1 | Indices

1.3 | Index Notation

1. Write in a form using indices:

   (a) $2 \times 2 \times 2 \times 2$ (b) $3 \times 3 \times 3$

   (c) $6 \times 6 \times 6 \times 6 \times 6$ (d) $7 \times 7 \times 7 \times 7 \times 7 \times 7$

   (e) $1 \times 1 \times 1 \times 1$ (f) $2 \times 2 \times 2 \times 5 \times 5$

   (g) $17 \times 17 \times 17 \times 17$ (h) $5 \times 5 \times 5 \times 6 \times 6 \times 6$

   (i) $3 \times 3 \times 7 \times 7 \times 7 \times 7 \times 7$ (j) $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

   (k) $5 \times 3 \times 3 \times 3 \times 5$ (l) $11 \times 11 \times 11 \times 11 \times 13 \times 13$

2. Find the value of the following:

   (a) 7^2 (b) 3^3 (c) 3^5

   (d) 2^8 (e) 7^0 (f) 5^3

   (g) 4^3 (h) 6^3 (i) 1^7

   (j) 10^6 (k) 2^{10} (l) 3^6

3. Simplify each of the following, leaving your answer in index notation.

   (a) $2^7 \times 2^5$ (b) $3^2 \times 3^4 \times 3^6$ (c) $5^2 \times 5^2 \times 5^2$

   (d) $4^3 \times 4^7$ (e) $7^2 \times 7^4 \times 7^3$ (f) $2^1 \times 2^5 \times 2^2 \times 2^1$

   (g) $2^3 \times 2^3 \times 5^1 \times 5^3$ (h) $3^2 \times 3^3 \times 4^5 \times 4^2 \times 3^1$ (i) $5^2 \times 5^4 \times 5^7$

4. Simplify each of the following, leaving your answer in index notation.

   (a) $3^4 \div 3^3$ (b) $7^5 \div 7^2$ (c) $\left(2^3 \times 2^5\right) \div 2^8$

   (d) $8^{12} \div 8^7$ (e) $\left(4^3 \times 4^3\right) \div 4^3$ (f) $2^6 \div \left(2^3 \times 2^2\right)$

   (g) $\left(9^{10} \times 9^4\right) \div 9^6$ (h) $\left(6^{11} \div 6^{10}\right) \times 6^3$ (i) $\left(10^9 \times 10^9\right) \div 10^{16}$

   (j) $\left(4^7 \div 4^3\right) \times 4^2$ (k) $\left(2^1 \times 2^2 \times 2^3\right) \div 2^4$ (l) $6^8 \div \left(6^1 \times 6^2 \times 6^3\right)$

5. Express each of the following numbers as a number to a power, e.g. $256 = 2^8$.

   (a) 1024   (b) 243   (c) 125   (d) 216

   (e) 512   (f) 169   (g) 343   (h) 1000

   (i) 625   (j) 2048   (k) 289   (l) 1331

6. Fill in the missing numbers.

(a) $\left(2^3\right)^2 = 2^?$ (b) $\left(3^3\right)^3 = 3^?$ (c) $\left(5^?\right)^3 = 5^{12}$

(d) $\left(2^4\right)^? = 2^8$ (e) $\left(5^3\right)^2 = 5^?$ (f) $\left(4^3\right)^? = 4^{15}$

(g) $\left(10^3\right)^? = 10^9$ (h) $\left(7^4\right)^2 = 7^?$ (i) $\left(2^?\right)^6 = 2^{12}$

(j) $\left(3^?\right)^7 = 3^{21}$ (k) $\left(2^4\right)^? = 2^{16}$ (l) $\left(6^?\right)^4 = 6^{20}$

7. Simplify the following expressions, leaving your answers in index notation.

(a) $a^4 \times a^3$ (b) $x^5 \div x^2$ (c) $\left(b^4 \times b^3\right) \div b^5$

(d) $a^4 \div a^3$ (e) $x^4 \times x^5$ (f) $\left(x^4 \times x^5\right)^2$

(g) $\left(a^5 \div a^2\right) \times a$ (h) $\left(a^3\right)^2 \times \left(a^2\right)^3$ (i) $\left(x^2 \times x^3\right)^2 \div x^4$

(j) $\left(b^4 \div b^2\right)^3$ (k) $\left(b^4\right)^3 \div \left(b^2\right)^3$ (l) $\left[a^4 \times \left(a^2\right)^3\right] \div a^8$

(m) $\dfrac{x^7 \times x^2}{x^4}$ (n) $\dfrac{a^4 \times \left(a^2\right)^2}{a^8}$ (o) $\dfrac{x^5}{x^2 \times x^2}$

8. (a) Fill in the missing number.
$$2^4 \times 2^? = 2^{10}$$

(b) 2^{10} is approximately equal to 1000.
1 000 000 is approximately equal to $2^?$

(SEG)

1.5 Prime Factors

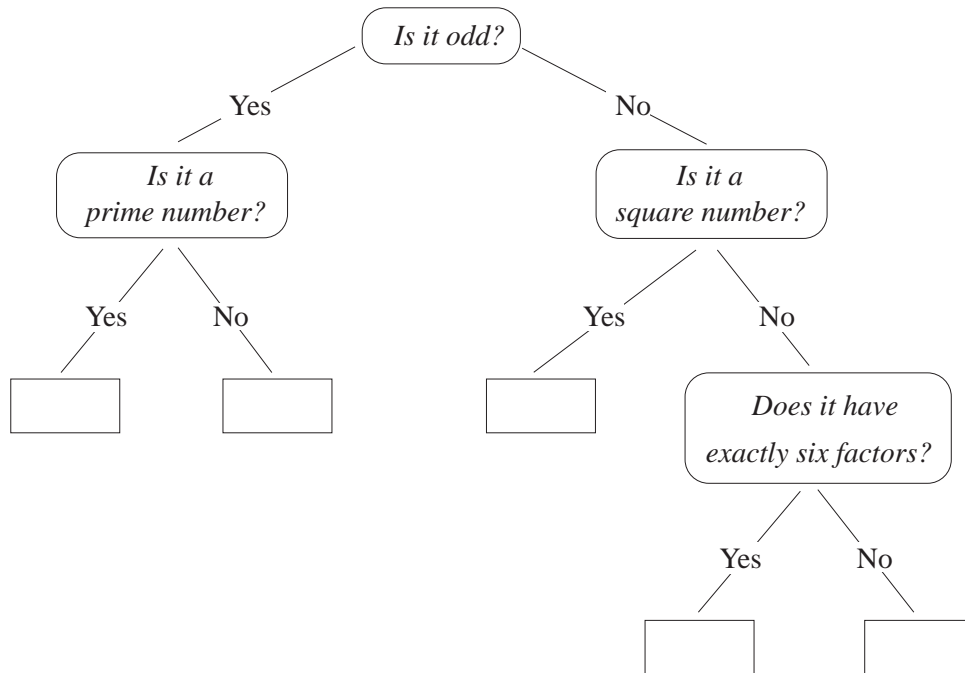1. Which of the numbers 2, 3, 5, 7, 11, 13 are prime factors of the following numbers?

(a) 189 (b) 264 (c) 490 (d) 770

(e) 165 (f) 288 (g) 2873 (h) 2310

(i) 3640 (j) 6006 (k) 1925 (l) 1815

2. Use the S*ieve of Eratosthenes* to find all prime numbers between 100 and 200.

3. Express the following numbers as products of prime numbers.

(a) 150 (b) 60 (c) 72 (d) 144

(e) 315 (f) 210 (g) 284 (h) 180

(i) 270 (j) 231 (k) 306 (l) 500

(m) 702 (n) 3234 (o) 8008 (p) 8190

4. Find the highest common factor of the following.

   (a) 16 and 24   (b) 45 and 63   (c) 56 and 70

   (d) 90 and 126   (e) 42, 66 and 78   (f) 84, 98 and 154

   (g) 189 and 84   (h) 315 and 720   (i) 616 and 392

   (j) 560, 140 and 224   (k) 132, 156 and 180   (l) 525, 1400 and 315

5. (a) Copy and put {9, 17, 28, 30} into the correct boxes.

Is it odd?

Yes   No

Is it a prime number?   *Is it a square number?*

Yes   No   Yes   No

Does it have exactly six factors?

Yes   No

   (b) Write down a number that could go into the empty box.

(SEG)

6. (a) Express the following numbers as products of their prime factors.

   (i) 72   (ii) 80

   (b) Two cars go round a race track. The first car takes 1 minute 12 seconds to complete a circuit and the second car takes 1 minute 20 seconds.
   They start level.
   Find the length of time before they are next level with one another.

(SEG)

1.6 Further Index Notation

1. *Without* using a calculator, find the value of each of the following.

   (a) $49^{\frac{1}{2}}$ (b) $27^{\frac{1}{3}}$ (c) $16^{\frac{1}{4}}$ (d) $8^{\frac{1}{3}}$

   (e) $100^{\frac{1}{2}}$ (f) $9^{1.5}$ (g) $16^{\frac{3}{4}}$ (h) $125^{\frac{2}{3}}$

   (i) $32^{\frac{4}{5}}$ (j) $36^{\frac{1}{2}}$ (k) $64^{\frac{1}{2}}$ (l) $121^{\frac{1}{2}}$

(m) $27^{\frac{2}{3}}$ (n) $81^{\frac{3}{4}}$ (o) $1000^{\frac{2}{3}}$ (p) $32^{0.6}$

(q) $4^{1.5}$ (r) $4^{2.5}$ (s) $81^{0.25}$ (t) 5^{-1}

(u) $\left(\dfrac{1}{4}\right)^{-2}$ (v) $8^{-\frac{1}{3}}$ (w) $9^{-0.5}$ (x) $16^{-0.25}$

(y) $32^{-0.8}$ (z) $16^{-1.5}$

2.   What is the value of each of the following expressions? *Do not use a calculator.*

(a) $\left(\dfrac{1}{4}\right)^{\frac{1}{2}}$ (b) $\left(\dfrac{1}{8}\right)^{-\frac{2}{3}}$ (c) $(25)^{-2.5}$

(d) $\left(\dfrac{1}{16}\right)^{-\frac{1}{4}}$ (e) 7^0 (f) $\left(\dfrac{4}{5}\right)^{-2}$

(g) $\left(\dfrac{4}{7}\right)^{-2}$ (h) $5^{-2} \times 4^3$ (i) $2^{-3} \div 3^{-3}$

(j) $3^4 \div \left(\dfrac{1}{3}\right)^{-4}$ (k) $\left(\dfrac{4}{9}\right)^{-2} \times \left(\dfrac{27}{8}\right)^{-3}$ (l) $78^{-1} \times 13^3$

3.   Express each of the following in index form.

(a) $\left(a^{16}\right)^{\frac{1}{2}}$ (b) $\left(a^{15}\right)^{\frac{1}{3}}$ (c) $\left(27a^9\right)^{\frac{1}{3}}$ (d) $\left(x^{32}\right)^{\frac{1}{4}}$

(e) $\left(x^{20}\right)^{\frac{1}{4}}$ (f) $a^{-2} \div a^{-4}$ (g) $a^4 \times a^{-5}$ (h) $\left(a^2\right)^{-\frac{1}{2}}$

(i) $\left(a^{-4}\right)^2$ (j) $a^{-4} \times a^4$ (k) $\left(a^{\frac{1}{2}}\right)^4$ (l) $\left(a^{16}\right)^{-\frac{1}{4}}$

4.   Solve the following equations for x.

(a) $3^x = 81$ (b) $4^x = 64$ (c) $5^x = 125$

(d) $7^x = 49$ (e) $2^x = 32$ (f) $2^x = 64$

(g) $x^3 = 27$ (h) $x^3 = 64$ (i) $x^5 = 32$

(j) $5x^2 = 45$ (k) $3x^3 = 24$ (l) $4x^4 = 324$

(m) $5x^3 = 320$ (n) $x^{\frac{1}{2}} = 3$ (o) $x^{-4} = 256$

(p) $x^{-\frac{1}{2}} = 5$ (q) $2^x = 1$ (r) $3^{-x} = 1$

5.   Simplify the following:

(a) $a^{\frac{1}{3}} \times a^{\frac{3}{5}}$ (b) $\left(a^{\frac{1}{3}}\right)^{\frac{3}{5}}$ (c) $a^{\frac{2}{5}} a^{-\frac{1}{3}}$

(d) $a^{\frac{5}{7}} \div a^{\frac{1}{4}}$ (e) $a^{\frac{7}{8}} \div a^{\frac{1}{4}}$ (f) $\left(a^2 b^{\frac{1}{3}}\right) \div \left(a^{\frac{2}{3}} b^{\frac{1}{6}}\right)$

(g) $a^{\frac{1}{2}} b^{\frac{2}{3}} \times a^{\frac{2}{3}} b^{\frac{1}{4}}$ (h) $\left(a^{\frac{2}{3}} a^{\frac{1}{2}}\right) \div a^{\frac{1}{4}}$ (i) $a^{\frac{6}{7}} \times a^{\frac{1}{14}}$

6.   Write these numbers in order of size, starting with the smallest.

$$\left(\frac{1}{4}\right)^{\frac{1}{2}}, \quad 2^{-3}, \quad 3^{-2}.$$

(SEG)

7.   Evaluate

   (a) $16^{\frac{1}{4}}$ (b) 2^{-4} (c) $27^{-\frac{1}{3}}$

(SEG)

8.   (a)   Calculate 2^n when $n = 5$.

   (b)   Calculate n when $2^n = \dfrac{1}{32}$

   (c)   Work out $9^{-\frac{3}{2}}$

(SEG)

9.   Solve the following equations.

   (a) $3^n \times 3^5 = 3^{11}$ (b) $2^{11} \div 2^n = 2^8$

   (c) $4^n = \dfrac{1}{16}$ (d) $3^n + 2^n = 97$

(SEG)

1.7 Standard Form

1.   Write the following numbers in standard form, $A \times 10^n$, where $1 \le A < 10$ and n is an integer.

   (a)   4000   (b)   560   (c)   700 000

   (d)   50   (e)   4213   (f)   2700

   (g)   236   (h)   2360   (i)   0.12

   (j)   0.007   (k)   0.1007   (l)   0.000 12

   (m)   2 million   (n)   0.1 million   (o)   562 005

   (p)   23.006   (q)   470.3   (r)   0.003 002

2.   Express the following in ordinary notation.

   (a) 3.2×10^2 (b) 4.67×10^3 (c) 1.30×10^1

   (d) 5.632×10^6 (e) 6.72×10^4 (f) 12.4×10^3

   (g) 3.612×10^{-2} (h) 1.47×10^{-1} (i) 65.3×10^2

   (j) 7.124×10^{-3} (k) 65.3×10^{-4} (l) 1.34×10^{-5}

   (m) 325×10^{-7} (n) 6.183×10^{-2} (o) 99.9×10^5

   (p) 2.75×10^8 (q) 2.75×10^{-3} (r) 4.216×10^{-2}

3. State whether or not the following numbers are in standard form.
If not, rewrite them in standard form.

(a) 2.157×10^{-1} (b) 42.76×10^2 (c) 5.672×10^{-5}

(d) 0.782×10^{-3} (e) 516×10^{-2} (f) 2.17×10^2

(g) 82.71×10^{-1} (h) 0.01×10^{-2} (i) 8.9×10^0

4. The area of the surface of the earth is about $510\,000\,000$ km².
Express this in standard form.

5. The population of the UK is estimated as $58\,700\,000$.
Write this in standard form.

6. The speed of light is approximately $300\,000$ km/s.

(a) Express this speed in m/s in standard form.

(b) The speed of sound is 300 m/s. How many times more than the speed of sound is the speed of light?

Give your answer in standard form.

7. The population of Singapore in a given year was 2.5×10^6.

Its total land area is estimated as 618 km². What was the average population per square km in that year?

Give your answer in standard form, correct to 2 significant figures.

1.8 Calculations with Standard Form

1. Without using a calculator, work out the following calculations.
Express your answers in standard form.

(a) $\left(2.8 \times 10^4\right) + \left(3 \times 10^3\right)$ (b) $\left(2.8 \times 10^4\right) - \left(3 \times 10^3\right)$

(c) $\left(6.3 \times 10^3\right) + \left(5.37 \times 10^4\right)$ (d) $\left(9.7 \times 10^2\right) + \left(0.3 \times 10^3\right)$

(e) $\left(4 \times 10^3\right) \times \left(2 \times 10^2\right)$ (f) $\left(5 \times 10^4\right) \times \left(3 \times 10^2\right)$

(g) $\left(8 \times 10^4\right) \div \left(4 \times 10^2\right)$ (h) $\left(1.5 \times 10^3\right) \times \left(2 \times 10^6\right)$

(i) $\left(6.5 \times 10^2\right) \times \left(2 \times 10^4\right)$ (j) $\left(9 \times 10^2\right) \div \left(3 \times 10^3\right)$

(k) $\left(6.4 \times 10^6\right) \div \left(1.6 \times 10^3\right)$ (l) $\left(8.4 \times 10^5\right) \div \left(2.1 \times 10^3\right)$

(m) $\left(2.5 \times 10^6\right) \times \left(4 \times 10^{-4}\right)$ (n) $\left(3.4 \times 10^{-4}\right) \times \left(2 \times 10^3\right)$

(o) $\left(5 \times 10^{-3}\right) \times \left(2 \times 10^{-2}\right)$ (p) $\left(2.4 \times 10^{-2}\right) \times \left(5 \times 10^2\right)$

2. Write $\left(4 \times 10^{-4}\right) + \left(8 \times 10^{-3}\right)$ as a single number expressed in standard form.

3. Given that $x = 2 \times 10^{-3}$ and $y = 7 \times 10^{-4}$, express $x + 8y$ in standard form.

4. Express $10^{-6} - \left(2.5 \times 10^{-7}\right)$ in standard form.

5. Given that $x = 3.2 \times 10^6$ and $y = 5 \times 10^7$, express in standard form:

   (a) $x\,y$ (b) $\dfrac{x}{y}$ (c) y^2 (d) $\left(\dfrac{x}{y}\right)^2$

6. In the formula $R = \dfrac{M}{EI}$, substitute

   $$M = 6 \times 10^4, \quad E = 4.5 \times 10^8, \quad I = 4 \times 10^2$$

   and evaluate R, giving your answer in standard form.

7. The radius of a circular micro-organism is $2.8 \times 10^{-7}\,$cm.

   Calculate the circumference and area of the micro-organism, giving your answer in standard form.

8.* Evaluate each of the following expressions, giving your answers in standard form.

   (a) $6.39^3 \times 7.8^4$ (b) $16.3^4 - 3.65^6$

   (c) $7.81 \times 10^3 - 0.13^{-2}$ (d) $6.3 \times 10^{14} \times 8.91^9$

   (e) $9.94 \times 10^{12} \div 23.5^4$ (f) $\sqrt{3.62 \times 10^{-4}}$

   (g) $4.5 \times 10^2 \times \sqrt{7.26 \times 10^{-6}}$ (h) $\dfrac{5.21 \times 10^{-4}}{\sqrt{2.26 \times 10^4}}$

   (i) $\sqrt{\dfrac{7.2 \times 10^8}{4.31 \times 10^6}}$ (j) $\left[\left(4.12 \times 10^3\right) \div \left(6.25 \times 10^2\right)\right]^{\frac{3}{2}}$

9. Given that $\dfrac{15.3 \times 12.4}{5.1 \times 31} = 1.2$, *without* using a calculator find the value of

   $$\dfrac{1.53 \times 1.24}{51 \times 3.1}$$

   and express it in standard form.

10.* If $x = 3.6 \times 10^{-2}$, evaluate and express your answer in standard form:

   (a) $4x^2 + x$ (b) $\sqrt{x + 1}$

11. Given that $x = 5 \times 10^5$, find the value of each of the following, giving your answer in standard form.

   (a) $5x$ (b) x^2 (c) $\dfrac{2}{x}$ (d) $\dfrac{3500}{x^2}$

12. Work out $4 \times 10^8 - 4 \times 10^6$. Give your answer in standard form.

(LON)

13. Saturn is approximately 1.43×10^9 km from the Sun.

Venus is approximately 1.08×10^8 km from the Sun.

How much further from the Sun is Saturn than Venus?
Give your answer in standard form.

(SEG)

14. A light year is the distance travelled by light in 365 days.

The speed of light is 3.9×10^5 kilometres per second.

(a) Calculate the number of kilometres in one light year.
Give your answer in standard form.

(b) The distance to the nearest star is 4.0×10^{13} kilometres.

How many light years is this?

(c) One kilometre = 0.625 miles.

Calculate the speed of light in miles per second.

(SEG)

15. The mass, M, of the planet Mars is 6.45×10^{23} kg.

The planet is a sphere with radius, r, equal to 2.28×10^{11} m.

Use this formula to find its density:

$$\text{Density} = \frac{M}{\frac{4}{3}\pi r^3}$$

Express your answer in standard form, correct to three significant figures.

(SEG)

16. The surface area of the Earth is approximately 1.971×10^8 square miles.
The surface area of the Earth covered by water is approximately
1.395×10^8 square miles.

(a) Calculate the surface area of the Earth not covered by water.
Give your answer in standard form.

b) What percentage of the Earth's surface is not covered by water?

(SEG)

17. The mass of a neutron is 1.675×10^{-24} grams. Calculate the total mass of
1500 neutrons.

Give your answer in standard form.

(LON)