10 Equations

10.4 Simple Equations

1. Solve each of the following equations:

   (a) $x + 5 = 8$ (b) $x - 5 = 4$ (c) $x + 5 = 10$

   (d) $x - 5 = 9$ (e) $6 + x = 7$ (f) $3x = 6$

   (g) $6x = 42$ (h) $7x = 14$ (i) $12x = 24$

   (j) $\frac{x}{2} = 6$ (k) $\frac{x}{5} = 5$ (l) $\frac{x}{2} - 1 = 4$

2. Solve each of these equations:

   (a) $x + 4 = 2$ (b) $5 + x = 3$ (c) $x - 3 = -7$

   (d) $3x = -12$ (e) $5x = -20$ (f) $2x + 1 = -3$

   (g) $3x - 1 = 14$ (h) $5x + 2 = -8$ (i) $2x - 4 = 8$

   (j) $4x - 7 = -9$ (k) $9 - 2x = 8$ (l) $3x + 7 = -10$

3. One number is greater than another by 4, and their sum is 32.
   Find the two numbers.

4. When a number is doubled and 5 is taken from the result, the answer is 37. What is the number?

5. The sum of two numbers is 120. If the larger number is four times the smaller number, what are the two numbers?

6. Andrew is 5 years older than Tim. If Tim is aged 21, then write down an equation for x, the age of Andrew. Solve this equation for x.

7. Morag thought of a number. She doubled this number and added 10 to give the result 52. What number did Morag think of?

8. The sum of three *consecutive* numbers is 120. If x is the smallest of the three numbers, write down the equation that x satisfies. Hence, solve for x.

9. When 42 is added to twice a number, the result is 346. Find the number.

10. A man was 26 years old when his son was born. Now, he is three times as old as his son. How old is the son now?

10.5 Solving Equations

1. Solve the following equations.

(a)	$2x - 7 = 3$	(b)	$3x - 4 = 8$	(c)	$5x + 2 = 7$
(d)	$3x + 9 = 0$	(e)	$15 - 2x = 9$	(f)	$17 + 3x = -3$
(g)	$5x = -15 + x$	(h)	$-2x - 7 = -4$	(i)	$5x - 4 = 3x - 1$
(j)	$7x - 14 = 18 - 4x$	(k)	$8x - 7 = 5 + 4x$	(l)	$9x + 4 = 3x - 9$

2 Solve the following equations.

(a)	$\frac{3}{4}x = 15$	(b)	$\frac{2}{5}x - 1 = 4$	(c)	$5 - \frac{x}{4} = 3$
(d)	$\frac{x}{3} + 5 = 15$	(e)	$2 + \frac{5}{7}x = 1\frac{1}{4}$	(f)	$\frac{2x + 4}{7} = 3$
(g)	$\frac{3x - 4}{5} - 7 = 0$	(h)	$\frac{3x + 4}{2} = x - 2$		
(i)	$\frac{2x - 1}{3} = 1 - x$	(j)	$7 + \frac{x - 1}{2} = x$		

3. Solve the following equations.

(a)	$3(x - 4) = 7$	(b)	$9(x - 4) = 3$
(c)	$5(2x + 3) = 35$	(d)	$8(2 + 3x) = 4$
(e)	$7(x + 4) = 2(x - 4)$	(f)	$5(3x + 5) = 2(7x - 4)$
(g)	$2(5 - 2x) = 4(2 - 3x)$	(h)	$2(x + 1) = 3(x - 5) + 9$
(i)	$\frac{1}{4}(5x + 4) = \frac{1}{3}(2x - 1)$	(j)	$2[2(x - 4) + 3] = 5$
(k)	$2x - [3 + (x - 5)] = 6$	(l)	$17(x - 3) = 3(7x - 15)$

4. When a number x is multiplied by 5, it gives the same result as when 48 is added to twice the number. Write down an equation for x, and find its solution.

5. Ahmad is twice as old as Bobby. John is 7 years younger than Ahmad. If the sum of their ages is 38, how old are the three boys?

6. Janet is three times as old as her daughter, Mary. Five years ago Janet was four times as old as Mary. How old is Janet now? How old will Mary be in 7 years' time?

7. Two boys, A and B, are 600 m apart. They walk towards each other at speeds of 36 m per minute and 25 m per minute respectively. After how many minutes will they meet each other?

8. Two men, P and Q, start at the same point and travel in opposite directions by motorcycle. The speed at which P's motorcycle travels is 4 km/h faster than Q's. After 5 hours, they are 580 km apart. Find the speed at which P travels.

9. Solve the equations.

   (a) $4x + 2 = 26$ (b) $19 + 4y = 9 - y$

10. Mrs. Root gave her maths class this problem:

    "When 8 is added to a certain number, the result is 3 times as large as when 2 is subtracted from the number."

    She asked the class to find the original number.

    Paul solved the problem using the equation

    $$x + 8 = 3(x - 2)$$

    Solve this equation.

11. Brenda went out walking and running. She travelled 7 km. She walked part of the way at 6 km/hour, and ran the rest of the way at 12 km/hour.

    The distance she ran was x km.

    (a) Write down an expression for the time taken running.

    (b) The time taken walking was $\frac{(7 - x)}{6}$ hours.

    The total time spent walking and running was one hour.

    (i) Write down an equation in terms of x.

    (ii) Find the value of x.

10.6 Trial and Improvement Method

1. Solve, using a trial and improvement method, each of the following equations, giving your answer correct to 1 decimal place.

   (a) $x^3 - 4 = 5$ (b) $x + \sqrt{x} = 10$

   (c) $x^3 - x = 6$ (d) $x^3 + x = 4$

2. Use trial and improvement methods to find the solution of each of these equations, giving your answer to 2 decimal places.

   (a) $x^2 + 4x - 3 = 0$ (b) $x^2 - 3x + 1 = 0$

   (c) $2x^2 + x - 4 = 0$ (d) $x^2 + 5x + 2 = 0$

3. George has to find a solution to the equation $x^2 + 2x = 10$, correct to one decimal place.

   First he tries $x = 3.0$ and finds that the value of $x^2 + 2x$ is 15.

By trying other values of x find a solution of the equation $x^2 + 2x = 10$, correct to one decimal place. You *must* show all your working.

(SEG)

4. (a) *Without using a calculator,* write down an estimate of the square root of 40. Give your estimate correct to one decimal place.

   (b) Explain how you obtained your estimate to the square root of 40.

   (c) Use a trial and improvement method to find the square root of 40 correct to two decimal places. Show your working clearly.

5. x is a number such that $x(x+1)(x-1) = 20$.

   (a) Find the two consecutive whole numbers between which x must lie.

   (b) Use the method of trial and improvement to find the solution correct to 3 significant figures.

(NEAB)

6. Dilip is using trial and improvement to solve equations.

   (a) He finds the solution of a certain equation lies between 2.731 and 2.734.

   Write down an approximation to the solution, correct to as many significant figures as are justified so far.

   (b) The solution to another equation lies between 4.62 and 4.67.

   Write down an approximation to the solution, correct to as many significant figures as are justified so far.

(SEG)

10.7 Expanding Brackets

1. Multiply out and simplify.

   (a) $x(1+x)$ (b) $2(2x+1)$ (c) $2x(x-1)$

   (d) $4x(2+x)$ (e) $5x(3-2x)$ (f) $x^2(1+x)$

   (g) $(x+1)(x+2)$ (h) $(x+1)(x-1)$ (i) $(x+2)(x-1)$

   (j) $(x-3)(x-2)$ (k) $(1+a)(1+2a)$ (l) $(x+y)(x-y)$

   (m) $(ax+b)(cx-d)$ (n) $(x+1)^2$

2. Expand the following:

(a)	$6(3x + y)$	(b)	$5z(z - 2y)$	(c)	$\frac{1}{2}(2xy - 4yz)$
(d)	$q(p + 2r - 3s)$	(e)	$(p + q)(r + s)$	(f)	$(x + y)(z + 2w)$
(g)	$(3a + b)(a + c)$	(h)	$(m + 2n)(2p + 3q)$	(i)	$(a - b)(c + d)$
(j)	$(2e - f)(2g - h)$	(k)	$(3p - 4q)(s + t)$	(l)	$(a + 7)(2b + 5)$
(m)	$(x + 3)(x + 4)$	(n)	$(a + 5)(a - 3)$	(o)	$(x - 7)(x - 6)$
(p)	$(3 + c)(6 - c)$	(q)	$(1 - 3x)(4 + 3y)$	(r)	$(2p + 3)(p + 5)$
(s)	$(4x + 5y)(2x + 3y)$	(t)	$(d - 7)(d - 5)$	(u)	$(a + 5)^2$
(v)	$(x - 3)^2$	(w)	$(b + 2)^2$	(x)	$(e - 4)^2$
(y)	$(2x + 1)^2$	(z)	$(3x - 2)^2$		

3. Simplify these expressions as far as possible.

(a)	$(3p + 2q)^2$	(b)	$(4m - 3n)^2$
(c)	$(x + 5)(x - 5)$	(d)	$(y + 7)(y - 7)$
(e)	$(5a + 3)(5a - 3)$	(f)	$(6x + 5y)(6x - 5y)$
(g)	$(x - 2)(x + 2)$	(h)	$(x - a)(x + a)$

4. (a) Multiply out and simplify

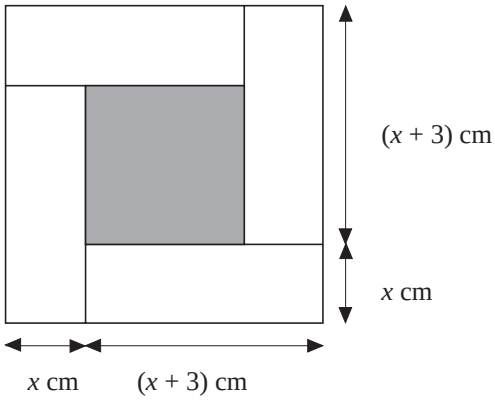
$$(3x - 1)(2x + 3)$$

(b) Show how you could use your answer to (a) to work out 29×23.

5. (a) (i) Multiply out $4x(x + 3)$.

(ii) Multiply out and simplify $(2x + 3)(2x + 3)$

(b) Four identical rectangular tiles are placed around a square tile as shown in the following diagram.



Using your answers to (a), or otherwise, find the area of the square tile.

10.8 Simultaneous Linear Equations

1. Solve each of the following pairs of simultaneous equations:

(a)	$x + y = 14$ $x - y = 4$	(b)	$x - y = -1$ $2x - y = 0$	(c)	$3x - y = 9$ $4x - y = -14$
(d)	$y - x = -1$ $3x - y = 5$	(e)	$5x + 4y = 4$ $3x + 4y = 8$	(f)	$3x + 5y = 5$ $3x + 9y = -3$
(g)	$3x + 2y = 0$ $-x + y = 5$	(h)	$3x - y = -2$ $x - 3y = 10$	(i)	$3x - 2y = 7$ $4x + y = 13$
(j)	$3a - b = 9$ $2a + 2b = 14$	(k)	$3x - 8y = 1$ $6x - 7y = 25$	(l)	$2m + 5n = 24$ $4m + 3n = 20$
(m)	$2x + 7y = 17$ $5x + 3y = -1$	(n)	$5u - 2v = 9$ $7u - 5v = 28$		

2. Solve the simultaneous equations

$$x + y = 4$$
$$15x + 25y = 76$$

3. Solve the following equations:

(a)	$x + y = 7$ $x - y = 3$	(b)	$x - 3y = 7$ $x - y = 3$	(c)	$3x + y = 13$ $5x - y = 35$
(d)	$3x + 3y = 15$ $3x - 5y = -41$	(e)	$3x + 2y + 7 = 0$ $5x - 2y + 1 = 0$	(f)	$3x + y = 17$ $3x - y = 19$
(g)	$3x + 2y = 8$ $2y - 5x = 8$	(h)	$2x = 5 - y$ $3y = 1 - 2x$	(i)	$7x - y + 23 = 0$ $x + 2y - 1 = 0$
(j)	$3x - 5y = 31$ $x + 3y = 1$	(k)	$x + 3y = 7$ $y - 4x = 11$	(l)	$3x + 7y = -15$ $x - 3y = 11$

4. Suvinder spends £26 on 100 postage stamps. If x of them are 20p stamps and the remaining y are 35p stamps, write down two equations in x and y and solve them.

5. Harry pays £8.50 for 5 kg of flour and 3 kg of sugar. Sarah pays £13.20 for 8 kg of flour and 4 kg of sugar. If the cost of flour is £x per kg and the cost of sugar is £y per kg, write down two equations in x and y and solve them.

6. John and David have £14.00 altogether. If John's money is doubled and David's tripled, they will have £34.00 altogether. How much does each boy have?

7. A retailer can buy either two television sets and three video-recorders for £3750, or four television sets and one video-recorder for £4250. What is the cost of a television set? What is the cost of a video-recorder?

8. A toothbrush and a tube of toothpaste cost £4.15; the toothbrush costs 25p less than the tube of toothpaste. Find the cost of each item.

9. A grocer wants to mix a type of spice which costs £22 per kilogram with another type which costs £12 per kilogram, to obtain 20 kilograms of mixture which will cost £15 per kilogram. What quantity of each spice must the grocer take?

10. Mrs Rogers bought 3 blouses and 2 scarves. She paid £26.

    Mrs Summers bought 4 blouses and 1 scarf. She paid £28.

    The cost of a blouse was x pounds.
    The cost of a scarf was y pounds.

    (a) Use the information to write down two equations in x and y.

    (b) Solve these equations to find the cost of one blouse.

10.9 Factorisation 1

1. Copy and complete the following:

    (a) $5a + 10b = 5(\ ? + ?\ )$ (b) $6p - 15q = ?\,(2p - ?\ )$

    (c) $16x - 32y = ?\,(x - ?\ y)$ (d) $14xy - 7yz = 7y(\ ? - z)$

    (e) $20x^2 - 16x = 4x(\ ? - 4)$ (f) $2a + 4ab = 2a(\ ? + ?\ )$

    (g) $2x^2 + 2xz = 2x(\ ? + ?\ )$ (h) $9mn - 27m^2r = ?\,(n - ?\ )$

    (i) $8pq - 12q^2 = ?\,(2p - ?\ )$ (j) $2ax^2 - 4a^2x = ?\,(\ ? - 2a)$

    (k) $9x^2y^2 - 3x = 3x(\ ? - ?\ )$ (l) $14m^2r - 7r = ?\,(2m^2 - ?\ )$

    (m) $12pq^2 + 16p^2q^2 = ?\,(\ ? + 4p)$

2. Factorise the following:

(a) $10a - 15b$ (b) $50py - 120p$ (c) $24abc - 8ab$

(d) $6abc + 12bcd$ (e) $16m^2 + 12n^2$ (f) $p^2y + p^2y^2$

(g) $18s^2t - 12st^2$ (h) $10a + 15a^2$ (i) $c - c^2$

(j) $2a^2b^2 - 8a^2b$ (k) $m^2n - mnl$ (l) $6xy - 3y + 9x$

(m) $pqr + p^2 + pr$ (n) $abc + a^2b + bc$ (o) $8abc + 6ab^2c + 4abc^2$

(p) $5s^2t - 3st - 4st^2$

3. Copy and complete the following:

(a) $m(x - y) + n(x - y) = (x - y)(\quad)$

(b) $k(a + b) + l(a + b) = (a + b)(\quad)$

(c) $a(2x + y) - b(2x + y) = (2x + y)(\quad)$

(d) $3x(c + d) - 2y(c + d) = (c + d)(\quad)$

(e) $(a + b)y + (a + b)x = (a + b)(\quad)$

(f) $(x - y)k + (x - y)l = (x - y)(\quad)$

(g) $(2x + 3y)a - (3y + 2x)b = (2x + 3y)(\quad)$

(h) $3b(p + q) - ab(q + p) = (p + q)(\quad) = b(p + q)(\quad)$

(i) $10a(c + d) - 5a^2(d + c) = (c + d)(\quad) = 5a(c + d)(\quad)$

(j) $4(m + n) - 8z(n + m) = (m + n)(\quad) = 4(m + n)(\quad)$

10.10 Factorisation 2

1. Factorise the following expressions:

(a) $x^2 - 3x + 2$ (b) $x^2 + 3x + 2$ (c) $x^2 - 1$

(d) $x^2 + x - 30$ (e) $x^2 + 6x + 9$ (f) $x^2 + x - 12$

(g) $x^2 - 2x - 15$ (h) $x^2 - 8x + 16$ (i) $x^2 + 10x + 21$

(j) $x^2 + 10x - 24$

2. Copy and complete the following:

(a) $16x^2 - 25 = (4x)^2 - 5^2 = (\quad + 5)(\quad - 5)$

(b) $a^2b^2 - c^2 = (\qquad)^2 - c^2 = (\quad + c)(\quad - c)$

(c) $4 - 9t^2 = 2^2 - (\qquad)^2 = (2 + 3t)(\qquad)$

(d) $49x^2 - 100 = (\qquad)^2 - 10^2 = (\quad + 10)(\qquad)$

(e) $36b^2 - 25a^2 = (6b)^2 - (\qquad)^2 = (6b + \quad)(\qquad)$

(f) $a^2 - \qquad = (a + 3)(\quad - 3)$

3. Use the fact that $x^2 - y^2 = (x + y)(x - y)$ to find the values of the following. Do them as quickly as possible mentally without using a calculator.

(a) $16^2 - 6^2 =$ (b) $91^2 - 9^2 =$

(c) $48^2 - 42^2 =$ (d) $2.5^2 - 1.5^2 =$

(e) $9.6^2 - 0.4^2 =$ (f) $\left(6\frac{4}{5}\right)^2 - \left(3\frac{1}{5}\right)^2 =$

4. Factorise the following expressions:

(a) $k^2 + 13k + 22$ (b) $m^2 + 12m + 32$ (c) $x^2 + 6x + 8$

(d) $y^2 - 3y + 2$ (e) $2x^2 + 11x + 15$ (f) $3a^2 + 5a + 2$

(g) $2y^2 + 7y + 6$ (h) $n^2 - 2n + 1$ (i) $5g^2 - 11g + 2$

(j) $r^2 - 8r + 12$ (k) $6b^2 + 11b + 5$ (l) $2d^2 - 13d + 15$

(m) $p^2 - 6p - 7$ (n) $h^2 - 5h - 36$ (o) $9x^2 - 12x + 4$

(p) $3s^2 - 7s - 6$ (q) $2f^2 - 5f - 3$ (r) $e^2 - e - 2$

5. Factorise the following expressions:

(a) $p^2 - 8pq + 15q^2$ (b) $y^2 - 2yz - 8z^2$

(c) $k^2 - 6kl + 5l^2$ (d) $m^2 - 4mn - 21n^2$

(e) $2x^2 + 5xy + 3y^2$ (f) $2x^2 + 7xy + 3y^2$

10.11 Solving Quadratic Equations by Factorisation

1. Solve the following equations by factorisation.

(a) $x^2 + 2x - 35 = 0$ (b) $x^2 - 15x - 54 = 0$

(c) $x^2 - x - 90 = 0$ (d) $x^2 + 15x + 54 = 0$

(e) $x^2 + 20x + 51 = 0$ (f) $x^2 - 12x + 32 = 0$

(g) $x^2 - 24x + 143 = 0$ (h) $x^2 - 17x + 60 = 0$

(i) $x^2 - 14x - 176 = 0$ (j) $x^2 - 26x + 133 = 0$

(k) $x^2 + 7x - 44 = 0$ (l) $x^2 + 2x - 195 = 0$

(m) $2x^2 - 5x + 3 = 0$ (n) $2x^2 - 7x - 9 = 0$

(o) $2x^2 + 13x + 6 = 0$

2. Solve the following equations:

(a) $x^2 - 6x + 8 = 0$ (b) $m^2 + 10m + 21 = 0$

(c) $p^2 - 7p - 30 = 0$ (d) $x^2 - 7x + 12 = 0$

(e) $x^2 - 9x + 20 = 0$ (f) $p^2 - 6p - 27 = 0$

(g) $a^2 - a - 56 = 0$ (h) $q^2 - 6q - 16 = 0$

(i) $2y^2 + 7y + 3 = 0$ (j) $6x^2 + x - 12 = 0$

(k) $4m^2 + 7m - 2 = 0$ (l) $4z^2 + 4z - 15 = 0$

3. Find the solutions of each of the following equations:

(a) $y^2 = y + 56$ (b) $12w^2 = 13w - 3$

(c) $11y = -4 - 6y^2$ (d) $c(c - 1) = 2$

(e) $q^2 = -2(q - 4)$ (f) $d(d + 2) = 3$

(g) $x(x - 5) = 84$ (h) $y(5y + 27) = 18$

(i) $3p^2 = 6p(2 + p)$ (j) $2x(4x + 5) = 3$

(k) $13x = 2(2x^2 + 5)$ (l) $2(10 - x^2) = 3x$

(m) $4y - 3 = 3y(y - 2)$ (n) $-12y - 9(y + 1) = 6y^2$

(o) $(a + 4)(a - 2) = -5$ (p) $(3x - 4)(x - 4) = -5$

4. Solve the following equations:

   (a) $x^2 - 16 = 0$ (b) $x^2 = 49$

   (c) $4x^2 - 81 = 0$ (d) $9x^2 = 64$

5. Solve the following equations:

   (a) $q^2 - 6q = -9$ (b) $x^2 + 81 = 18x$ (c) $y^2 = 22y - 121$

   (d) $4(3x - 1) = 9x^2$ (e) $-25 = 4y(y - 5)$

6. Solve the following equations:

   (a) $x^2 = 25$ (b) $a^2 = 36$ (c) $y^2 = \frac{49}{4}$

   (d) $b^2 - 16 = 0$ (e) $a^2 - 64 = 0$ (f) $x^2 - \frac{4}{81} = 0$

   (g) $4y^2 = 9$ (h) $2x^2 = 32$ (i) $3p^2 - 27 = 0$

   (j) $5p^2 - 20 = 0$ (k) $25b^2 - 40 = 9$ (l) $3b^2 - 8 = 4$

7. The area of a parallelogram is 50 cm^2. If the base is twice its height, calculate the height.

8. The breadth of a rectangular plot of land is 5 m less than its length. If the area of the plot is 104 m^2, find the dimensions of the plot.

9. A circle has an area of 154 cm^2. Find its radius.

10. In a triangle, its base is 3 cm less than its height. If its area is 14 cm^2, find its height.

11. The area of a rectangle is 51 cm^2. Find the length and the breadth of this rectangle if their difference is 14 cm.

10.12 Solving Quadratic Equations using the Formula

1. Solve the following equations by using the formula. Give your answers correct to 3 decimal places.

   (a) $x^2 + 6x + 4 = 0$ (b) $y^2 - 5y + 5 = 0$

   (c) $x^2 + 2x - 1 = 0$ (d) $x^2 - 5x + 1 = 0$

   (e) $x^2 = -5x - 3$ (f) $y^2 = 10y - 18$

2. Solve the following equations by using the formula. Give your answers correct to 3 decimal places.

(a) $2t^2 - 4t - 3 = 0$ (b) $2y^2 + 3y - 4 = 0$

(c) $2x^2 - 14x + 7 = 0$ (d) $4x^2 - 6 = -7x$

(e) $7 = 3t^2 + 6t$ (f) $5t^2 = 11t + 7$

3. Bill sets Ben this problem.

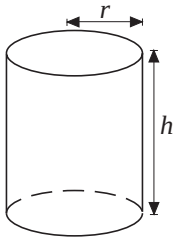
"When 2 is added to a certain number, the result is the same as dividing 8 by the number."

Ben uses this equation to solve the problem:

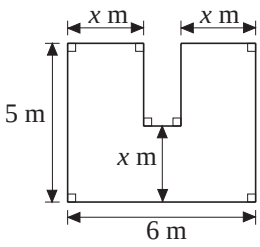
$$x + 2 = \frac{8}{x}$$

Solve this equation, and show that there are two possible solutions to Bill's problem.

4. The surface area of a circular cylinder can be calculated using the formula $A = 2\pi r^2 + 2\pi rh$, where r is the radius and h the height. Find the radius of the cylinder, correct to 2 decimal places, if the cylinder has a surface area of 322π cm^2 and a height of 10 cm.



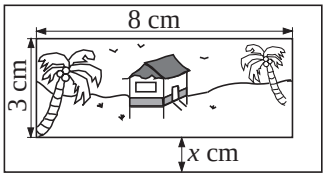
5. If the area of the field shown is 26 m^2, what is the value of x correct to 1 decimal place?



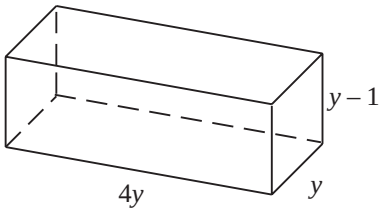
6. A cuboid has dimensions $(x + 3)$ cm by $(x + 3)$ cm by 5 cm. If its volume is 555 cm^3, find x correct to 2 decimal places.

7. The area of a triangular plate is 16 cm^2. If the base is 1 cm longer than the height, what are the lengths of its base and height? Give your answers correct to 1 decimal place.

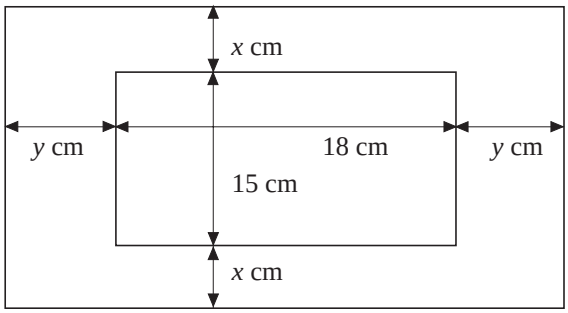
8. A picture 8 cm by 3 cm has a border x cm wide all round it. The area of the border is equal to the area of the picture. Find the value of x correct to 2 decimal places.



9. The diagram shows a closed rectangular box whose dimensions are in cm. Given that the area of material used to make the box is 146 cm^2, find the value of y correct to 2 decimal places.



10. A rectangular photograph, 15 cm by 18 cm, is mounted on a large rectangular piece of card so as to leave a border, x cm wide along both the top and bottom and y cm wide along each side.



The perimeter of the mount is 102 cm. The cost of mounting this photograph is £12.88.

(a) Show that $x = 9 - y$.

The cost of mounting a photograph is 2 pence per cm^2 of card.

(b) Show that $(18 + 2y)(15 + 2x) = 644$.

(c) Use the expression for x in part (a) to show that the equation in part (b) can be written as $2y^2 - 15y + 25 = 0$.

(d) Solve this equation to determine the possible dimensions of the piece of card.

10.13 Algebraic Fractions

1. Simplify each of the following:

(a) $\dfrac{xy}{y}$ (b) $\dfrac{12xy}{6y}$ (c) $\dfrac{15ab}{3a}$ (d) $\dfrac{6xy}{3yz}$

(e) $\dfrac{12cde}{3cd}$ (f) $\dfrac{a+b}{a+b}$ (g) $\dfrac{4(s-t)}{s-t}$ (h) $\dfrac{b-a}{a-b}$

(i) $\dfrac{b-4}{4-b}$ (j) $\dfrac{8-c}{c-8}$ (k) $\dfrac{2x+6}{2x}$ (l) $\dfrac{4p}{8+16p}$

(m) $\dfrac{6-3r}{6r-12}$ (n) $\dfrac{b+c}{cd+bd}$ (o) $\dfrac{cy-cz}{2y-2z}$

2. Simplify each of the following:

(a) $\dfrac{6z}{4z-6}$ (b) $\dfrac{ax}{ax-a^2}$ (c) $\dfrac{8p+16p^2}{4p}$

(d) $\dfrac{x-2}{x^2-5x+6}$ (e) $\dfrac{x+1}{x^2+2x+1}$ (f) $\dfrac{2y^2+y-3}{y-1}$

(g) $\dfrac{c^2-1}{c-1}$ (h) $\dfrac{m^2-n^2}{m-n}$ (i) $\dfrac{3+q}{9-q^2}$

(j) $\dfrac{6x-8}{4-3x}$ (k) $\dfrac{4a-8}{16-8a}$ (l) $\dfrac{mn-m^2}{m-n}$

(m) $\dfrac{2-x}{x^2+x-6}$ (n) $\dfrac{3-x}{x^2-5x+6}$ (o) $\dfrac{3-2x}{2x^2-x-3}$

(p) $\dfrac{m^2-1}{1-m}$ (q) $\dfrac{q^2-36}{6-q}$ (r) $\dfrac{25-p^2}{p-5}$

3. (a) $\dfrac{4a+8b}{6a+12b}$ (b) $\dfrac{a^2-b^2}{(a-b)^2}$ (c) $\dfrac{(a+b)^2}{a^2-b^2}$

(d) $\dfrac{x^2-4x}{x^2-16}$ (e) $\dfrac{a^2+ab}{a^2-b^2}$ (f) $\dfrac{ab-b^2}{(a-b)^2}$

(g) $\dfrac{8a^2-16ab}{5a-10b}$ (h) $\dfrac{a^2-4a}{a^2-16}$ (i) $\dfrac{4x^2-y^2}{12x^2-4xy-y^2}$

(j) $\dfrac{x^2+x-6}{x^2-9x+14}$ (k) $\dfrac{x^2-2x-15}{x^2-5x}$ (l) $\dfrac{5x-15}{3x^2-13x+12}$

4. Simplify each of the following:

(a) $\dfrac{a^2}{a^2+3a}\times\dfrac{a+3}{a}$ (b) $\dfrac{3x-2}{4x}\times\dfrac{24}{18x-12}$

(c) $\dfrac{49-b^2}{b^2}\times\dfrac{b}{7-b}$ (d) $\dfrac{3u^2+7u+2}{21}\times\dfrac{7u}{u+2}$

(e) $\dfrac{2y-1}{y^2}\times\dfrac{y}{2y^2+5y-3}$ (f) $\dfrac{2p^2}{p^2+8p+7}\times\dfrac{p+7}{4p}$

(g) $\dfrac{2x-6}{4}\times\dfrac{x}{x^2-x-6}$ (h) $\dfrac{3}{y^2+y-2}\times\dfrac{3y+6}{y}$

(i) $\dfrac{3}{p^2-9} \div \dfrac{4}{p+3}$ (j) $\dfrac{25-n^2}{5} \div \dfrac{5-n}{2}$

(k) $\dfrac{a^2-16}{a^2} \div \dfrac{a+4}{5a}$ (l) $\dfrac{2y^2-y}{4y} \div \dfrac{2y-1}{10}$

5. Express each of the following as a single fraction in its simplest form.

(a) $\dfrac{4}{y-2} + \dfrac{y}{y+3}$ (b) $\dfrac{2}{a-2} + \dfrac{a}{3a-2}$

(c) $\dfrac{x}{x+4} - \dfrac{3}{x-4}$ (d) $\dfrac{5}{1-3c} - \dfrac{c}{2+c}$

(e) $\dfrac{u}{u+2} - \dfrac{4}{3u-2}$ (f) $\dfrac{x}{1-5x} + \dfrac{6}{3+x}$

6. Express each of the following as a single fraction in its simplest form.

(a) $\dfrac{1}{a+b} + \dfrac{a}{4(a+b)}$ (b) $\dfrac{2}{x+1} + \dfrac{3}{(x+1)(x-2)}$

(c) $\dfrac{7}{x+3} - \dfrac{4}{(x+3)(x-5)}$ (d) $\dfrac{3}{x-2} - \dfrac{5}{(x-2)(x+4)}$

(e) $\dfrac{5}{3-a} + \dfrac{6}{(3-a)(3+a)}$ (f) $\dfrac{3}{a-4} + \dfrac{a}{a^2-16}$

(g) $\dfrac{p}{p^2-9} + \dfrac{2}{p+3}$ (h) $\dfrac{7}{5a-5b} + \dfrac{4}{2a-2b}$

(i) $\dfrac{c}{25-c^2} - \dfrac{1}{5-c}$ (j) $\dfrac{6}{x-3} + \dfrac{x}{x^2-3x}$

10.14 Completing the Square

1. Solve the following equations:

(a) $(a-5)^2 = 36$ (b) $(2m+1)^2 = 16$

(c) $(3p-2)^2 = 4$ (d) $5(y+2)^2 - 20 = 0$

(e) $(x+2)^2 - 9 = 0$ (f) $(a-3)^2 - 16 = 0$

(g) $(2y-1)^2 - 25 = 0$ (h) $49 = (2a+3)^2$

(i) $(2x-5)^2 - 4 = 0$ (j) $(3b+4)^2 = 81$

2. Solve the following equations by completing the square. Give your answers correct to 1 decimal place.

(a) $x^2 + 2x - 2 = 0$ (b) $x^2 - 4x + 2 = 0$

(c) $y^2 - 3y - 4 = 0$ (d) $y^2 + 9y + 1 = 0$

(e) $t^2 - 4t - 4 = 0$ (f) $p^2 - 5p + 1 = 0$

3. Solve the following equations by completing the square. Give your answers correct to 2 decimal places.

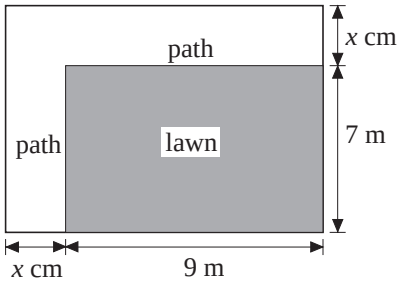
(a) $2y^2 + 3y - 4 = 0$ (b) $2t^2 - 2t - 3 = 0$

(c) $2p^2 - 3p - 1 = 0$ (d) $3x^2 + 2x - 3 = 0$

(e) $15x^2 - 30x + 10 = 0$ (f) $5y^2 = 9y - 1$

(g) $2x^2 = 11x + 7$ (h) $2x = 8 - 5x^2$

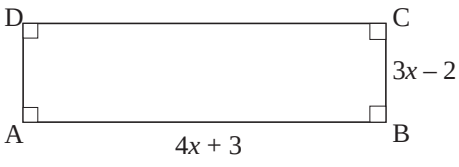
4. A rectangular garden consists of a lawn 9 m by 7 m, and a path x m wide along two sides of the lawn as shown.

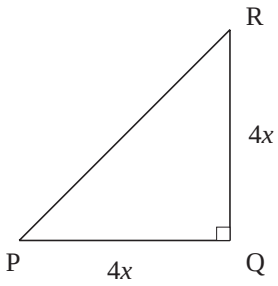


(a) Write down the perimeter of the garden, in terms of x.

(b) If the area of the garden is 96 m^2, write down an equation in x. Find x by solving the equation, completing the square and giving your answer correct to 2 decimal places.

5.





ABCD is a rectangle in which AB = (4x + 3) cm and BC = (3x – 2) cm.

PQR is a triangle in which PQ = QR = 4x cm and $\angle PQR = 90°$.

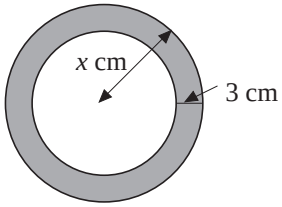
The area of the rectangle ABCD is equal to the area of the triangle PQR.

(a) Form an equation in x and show that it reduces to $4x^2 + x - 6 = 0$.

(b) Solve this equation by completing the square, giving your answers correct to 2 decimal places.

(c) Hence find the length of PQ.

6. The diagram shows the cross-section of an x m long concrete pipe of external radius x cm and thickness 3 cm.



(a) Write down the area of the shaded region of the pipe in terms of x in cm^2.

(b) Find the volume of concrete required to make the pipe in terms of x in cm^3.

(c) If the volume of concrete required to make the pipe is $3000\pi\ \text{cm}^3$, write down an equation in x and show that it reduces to $2x^2 - 3x - 10 = 0$.
Solve the equation by completing the square to find the value of x correct to 1 decimal place.

10.15 Algebraic Fractions and Quadratic Equations

1. Solve the following equations:

(a) $\frac{9}{x} = \frac{x}{4}$ (b) $\frac{x}{32} = \frac{1}{18x}$

(c) $\frac{y}{6} \div \frac{2}{3y} = 0$ (d) $\frac{5}{4x} \div \frac{x}{5} = 0$

2. Solve each of the following equations:

(a) $\frac{7}{a-3} - \frac{12}{a+3} = 2$ (b) $\frac{3}{x-2} - \frac{12}{x-3} = 2$

(c) $\frac{6}{y+2} - \frac{1}{y-2} = 3$ (d) $\frac{3}{c+2} + \frac{4}{c-3} = 2$

(e) $\frac{2}{t-3} - \frac{3}{t+1} = \frac{1}{2}$ (f) $\frac{8}{x+4} - \frac{1}{x-1} = \frac{1}{3}$

(g) $\frac{1}{x+3} - \frac{1}{x-2} = 1\frac{1}{4}$ (h) $\frac{3}{c+2} + \frac{4}{c+3} = 3$

3. Solve each of the following equations:

(a) $\dfrac{x}{x-2} - \dfrac{3}{x+1} = 1$ (b) $\dfrac{9x}{4} + \dfrac{x-9}{x} = 1$

(c) $\dfrac{3x}{x-1} = \dfrac{4}{x-3}$ (d) $x + 4 = \dfrac{x+4}{x-5}$

4. Solve each of the following equations:

(a) $\dfrac{x}{2x-3} = \dfrac{5}{x-4}$ (b) $\dfrac{1}{x-2} - \dfrac{1}{x+2} = 5$

Give your answers correct to 2 decimal places.