$\frac{2}{3}\times\frac{1}{4}$
(d)	$\frac{5}{2}\times\frac{2}{7}$	(e)	$\frac{1}{4}\times\frac{2}{9}$	(f)	$\frac{5}{7}\times\frac{14}{3}$
(g)	$\frac{2}{5}\times\frac{10}{9}$	(h)	$\frac{3}{7}\times\frac{7}{3}$	(i)	$\frac{1}{10}\times\frac{2}{9}$
(j)	$\frac{5}{9}\times\frac{3}{4}$	(k)	$\frac{7}{10}\times\frac{3}{14}$	(l)	$\frac{9}{4}\times\frac{2}{3}$

2. Evaluate the following:

(a) $\frac{2}{3} \div \frac{1}{3}$ (b) $\frac{5}{7} \div \frac{5}{14}$ (c) $\frac{5}{8} \div \frac{1}{8}$

(d) $\frac{3}{4} \div \frac{1}{4}$ (e) $\frac{1}{2} \div \frac{1}{8}$ (f) $\frac{4}{9} \div \frac{5}{9}$

(g) $\frac{5}{2} \div \frac{1}{2}$ (h) $\frac{7}{3} \div \frac{2}{3}$ (i) $\frac{10}{9} \div \frac{5}{3}$

3. Simplify the following:

(a) $7 \times 2\frac{6}{7}$ (b) $1\frac{1}{9} \times 4\frac{1}{2}$ (c) $8\frac{2}{3} \div 2\frac{1}{6}$

(d) $5\frac{1}{4} \div 3\frac{1}{2}$ (e) $\frac{7}{10} \div 4\frac{1}{5}$ (f) $1\frac{1}{8} \times 1\frac{1}{3}$

4. Evaluate each of the following:

(a) $18 \times 3\frac{2}{9}$ (b) $2\frac{1}{8} \times 3$ (c) $-6\frac{3}{4} \times \frac{4}{3}$

(d) $6\frac{1}{3} \times 4\frac{1}{5}$ (e) $\frac{2}{25} \times 12\frac{1}{2}$ (f) $1\frac{10}{11} \times \left(-2\frac{1}{7}\right)$

(g) $200 \times \frac{3}{4} \times \frac{1}{100}$ (h) $2\frac{1}{2} \times \frac{11}{100} \times 1000$

5. Evaluate the following:

(a) $\frac{1}{16} \div \frac{1}{4}$ (b) $\frac{3}{4} \div \frac{7}{8}$ (c) $\frac{4}{27} \div 6$

(d) $\frac{3}{16} \div \frac{2}{9}$ (e) $2 \div \frac{3}{4}$ (f) $\frac{7}{8} \div 1\frac{3}{4}$

(g) $3\frac{2}{3} \div 2\frac{1}{4}$ (h) $7\frac{1}{5} \div 2\frac{1}{4}$

6. You have to walk $1\frac{3}{4}$ km to school. How far have you walked when you are halfway?

7. A recipe for 6 buns requires $1\frac{1}{2}$ kg of sugar. How much sugar is needed for 1 bun?

11.8 Compound Interest and Depreciation

1. Matthew invests £240 in a bank account which earns interest at a rate of 5% per annum. Find the value of the investment after:

(a) 1 year, (b) 2 years, (c) 10 years.

2. Using the compound interest formula, calculate the value of the following accounts:

(a) £500 invested for 5 years at 8% interest per annum,

(b) £1000 invested for 7 years at $7\frac{1}{2}$% per annum,

(c) £4000 invested for 10 years at 9% per annum.

3. A new network of computers costs a firm £15 000. The value of this computer network depreciates at a rate of 20% per annum.

   What is the value of the network after:

   (a) 4 years, (b) 8 years?

4. Louise has £50 to invest, and wants to invest this money for as long as it takes to reach a value of £100. If the account pays 5% interest per annum, how long will it take for Louise to reach her target?

5. Fare prices on a newly privatised railway are only allowed to rise in line with inflation. Assuming constant inflation at a 2% rate per annum, how much will a £40 fare cost after:

   (a) 1 year, (b) 2 years, (c) 5 years, (d) 10 years?

6. A car costs £12 000 when new. It depreciates 20% in the first year, and at a 10% constant rate for each subsequent year. What is its value after:

   (a) 1 year (b) 2 years (c) 5 years?

7. Jim borrows £2000 to furnish a new flat. He has to pay interest at the rate of 15% per annum on this amount.

   (a) Find the amount of interest to be paid at the end of the first year.

   (b) If he actually pays £500 back at the end of each year, how much will he still owe at the end of the fourth year?

11.9 Reverse Percentage Problems

1. A stereo system is sold for £1998 and an 11% profit is made. Find the original cost of the stereo.

2. A dealer sells a television set to a man and makes a 15% profit. The man sells it to another man for £414 at a loss of 10%. Find the original price of the television set.

3. At what price must an article which costs £450 be sold in order to make a profit of $16\frac{1}{2}$%?

4. A cash discount of 8% is allowed on an item which costs £45. How much money is saved if a customer decides to pay in cash? How much more can he save if the discount is 9%?

5. A dealer gains $18\frac{3}{4}$% by selling a washing machine for £950. Find the cost price of the washing machine. What percentage profit would he get if he were to sell it for £1050?

6. A second-hand car dealer bought a second-hand car and spent £650 on repairs. He sold the car for £18 650, gaining 20% on the purchase price. For how much did he purchase it?

7. During a sale, a dress marked '50% off usual price' sells for £70. What is the usual price?

8. A man bought a flat for £76 000 and a second-hand car for £27 500. He sold the flat at a gain of 15% and the car at a loss of 12%. Find the total amount gained or lost from the two transactions.

9. By selling a particular set of books for £408, a bookseller suffers a loss of 4%. Find the cost price of the books. What is the percentage gain or loss if the books are sold for £510?

10. Many articles are subject to VAT at $17\frac{1}{2}\%$. Normally the quoted price of such articles includes VAT, but businesses can often obtain refunds on any VAT paid. It is therefore important to be able to determine the amount of VAT paid, given the quoted price of the article.

    (a) The quoted price of an article is £58.75. How much VAT is included in the quoted price?

    (b) An approximate method of finding the amount of VAT is to divide the quoted price by the number 6.71. This gives an answer that is not always accurate to the nearest penny. Find a more accurate number to use in place of 6.71, correct to 5 significant figures.

    (c) If VAT rises to 19%, determine, to 5 significant figures, the number by which the quoted price should be divided to find the amount of VAT paid.